Regulated Wage Economy and Taxation Systems: A Long-Run Welfare and Growth Theoretical Analysis and a Policy Exercise

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Abstract The debate on the macroeconomic effects of the regulation of wages and of unemployment benefits - which has also occurred in the recent years in Italy - is long lasting. Conventional wisdom holds that both may be harmful for efficiency, although often advocated for equity reasons. Another controversial debate concerned the taxation of capital income. In this paper we have shown that, despite the common wisdom which considers harmful the introduction of a minimum wage in that source of output losses, the introduction of a binding minimum wage, although on the one hand it generates market inefficiencies and unemployment, on the other hand it may also, surprisingly, generate production as well as welfare gains in the long run. Moreover, it may generate welfare gains even when it generates a production loss. From a policy point of view, motivated by the recent political debate about the increase from 12.5% to 20% in the capital income tax rate, the aim of this paper was to investigate, for the Italian case, whether the introduction of a minimum wage and of an unemployment insurance system may eventually enhance long run welfare, if the increase of the tax burden on capital income is used for preserving the balanced budget instead of financing other public expenditures. Our findings showed that, in contrast with the prevailing wisdom, positive effects always appear with a "calibration" of parameters largely corresponding to the current Italian situation. Therefore these findings offer some important policy implications so far not explored.

Keywords Minimum wage; Unemployment; Home production; Capital income tax; Neoclassical economic growth; welfare

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1 Introduction

The debate on the macroeconomic effects of the regulation of wages and of the introduction of unemployment benefits is long lasting. Conventional wisdom, dating back to Stigler (1946), holds that both may be harmful, mainly because they tend to reduce output, employment, labour supply and welfare. However, the necessity of higher wages for non-specialised workers and benefits for the unemployed is often advocated mainly for equity reasons. Another controversial debate has concerned the taxation of capital income. These debates also occurred in the recent years in Italy. Although some recent literature[1] has relaxed the efficiency-equity trade-off showing that in some cases a minimum wage could even be welfare improving regardless of its negative effect on the unemployment rate, we note that, however, these models neither are concerned on standard public policies as unemployment insurance system and income taxation nor are framed in the basic dynamic OLG model used in this paper. Therefore to our knowledge there is no literature that formally explores the joint roles played by the interventions on the labour market on the one side – e.g. regulation of wage, unemployment insurance scheme - and on the other hand on the taxation system. More in detail, we only explore the use of the taxation for financing an unemployment benefit scheme plus an unproductive public spending. We show that while taxation systems burdening on the income of the young people (as a wage tax or a lump-sum tax) as well as burdening on the firms (as a contribution proportional to the net wage) always result in a reduction of output and welfare, taxation systems based on consumption taxes (on the consumption of both periods or even only on that of one period) as well as on the taxation of the capital income and on the lump sum tax on the old people imply that output and welfare may be greater with regulated wages than with competitive wages. As an illustrative exercise, in this paper we only focus on the effects of an increased tax burden on capital income.

In this paper we try to fill that gap by developing a standard neoclassical OLG growth model à la Samuelson – Diamond embodying such features.

In particular this paper is motivated by the recent (2006) Italian "DPEF (Document for Economic and Financial Planning)" which has increased the capital income tax rate from 12.5% to 20%. In this work we evaluate an alternative fiscal reform according to which such an increase of the capital income tax is accompanied by the introduction of both binding minimum wages and unemployment insurance benefit schemes with a fixed replacement ratio. For that purpose we compare the situation before and after the assumed reform. The assumed reform is supposed to be implemented at balanced budget: the revenues from the increasing capital income taxation are used to finance the unemployment benefit scheme. Under plausible values of technology and preferences parameters, we show that our fiscal exercise provides significant improvements as regards the steady-state representative individual's lifetime welfare even for minimum wages slightly higher than market-clearing wages. Moreover, our results imply that the introduction of minimum wages should be accompanied by a replacement ratio as high as possible.

However we should remark that the increased welfare in presence of minimum wage and unemployment reflects only a part of the story. In fact in the present context the hours of unemployment should be considered as an additional resource instead of a damage. To see this, it is sufficient to say that so far in our model we have not taken account for the important leisure values associated with unemployment (for instance leisure time, self-enrichment activities, education, home production and so on). In other words in our model an unemployment rate, namely for example, of the 50% simply means that each individual works only for six month instead of the entire year and even reaches a higher welfare: in this sense the large amount of free time created by the minimum wage may be thought as a further step towards the realization of an "utopia" such as the liberation from the pain of work. In particular leisure associated with the unemployment may have straightforward economic effects as in the case of its use either for education or for exploiting an existing "backyard" technology. For exploring the further effects of the our proposal for reform in presence of an economic use of the

[1] For instance models introducing monopsonistic labour markets (e.g. West and McKee (1980)), education as a signalling device (Lang (1987)), schooling (Cahuc and Michel (1996)) or training on the job (Ravn and Sorensen (1999)) as motors of human capital accumulation, efficiency wages (Rebitzer and Taylor (1985)), imperfect information and job search (Swinnerton (1996)).

leisure associated with the unemployment, we assume that a home production technology with constant productivity does exist and that such a productivity is lower than both the marginal productivity in the firms sector and the binding minimum wage, so that nobody makes use of the home production technology unless it is unemployed. We show in this case that the higher the productivity of the home technology is, either the more likely or higher the welfare gain is.
To sum up, our findings show that accompanying the increase in the capital income tax with the introduction of a minimum wage and a sufficiently high replacement ratio, keeping the balanced budget, may be beneficial for the long-run lifetime welfare. This latter result is magnified when a "backyard" technology exists. The plan of the paper is as follows. In section 2 and 3 we present the competitive wage and the regulated wage models, respectively, and the corresponding steady state results are derived and discussed. Section 4 introduces our proposal for reform and compares the results, showing a numerical illustration. Section 5 introduces a home technology and discusses the results comparatively to the model of the previous sections. Finally, section 6 concludes.

2 The Market-Wage Economy

In this section we consider a standard dynamic general equilibrium OLG economy (as in Samuelson (1958) and Diamond (1965)) with young population N_t growing at the constant rate n and closed to international trade, and where goods, capital and labour markets are competitive.[2]
Individuals. Each generation is represented by identical individuals who live for two periods. Only young individuals work. In the first time-period they supply inelastically one unit of labour and receive wage income. This income is used to consume and to save. During the second period of life they are retired and live on the proceeds of their savings. Old individuals earn a return (net of taxes) of $1+r_{t+1}(1-\tau_{t+1,pc})$ on their investments when young, where r_{t+1} is the gross rate of return on savings from t to $t+1$ and $\tau_{t+1,pc}$ the capital income tax.[3] Savings by the young at time t are denoted by s_t. The lifetime utility of the representative individual born at time t is $U_t(c_t^{\ y}, c_{t+1}^{\ o}) = (c_t^{\ y})^{1-\phi}(c_{t+1}^{\ o})^{\phi}$, where $c_t^{\ y}$ and $c_{t+1}^{\ o}$ represent young and old age consumption, and $\phi \in (0,1)$ is the (constant) propensity to save (i.e., $\beta := \dfrac{\phi}{1-\phi}$ is the rate of time preference). Each generation takes the time-t real wage (w_t) and the real interest rate on savings as given. Therefore, the maximisation of $U_t(c_t^{\ y}, c_{t+1}^{\ o})$ under the constraints $c_t^{\ y} + s_t = w_t$, $c_{t+1}^{\ o} = [1+r_{t+1}(1-\tau_{t+1,pc})]s_t$, $c_t^{\ y} \geq 0$ and $c_{t+1}^{\ o} \geq 0$ implies the optimal young and old age consumption functions are the following:

$$c_t^{\ y} = (1-\phi)w_t, \tag{1}$$

$$c_{t+1}^{\ o} = \phi[1+r_{t+1}(1-\tau_{t+1,pc})]w_t. \tag{2}$$

The solution of the problem may also be expressed in terms of the savings function[4] as:

$$s_t = \phi w_t. \tag{3}$$

Firms. All the firms on the economy are identical and own a constant returns to scale Cobb-Douglas production technology by which physical capital and labour are transformed into consumption good.[5] Hence the representative profit-maximising firm hires aggregate capital stock (K_t) and demands labour

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[2] Two reference textbooks are Azariadis (1993) and De La Croix-Michel (2002).

[3] The subscript pc means perfect competition.

[4] In this context taxing capital income does not affect saving decisions, since the Cobb-Douglas utility specification implies that the elasticity of savings with respect to the interest rate is equal to zero. Anyway, by considering a more general CIES utility function, where the capital income tax may distort agents' decisions, it can be seen via numerical simulations (for economy of space not reported here) that the main findings of this paper are confirmed, provide that the elasticity of savings with respect to the interest rate is not too much positive.

[5] For simplicity we assume physical capital totally depreciates over time, i.e. $\delta = 1$.

supplied by young agents ($L_t = N_t$) to determine aggregate production, that is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where $A > 0$ is a technology scale parameter and $\alpha \in (0,1)$ is the capital's share on total output.[6] Factor prices are taken as given. Thus profit maximisation leads to the following marginal conditions for capital and labour:[7]

$$r_t = \alpha A k_t^{\alpha-1} - 1, \tag{4}$$

$$w_t = (1-\alpha) A k_t^{\alpha}. \tag{5}$$

Government. For simplicity we assume that only a capital income tax is levied by the government. Capital income taxes are levied by the government and used to finance a fixed amount of unproductive public expenditure (g_t) per young individual. The government strategy is to adjust the capital income tax rate such as to balance out per-capita public expenditure with per-capita tax revenues in each period, that is:

$$g_t = T_t, \tag{6}$$

where $T_t := \tau_{t,pc} r_t k_t$ represents per-capita tax revenues.

The long-run equilibrium. Given eq. (6), the equilibrium in goods as well as in capital markets is given by the following condition:

$$(1+n) k_{t+1} = s_t, \tag{7}$$

and combining (7) with (3) and (5), capital evolves over time according to:

$$(1+n) k_{t+1} = \phi (1-\alpha) A k_t^{\alpha}. \tag{8}$$

Steady-state implies $k_{t+1} = k_t := k^*$. Hence, the long-run (per-capita) stock of capital is:

$$k^*_{pc} = \left(\frac{\phi(1-\alpha)A}{1+n} \right)^{\frac{1}{1-\alpha}}. \tag{9}$$

Substitution of k^*_{pc} into the intensive form production function and into eqs. (4) and (5) yields the long-run per-capita output, and the long-run interest rate and market clearing wage respectively:

$$y^*_{pc} = A \left(\frac{\phi(1-\alpha)A}{1+n} \right)^{\frac{\alpha}{1-\alpha}}, \tag{10}$$

$$r_{pc} = \alpha A \left(k^*_{pc} \right)^{\alpha-1} - 1 = \frac{\alpha(1+n)}{(1-\alpha)\phi} - 1, \tag{11}$$

$$w_{pc} = (1-\alpha) A \left(k^*_{pc} \right)^{\alpha} = \left(\frac{\phi}{1+n} \right)^{\frac{\alpha}{1-\alpha}} \left((1-\alpha)A \right)^{\frac{1}{1-\alpha}}. \tag{12}$$

As regards stability, the analysis of eq. (8) implies:

$$\left. \frac{\partial k_{t+1}}{\partial k_t} \right|_{k_t = k^*_{pc}} = \alpha. \tag{13}$$

Since $0 < \alpha < 1$, eq. (13) says that in the neighbourhood of the steady-state the trajectory will always be monotonic and convergent towards the equilibrium.

Given eq. (8), the long-run rate of growth of the economy is computed as follows:

$$1 + g_{pc}(k) := \frac{k_{t+1}}{k_t} = \frac{\phi}{1+n} (1-\alpha) A k_t^{\alpha-1}, \tag{14}$$

[6] By defining $k_t := K_t / N_t$ and $y_t := Y_t / N_t$ as capital and output per-capita respectively, the intensive form production technology is simply $y_t := A k_t^{\alpha}$.

[7] The price of output has been normalised to unity.

Where $g_{pc}(k)$ is the growth rate and k_{t+1}/k_t the growth factor. An increase in the stock of capital reduces the rate of long-run growth, and $g_{pc}(k)$ converges asymptotically to zero as k_t approaches to the steady-state.

3 The Regulated-Wage Economy

We now suppose the existence of an economy where goods and capital markets are both competitive and where the only departure from the model of the previous section is an imperfect market for labour in which a minimum wage per hour worked ($\underline{w}$) is introduced by law.[8] As known, when a binding minimum wage is introduced in the economy, the labour market does not clear and unemployment occurs.

Individuals. Only young individuals work, assuming a unitary constant labour supply. Depending on the demand for labour, the supplied labour force may be, entirely or partially, employed or unemployed. If employed wage income is $\underline{w}$. If unemployed the government pays an unemployment subsidy indexed with the minimum wage, i.e. $b(\underline{w}) := \gamma\underline{w}$, where $\gamma \in (0,1)$ is the so-called replacement ratio. We treat $\underline{w}$ and γ as policy parameters, whereas the quantity of employed labour force is endogenous. The aggregate unemployment rate is defined as $u_t = (N_t - L_t)/N_t$, where $L_t = (1-u_t)N_t$ is the total number of hours worked by young agents. In this context a capital income tax ($\tau_{mw} > \tau_{pc}$) is levied by the government and used to guarantee the financing of the unemployment benefit system plus the same amount of per-capita public spending as the one of the market-wage economy (g_t) at balanced budget. Thus, the individual maximisation problem for agents of generation t modifies to:

$$\max_{\{c_t^{\,y}, c_{t+1}^{\,o}\}} U_t\left(c_t^{\,y}, c_{t+1}^{\,o}\right) = \left(c_t^{\,y}\right)^{1-\phi}\left(c_{t+1}^{\,o}\right)^{\phi},$$

subject to

$$c_t^{\,y} + s_t = \underline{w}(1-u_t) + \gamma\underline{w}u_t,$$
$$c_{t+1}^{\,o} = \left[1 + r_{t+1}\left(1-\tau_{t+1,mw}\right)\right]s_t,$$
$$c_t^{\,y}, c_{t+1}^{\,o} \geq 0.$$

The optimal young and old age consumption functions become:

$$c_t^{\,y}(\underline{w}) = (1-\phi)W_t(\underline{w}), \tag{15}$$

$$c_{t+1}^{\,o}(\underline{w}) = \phi\left[1 + r_{t+1}\left(1-\tau_{t+1,mw}\right)\right]W_t(\underline{w}), \tag{16}$$

where $W_t(\underline{w}) := \underline{w}[1-u_t(1-\gamma)]$ is the total income of the young (given by the sum of labour income, $\underline{w}$, plus the unemployment insurance benefit, $b(\underline{w})$).

The savings function is the following:

$$s_t(\underline{w}) = \phi W_t(\underline{w}). \tag{17}$$

Firms. Goods and capital markets are both competitive. The labour market is imperfect and regulated via the introduction of a binding minimum wage per hour worked. The per-capita Cobb-Douglas technology transforms to:

$$y_t = A(1-u_t)\left(\frac{k_t}{1-u_t}\right)^{\alpha}. \tag{18}$$

Standard profit maximisation leads to the following marginal conditions:

$$r_t = \alpha A\left(\frac{k_t}{1-u_t}\right)^{\alpha-1} - 1, \tag{19}$$

[8] For simplicity we assume $\underline{w}$ to be constant over time.

$$\underline{w} = (1-\alpha)A\left(\frac{k_t}{1-u_t}\right)^{\alpha}. \qquad (20)$$

Once the wage has been fixed the real interest rate is exogenous, that is, it does not depend on the capital stock. Substituting (20) into (19) for $k_t/(1-u_t)$ we find:

$$r(\underline{w}) = \alpha A(\psi/\underline{w})^{\frac{1-\alpha}{\alpha}} - 1, \qquad (21)$$

where $\psi := (1-\alpha)A$. An increase in $\underline{w}$ always reduces the real interest rate. Moreover, $r(\underline{w}) < r_{pc}$ for any $\underline{w} > w_{pc}$. The short-run unemployment rate is endogenous, and solving eq. (20) for u_t we get:

$$u_t(k_t, \underline{w}) = 1 - (\psi/\underline{w})^{\frac{1}{\alpha}} \cdot k_t, \qquad (22)$$

which is positively related with the minimum wage and strictly decreasing in the capital per-capita.

Government. An effect of the introduction of the minimum wages is to cause a positive level of unemployment. Therefore, in presence of an unemployment benefit scheme, there is need to finance the payment of benefits. There are many ways to raise the revenue for financing the benefits. The extent to which a long run welfare improvement will be successful depends crucially upon the type of taxation used. Although without exploring in a exhaustive way the entire range of possible taxation systems, we show that taxation systems burdening on the income of the young people (as a wage tax or a lump-sum tax) as well as burdening on the firms (as a contribution proportional to the net wage) always result in a reduction of output and welfare (see Appendix 1). On the contrary, taxation systems based on consumption taxes (on the consumption of both periods or even only on that of one period) as well as on the taxation of the capital income and on the lump sum tax on the old people imply that output and welfare may be greater with regulated wages than with competitive wages.[9] We focus here only on the capital income taxes in order to evaluate an alternative fiscal reform according to which such an increase of the capital income tax – as that experienced for the year 2007 in Italy with an increase of the capital income tax rate from 12.5% to 20% - is accompanied by the introduction of both binding minimum wages and unemployment insurance benefit schemes with a fixed replacement ratio, by comparing the macroeconomic outcomes before and after the assumed reform. Since we have supposed that the additional revenues from the increased capital income taxation are used to finance the unemployment benefit scheme, under balanced budget, therefore this means that the government strategy is to adjust the capital income tax rate such as to balance out unemployment benefit expenditure and per-capita public spending with tax receipts in each period. In this case, the time-t government constraint is the following:

$$\gamma \underline{w} u_t + g_t = \tau_{t,mw} r(\underline{w}) k_t, \qquad (23)$$

where $g_t = T_t = \tau_{t,pc} r_t k_t$.

The long-run equilibrium. Given eq. (23), the market clearing condition in goods as well as in capital markets is:

$$(1+n)k_{t+1} = s_t(\underline{w}), \qquad (24)$$

and combining (24) with (17), we obtain that:

$$(1+n)k_{t+1} = \phi \underline{w}[1 - u_t(k_t, \underline{w}) \cdot (1-\gamma)]. \qquad (25)$$

Substituting out for $u_t(k_t, \underline{w})$ from eq. (22), capital evolves over time according to the following first order linear difference equation:

$$k_{t+1} = \frac{\phi}{1+n}(1-\gamma)\psi^{\frac{1}{\alpha}} \underline{w}^{-\frac{1-\alpha}{\alpha}} k_t + \frac{\phi}{1+n}\gamma \underline{w}. \qquad (26)$$

Steady-state implies $k_{t+1} = k_t := k^*$. When the minimum wage is binding ($\underline{w} > w_{pc}$), the per-capita long-run unemployment rate, capital stock and income are given by the following equations:

[9] For the sake of brevity we do not report here the investigation of the cases with consumption taxes and with lump-sum tax on the old persons.

$$u^*(\underline{w}) = \frac{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n) - \phi\psi^{\frac{1}{\alpha}}}{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n) - \phi(1-\gamma)\psi^{\frac{1}{\alpha}}}, \tag{27}$$

$$k^*(\underline{w}) = \frac{\phi\gamma\underline{w}^{\frac{1}{\alpha}}}{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n) - \phi(1-\gamma)\psi^{\frac{1}{\alpha}}}, \tag{28}$$

$$y^*(\underline{w}) = \frac{A\phi\gamma\psi^{\frac{1-\alpha}{\alpha}}\underline{w}}{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n) - \phi(1-\gamma)\psi^{\frac{1}{\alpha}}}, \tag{29}$$

which are defined for any $\underline{w} \neq w_T$, where $w_T := (1-\gamma)^{\frac{\alpha}{1-\alpha}} \cdot w_{pc} < w_{pc}$.[10]

It is possible to demonstrate[11] that: 1) a binding level of a minimum wage which causes a higher steady-state capital stock that the competitive-wage case does always exists. In particular, we may show that for sufficiently high levels of minimum wage the long-run capital accumulation is always higher; 2) furthermore, provided that $\alpha > 1-\gamma$ for whatever binding level of minimum wage, capital stock is always higher than in the market-wage case. The following Figures 1.A and 1.B display the behaviours of the long-run capital stock and income for increasing values of the minimum wage, for the parametric configuration above discussed.

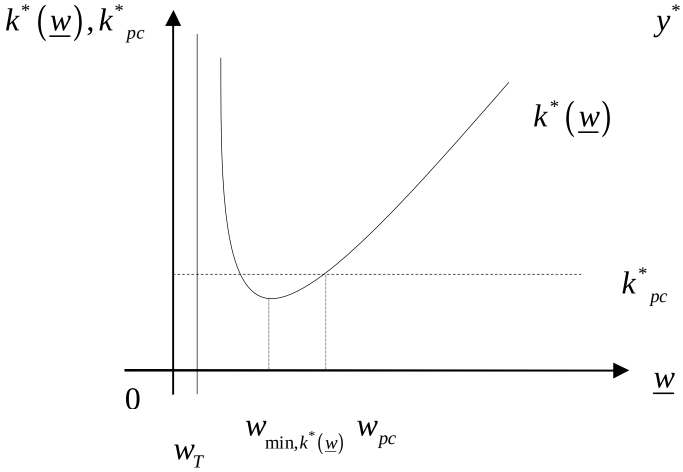

Figure 1.A. *Steady-state capital stock in the regulated and market wage economies.*

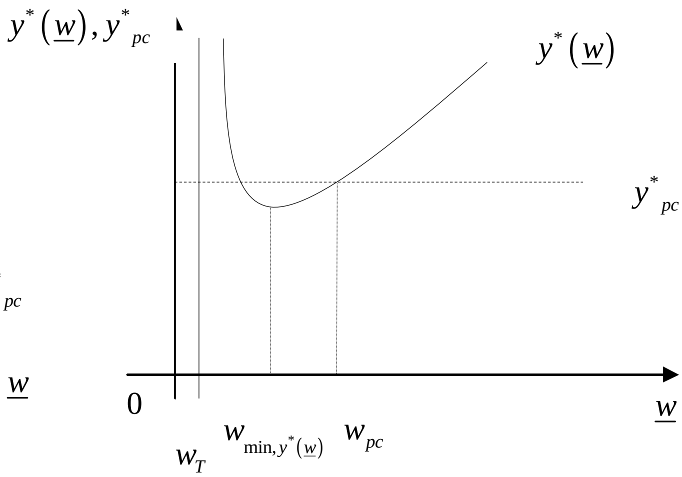

Figure 1.B. *Steady-state income in the regulated and market wage economies.*

Solving eq. (26) yields:

$$k_t = k_0 d^t + k^*(\underline{w}). \tag{30}$$

[10] If the minimum wage is not binding, i.e. $\underline{w} = w_{pc}$, eqs. (28) and (29) collapse to eqs. (9) and (10) respectively, and $u^*(w_{pc}) = 0$.

[11] For economy of space we do not report here the complete proof, which is available in request.

with $k_0 > 0$ given and $d := \frac{\phi(1-\gamma)\psi^{\frac{1}{\alpha}}}{(1+n)\underline{w}^{\frac{1-\alpha}{\alpha}}}$. Stability requires $d < 1$, that is $\underline{w} > w_T$. In this case $\lim_{t\to+\infty} k_t = 0^+$ and the economy converges towards the steady-state equilibrium, $k^*(\underline{w})$.
Multiplying both sides of (26) by $1/k_t$, the long-run rate of growth of the economy is given by:

$$1+g(k,\underline{w}) := \frac{k_{t+1}}{k_t} = \frac{\phi\gamma\underline{w}}{(1+n)k_t} + \frac{\phi(1-\gamma)\psi^{\frac{1}{\alpha}}}{(1+n)\underline{w}^{\frac{1-\alpha}{\alpha}}}, \tag{31}$$

where k_{t+1}/k_t is the growth factor and $g(k,\underline{w})$ the growth rate of the economy. As k_t approaches to the steady-state, $g(k,\underline{w})$ converges asymptotically to zero.
The following Figure 2 shows the locus of dynamic capital accumulation equation $k_{t+1} = f(k_t)$ in both cases of competitive wage an minimum wage (eqs. (8) and (26) respectively).

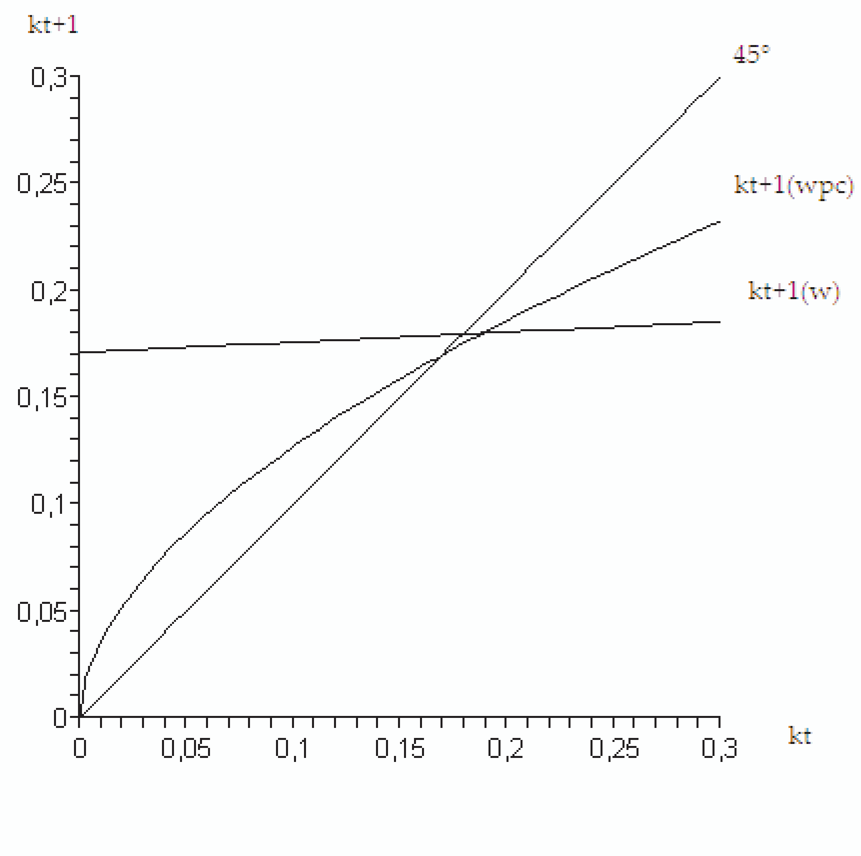

Figure 2. The behaviour of the capital accumulation equations in both cases of competitive wage and minimum wage, kt+1(wpc) and kt+1(w) respectively. Parameter set: $A = 10$, $\alpha = 0.55$, $\phi = 0.10$, $\gamma = 0.95$ and $n = 0$.

The following Figure 3 shows the behaviours of the growth rates in both the competitive-wage and regulated-wage economies (see eqs. (14) and (31) respectively): as it can be easily seen, during the transition towards the steady-state the regulated wage economy always grows at a higher rate then the market-wage economy. The function kt+1(w) is constructed assuming a minimum wage equal to 1.79. The locus of the minimum wage case always lies above that of the market-wage economy and, as expected, the corresponding steady-state is higher. Notice that along all the points of the transitional paths, the minimum wage is always binding, that is at any time *t* the condition $\underline{w} \geq w_{t,pc}$ holds.

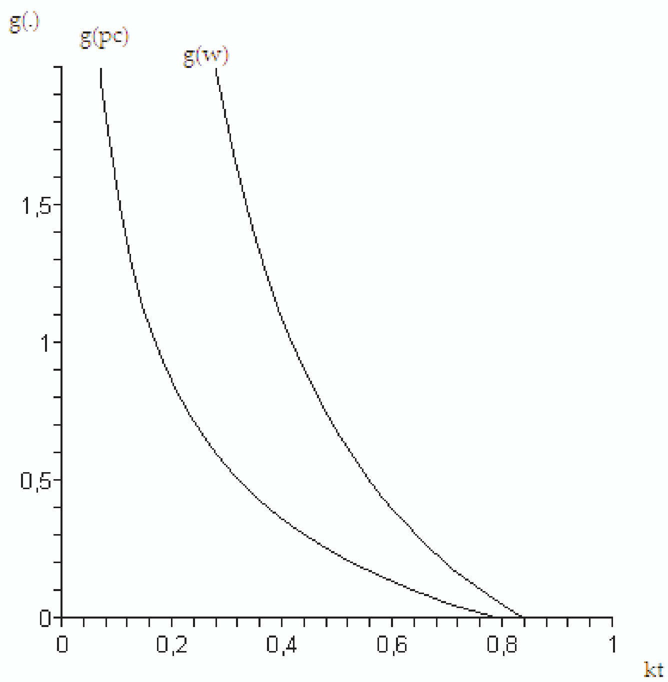

Figure 3. The growth rates of k in both the regulated-wage and market-wage economies, where g(pc) is the rate of long-run growth in the competitive case, while g(w) is the one of the regulated-wage economy. Parameter set: $A = 10$, $\alpha = 0.55$, $\phi = 0.20$, $\gamma = 0.95$ and $n = 0$. The curve g(w) is constructed assuming a minimum wage equal to 4.19 (of course, along all the points of the transitional paths, the minimum wage is always binding, that is at any time *t* the condition $\underline{w} \geq w_{t,pc}$ holds).

Thus, the introduction of a binding minimum wage always brings about: 1) higher transitional rate of growth and 2) higher levels of steady-state capital per-capita as compared with the market-wage economy. Moreover, higher minimum wage values imply higher growth rate during transition as illustrated in the following Figure 4.

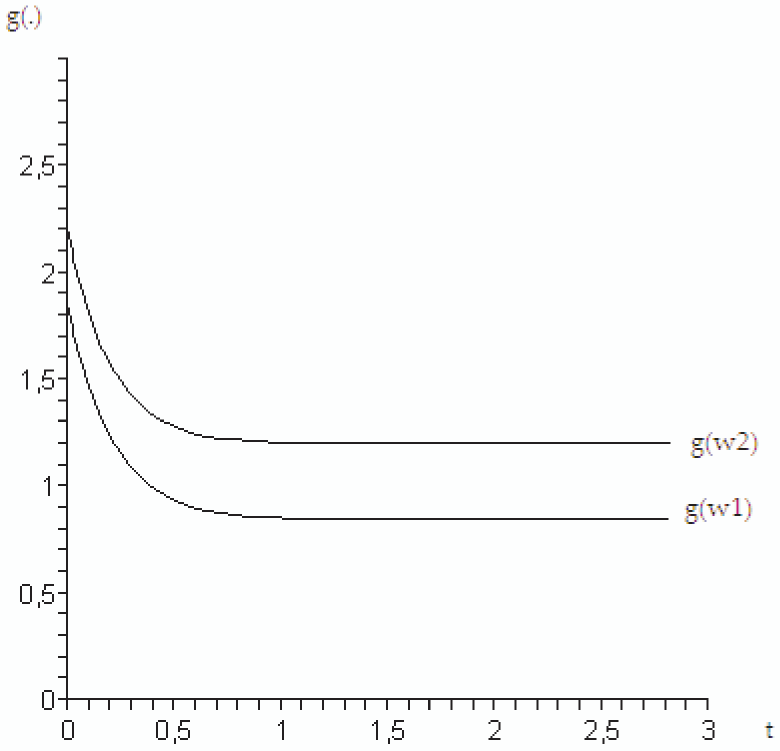

Figure 4. The behaviours of the growth rate in the regulated-wage economy as a function of time for two binding values of minimum wage, w1=4.19 and w2=6.1. Parameter set: $A = 10$, $\alpha = 0.55$, $\phi = 0.20$, $\gamma = 0.95$ and $n = 0$.

The above analysis has the following economic content: provided that the capital weight and the replacement ratio are sufficiently high, then capital accumulation is increased by the introduction of the minimum wage. But the increased accumulation is not only the end of the story. It is possible to show that another condition ensuring even a greater long-run output does exist, and it may be satisfied for many real economies, that is $\alpha > \frac{1}{1+\gamma}$, then any value of the regulated wage higher than the market wage is associated with a steady-state capital stock and income functions both increasing. An illustration of an increasing income function is in Figure 1.B. Therefore, a sufficiently high capital weight in the production technology as well as a replacement ratio sufficiently high are a sufficient condition for the introduction of a regulated wage brings about values of savings, capital and income always greater than the ones obtained in the competitive-wage economy. Indeed, given the relatively high capital weight in the production technology, the increasing accumulation induced by the increasing minimum wage leads to a raising output so to create a virtuous growth mechanism. Moreover the higher the minimum wage, the higher the long-run capital stock and income.
To sum up, under some plausible conditions: 1) an increase in the regulated wage is always beneficial for the long-run income, and 2) interestingly this beneficial effect may lead to a higher economic growth than in the market-wage economy.
These results are in contrast with the common wisdom attributing to the minimum wage a negative role on economic growth.

4 Welfare Analysis and Numerical Examples

In this section we compare two different capital income tax regimes (as the Italian fiscal reform implies) evaluating their effects on the steady-state representative individual's lifetime welfare in both the market-wage and regulated-wage economies.

The market-wage economy. The steady-state young and old age consumption functions are given by:

$$c^y(w_{pc}) = (1-\phi)w_{pc}, \tag{32}$$

$$c^o(w_{pc}) = \phi[1 + r_{pc}(1-\tau_{pc})]w_{pc}. \tag{33}$$

The pre-reform long-run lifetime welfare is evaluated as follows:

$$V(w_{pc}) = (1-\phi)\ln(c^y(w_{pc})) + \phi\ln(c^o(w_{pc})), \tag{34}$$

where $V(\cdot) = \ln(U(\cdot))$. Assuming the government levies a capital income tax $\tau_{pc} = 0.125$, eq. (34) may be rewritten as:

$$V(w_{pc}) = (1-\phi)\ln((1-\phi)w_{pc}) + \phi\ln(\phi[1 + r_{pc}(1-0.125)]w_{pc}). \tag{35}$$

The regulated-wage economy. In this case employees receive a minimum wage per hour worked higher than the market-clearing one, i.e. $\underline{w} > w_{pc}$. As a consequence workers remain unemployed for $u^*(\underline{w})$ hours in the long-run and receive an unemployment subsidy as we have previously explained. The long-run young and old age consumption functions are given by:

$$c^y(\underline{w}) = (1-\phi)W(\underline{w}), \tag{36}$$

$$c^o(\underline{w}) = \phi[1 + r(\underline{w})(1-\tau_{mw})]W(\underline{w}). \tag{37}$$

We assume that after the introduction of the minimum wage and the new capital income tax rate, g_t is keeping at the same level preceding the reform, that is at the constant value $\overline{g} = \tau_{pc} r_{pc} k^*_{pc}$. This means that the additional revenue generated by the increase in tax rate is used to finance the unemployment insurance scheme holding the preceding public expenditure under balanced budget. The government balanced budget equation evaluated at the steady-state becomes:

$$\gamma \underline{w} u^*(\underline{w}) + \overline{g} = \tau_{mw} r(\underline{w}) k^*(\underline{w}) \Rightarrow \tau(\underline{w}) = \frac{\gamma \underline{w} u^*(\underline{w})}{r(\underline{w}) k^*(\underline{w})} + \frac{\overline{g}}{r(\underline{w}) k^*(\underline{w})}. \tag{38}$$

Eq. (38) is a non-linear function of $\underline{w}$. Therefore, it is not possible to obtain a closed-form solution for the minimum wage balancing the government budget. The implicit solution of (38), however, implies:

$$w° = w°(\cdot, \tau_{mw}, \gamma). \tag{39}$$

Eq. (39) gives us the wage rate at which the government balances its budget as a function of the key parameters of the model, the capital income tax and the replacement ratio. Thus, the post-reform representative individual's indirect utility function evaluated at $w°$ is the following:

$$V(w°) = (1-\phi)\ln(c^y(w°)) + \phi\ln(c^o(w°)). \tag{40}$$

Assuming $\tau_{mw} = 0.20$, eq. (40) becomes:

$$V(w°) = (1-\phi)\ln((1-\phi)W(w°)) + \phi\ln(\phi[1 + r(w°)(1-0.20)]W(w°)). \tag{41}$$

The behaviour of the welfare function (41) is dependent on the technology and the preference parameters, α and ϕ, as well as on the replacement ratio. Such a dependence is highly non-linear, so that analytical results are prevented. Since (38) and (39) are difficult to handle analytically, we shall run simulations that basically make use of equations (39) and (41). In fact, numerical simulations using typical values of the weight of capital in technology and of the propensity to save for Italy leads to clear cut results as regards the effects of the introduction of the assumed reform. Our proposal for the reform is, obviously, only illustrative: our purpose is to demonstrate that within a "calibrated" standard OLG model an increase in the capital tax together with regulated wages and unemployment benefits could produce an improvement of the lifetime welfare in the long run. Now we are concerned with the choose of the parameter values for the simulations. Recently Jones (2003, 2005) provides estimates of the capital's share in OECD countries. He reports two types of measures for the capital's share: 1) a measure constructed as one minus employee compensation divided by GDP and 2) the employee compensation share corrected for self-employment. As regards Italy, the evidence reported by Jones

((2003), Figure 1, p. 8) shows that in the recent period the capital's share is between 0.55 and 0.60 according to the first measure and among 0.37 and 0.42 according to the second measure.
As regards the propensity to save, Italy experienced a decrease in the recent decade from about 20 per cent to about 10 per cent, as shown in the following Table 1.

Table 1. *Household Saving Rates 1990-2000 (in percentage points) for some OECD countries.*

Household Saving Rates 1990-2000 (in percentage points)							
	Canada	United States	Italy	United Kingdom	Germany	Spain	France
1990s peak	*13.2*	*8.7*	*18.7*	*11.4*	*13.1*	*14.4*	*16.2*
Year of peak	1991	1992	1991	1992	1991	1993	1997
2000	*3.2*	*-0.1*	*10.4*	*4.4*	*9.8*	*11.6*	*15.8*
Change[1]	*-10.0*	*-8.8*	*-8.3*	*-7.0*	*-3.3*	*-2.8*	*-0.4*
[1] From 1990s peak to 2000.							

Source: OECD national data; our elaboration from Table II.3, Bank of International Settlement (BIS), 71st annual report, p. 30, (2001).

Finally, as regards the replacement ratio Italy shows an hybrid form of unemployment benefits including various type of subsidies, which only in an approximated way could be summarised in a single "replacement ratio" value (see the appendix 2). In any case, without loss of realism, we have assumed values of the replacement included among 0.50 and 0.99.
In what follows we resort to numerical simulations to compare the pre and post reform representative individual's steady-state lifetime welfare, i.e. eq. (35) versus eq. (41), for increasing values of γ. We used for the technological capital weight the higher estimates of Jones (2003) in that we think that in our model the labour input interested to the wage-regulation only includes non-specialised labour and the capital stock may be thought in its broad concept, including physical and human components. In such a case, as known, the coefficient α may be fairly about 0.6 and 0.8.[12]
In Tables 2-5 we present values for both the market-wage and the minimum-wage economies for the plausible parametric cases above mentioned. The only difference among the tables[13] regards the assumed values for α and ϕ, which are respectively the minimum and the maximum approximated values evidenced for the recent decade. It is easy to see that in all cases both accumulation and welfare are significantly improved,[14] by introducing a minimum wage quite close to the market one (from about +8 per cent to about +15 per cent). Moreover the higher is the replacement ratio the higher the welfare and capital accumulation improvements are.
In this paper we only focus on the welfare effects of the assumed reform. However, we note that such a reform would also enhance the long run per-capita output, but the corresponding results are not presented here for brevity.
Note that, although the regulation of wages has caused rates of unemployment between 7 per cent and 13 per cent, the long run welfare is higher than in the competitive wage case.

[12] In fact Mankiw et al. (1992), p. 417, suggest that: i) since the minimum wage is a proxy of the return to labour without human capital, and ii) since the minimum wage has averaged about 30 to 50 percent of the average wage in manufacturing, then 50 to 70 percent of total labour income represents the return to human capital, then if the physical capital's share of income is expected to be about 1/3, the human capital's share of income should be between 1/3 and one half. In sum, with the broad view of capital the coefficient α may be fairly about 0.6 and 0.8.
[13] Since the purpose of this paper is limited to the current increase of the capital income tax from 12.5 per cent to 20 per cent, we don't report here simulations for different tax rates. However, it is worth noting that higher capital income taxes may bring to higher lifetime welfare levels.
[14] In some cases, as for instance the ones shown in Table 4, the improvement is really large: capital accumulation and utility increase respectively from 0.113 to 0.131 and from 0.05 to 0.18 with a minimum wage higher than the competitive one of about 15 per cent.

Despite the common wisdom which considers harmful the introduction of a minimum wage in that cause of output losses, we have shown that the introduction of a binding minimum wage, although on the one hand it generates market inefficiencies and unemployment, on the other hand it may also, surprisingly, generate production as well as welfare gains in the long run. Moreover, it may generate welfare gains even when it generates a production loss.
To better understand the economic reasons of why the introduction of a minimum wage may favour long run economic growth and welfare it is sufficient to say that it acts as a reversed social security scheme: that is, in principle, it transfers resources over time from the old to the young by raising the labour income and decreasing the interest rate. Moreover, if the unemployment insurance scheme, which supports the income of the young, is financed with a tax burden on the old people, then this mechanism of intergenerational transfer is even more strong.
Finally, it is worth noting that, when not only the long run, but also the short run, would be considered another important question can be asked: is there any possibility for the fiscal reform not to imply any welfare loss for the generations bearing it? First, the young generation is not harmed by the fiscal reform to the extent that the capital accumulation effect is sufficiently positive. Second, it is easy to see that, even if the welfare of the young generation is increased, the old generation living at the moment of the reform incurs in a loss due to the increased capital income tax and the decreased interest rate. However, both generations could be better off after the reform by designing a transfer policy between the two generations living at the moment of the reform through which young people compensate old people for the welfare loss. Anyway, this exercise is beyond the scope of the present paper and is left for future research. Note that the model of this section has not taken into account the possibly important values of leisure associated with unemployment. In the next section we will fill this gap.

Table 2. *Behaviours of the capital stock, unemployment, output, minimum wages, young and old age consumption and lifetime utility in the market-wage economy and in the regulated-wage economy. Parameter set:* $A = 10$, $\alpha = 0.55$, $\phi = 0.10$, $n = 0$, $\tau_{pc} = 0.125$ *and* $\tau_{mw} = 0.20$.

	$\gamma = 0.60$	$\gamma = 0.80$	$\gamma = 0.90$	$\gamma = 0.99$
$w°$	1.8593	1.8679	1.8709	1.8732
w_{pc}	1.6957	1.6957	1.6957	1.6957
$V(w°)$	0.4705	0.5032	0.5149	0.5238
$V(w_{pc})$	0.4411	0.4411	0.4411	0.4411
$w° / w_{pc}$	+9.64%	+10.15%	+10.33%	+10.46%
$u^*(w°)$	0.1153	0.093	0.085	0.078
$u^*(w_{pc})$	0	0	0	0
$k^*(w°)$	0.1773	0.1833	0.1855	0.1871
$k^*(w_{pc})$	0.1695	0.1695	0.1695	0.1695

Table 3. *Behaviours of the capital stock, unemployment, output, minimum wages, young and old age consumption and lifetime utility in the market-wage economy and in the regulated-wage economy. Parameter set:* $A = 10$, $\alpha = 0.55$, $\phi = 0.20$, $n = 0$, $\tau_{pc} = 0.125$ *and* $\tau_{mw} = 0.20$.

	$\gamma = 0.60$	$\gamma = 0.80$	$\gamma = 0.90$	$\gamma = 0.99$
$w°$	4.2986	4.3148	4.3206	4.3248
w_{pc}	3.9562	3.9562	3.9562	3.9562
$V(w°)$	1.2274	1.2564	1.2668	1.2747
$V(w_{pc})$	1.2148	1.2148	1.2148	1.2148

$w°/w_{pc}$	+8.65%	+9.06%	+9.20%	+9.31%
$u^*(w°)$	0.104	0.084	0.076	0.070
$u^*(w_{pc})$	0	0	0	0
$k^*(w°)$	0.8236	0.8484	0.8574	0.8643
$k^*(w_{pc})$	0.7912	0.7912	0.7912	0.7912

Table 4. *Behaviours of the capital stock, unemployment, output, minimum wages, young and old age consumption and lifetime utility in the market-wage economy and in the regulated-wage economy. Parameter set:* $A=10$, $\alpha=0.59$, $\phi=0.10$, $n=0$, $\tau_{pc}=0.125$ *and* $\tau_{mw}=0.20$.

	$\gamma=0.60$	$\gamma=0.80$	$\gamma=0.90$	$\gamma=0.99$
$w°$	1.2963	1.3064	1.3100	1.3127
w_{pc}	1.1365	1.1365	1.1365	1.1365
$V(w°)$	0.1149	0.1559	0.1709	0.1822
$V(w_{pc})$	0.0571	0.0571	0.0571	0.0571
$w°/w_{pc}$	+14.06%	+14.94%	+15.26%	+15.50%
$u^*(w°)$	0.137	0.1127	0.103	0.096
$u^*(w_{pc})$	0	0	0	0
$k^*(w°)$	0.1224	0.1276	0.1296	0.1311
$k^*(w_{pc})$	0.1136	0.1136	0.1136	0.1136

Table 5. *Behaviours of the capital stock, unemployment, output, minimum wages, young and old age consumption and lifetime utility in the market-wage economy and in the regulated-wage economy. Parameter set:* $A=10$, $\alpha=0.59$, $\phi=0.20$, $n=0$, $\tau_{pc}=0.125$ *and* $\tau_{mw}=0.20$.

	$\gamma=0.60$	$\gamma=0.80$	$\gamma=0.90$	$\gamma=0.99$
$w°$	3.4747	3.4974	3.5055	3.5115
w_{pc}	3.0814	3.0814	3.0814	3.0814
$V(w°)$	1.0338	1.0706	1.0840	1.0941
$V(w_{pc})$	0.9969	0.9969	0.9969	0.9969
$w°/w_{pc}$	+12.76%	+13.49%	+13.76%	+13.95%
$u^*(w°)$	0.126	0.103	0.094	0.087
$u^*(w_{pc})$	0	0	0	0
$k^*(w°)$	0.6597	0.6850	0.6944	0.7017
$k^*(w_{pc})$	0.6162	0.6162	0.6162	0.6162

5 The Home Production Model

In this section we relax the assumption that unemployed hours are without economic value. In particular, we suppose there exists a home production technology employing only labour factor. We assume per-capita home-produced goods are created, for simplicity, with the following linear technology: $h_t := Bu_t$, where B is the constant average and marginal productivity.[15] The reason for which the labour input in home production is given by u is simple: if we assumed that the return to labour in home production is always smaller than the regulated wage in the firms sector (that is $B < \underline{w}$), , then the hours of work employed in the home production are only those left unemployed by the introduction of the minimum wage.[16] Furthermore we also make the technical assumption that the physical labour marginal productivity of the home production is always smaller than the physical labour marginal productivity in the "firms" sector with competitive labour market, that is $B \in (0, w_{pc} \leq \underline{w})$.

As regards the production side, firms behave just as explained in the previous section.

In this case, the individual maximisation problem faced by agents of generation t is given by:

$$\max_{\{c_t^y, c_{t+1}^o\}} U_t(c_t^y, c_{t+1}^o) = (c_t^y)^{1-\phi}(c_{t+1}^o)^{\phi},$$

subject to

$$c_t^y + s_t = \underline{w}(1-u_t) + \gamma \underline{w} u_t + h_t,$$
$$c_{t+1}^o = [1 + r_{t+1}(1-\tau_{t+1,mw})]s_t,$$
$$c_t^y, c_{t+1}^o \geq 0.$$

The optimal young and old age consumption functions become:

$$c_t^y(\underline{w}, B) = (1-\phi)W_t(\underline{w}, B) \tag{42}$$

$$c_{t+1}^o(\underline{w}, B) = \phi[1 + r_{t+1}(1-\tau_{t+1,mw})]W_t(\underline{w}, B) \tag{43}$$

where $W_t(\underline{w}, B) := \underline{w}(1-u_t) + (B + \gamma \underline{w})u_t = W_t(\underline{w}) + Bu_t$ is the total income of the young, as given by the sum of labour income, $\underline{w}$, plus the unemployment insurance benefit and the income received by the home-produced goods activities.

The savings function, instead, is the following:

$$s_t(\underline{w}, B) = \phi W_t(\underline{w}, B). \tag{44}$$

Government. The government strategy is to adjust the capital income tax rate such as to balance out unemployment benefit expenditures with tax receipts in each period. Thus, the per-capita time-t government constraint is given exactly by eq. (23).

The long-run equilibrium. Given the government constraint, the market clearing condition in goods as well as in capital markets is simply given by:

$$(1+n)k_{t+1} = s_t(\underline{w}, B), \tag{46}$$

and combining (46) with (44) we find:

$$(1+n)k_{t+1} = \phi[\underline{w}(1-u_t(k_t, \underline{w})) + (B + \gamma \underline{w})u_t(k_t, \underline{w})]. \tag{47}$$

Substituting out for $u_t(k_t, \underline{w})$ from eq. (22), capital evolves over time according to the following first order linear difference equation:

$$k_{t+1} = \frac{\phi}{1+n}(B + \gamma \underline{w}) + \frac{\phi}{1+n}\psi^{\frac{1}{\alpha}} \underline{w}^{-\frac{1}{\alpha}}(\underline{w}(1-\gamma) - B)k_t. \tag{48}$$

Steady-state implies $k_{t+1} = k_t := k^*$. When the minimum wage is binding ($\underline{w} > w_{pc}$), the per-capita long-run unemployment rate, capital stock and total production $q = y + h$, where now in addition to the output of the "firms" sector there is the home production, are given by the following conditions:

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[15] For an important contribution on home production in macroeconomic models see Benhabib et al. (1991).

[16] It is important to note that since i) the unemployment created by the introduction of the minimum wage is only involuntary, and ii) the government pays a wage-indexed unemployment insurance subsidy for the hours left unemployed, then the condition preserving the incentive to work in the "firms" is that the average and marginal productivity, B, must be not greater than the regulated wage, $\underline{w}$. It would be wrong to consider the total income earned by the unemployed people imposing the condition $B + \gamma \underline{w} < \underline{w}$ since the benefit is paid only if unemployment is involuntary and then a home production regime, in which only home-produced goods would be produced, is always prevented to occur.

$$u^*(\underline{w},B)=\frac{\underline{w}^{\frac{1}{\alpha}}(1+n)-\phi\underline{w}\psi^{\frac{1}{\alpha}}}{\underline{w}^{\frac{1}{\alpha}}(1+n)-\phi(\underline{w}(1-\gamma)-B)\psi^{\frac{1}{\alpha}}},\tag{49}$$

$$k^*(\underline{w},B)=\frac{\phi(B+\gamma\underline{w})\underline{w}^{\frac{1}{\alpha}}}{\underline{w}^{\frac{1}{\alpha}}(1+n)-\phi(\underline{w}(1-\gamma)-B)\psi^{\frac{1}{\alpha}}},\tag{50}$$

$$q^*(\underline{w},B)=y^*(\underline{w},B)+h^*(\underline{w},B)=\frac{\underline{w}^{\frac{1}{\alpha}}(1+n)B+\phi\psi^{\frac{1}{\alpha}}\underline{w}\left(\frac{\alpha B+\gamma\underline{w}}{1-\alpha}\right)}{\underline{w}^{\frac{1}{\alpha}}(1+n)-\phi(\underline{w}(1-\gamma)-B)\psi^{\frac{1}{\alpha}}},\tag{51}$$

If $B=0$, eqs. (49), (50) and (51) collapse to (27), (28) and (29) respectively.
As regards stability, the analysis of the difference equation (48) gives us more interesting results than in the case without home production (see eq. (26)). In particular, as known, the general condition for stability requires that $-1<\partial k_{t+1}/\partial k_t<1$. Differentiating eq. (48) with respect to k_t yields $\partial k_{t+1}/\partial k_t=\frac{\phi}{1+n}\psi^{\frac{1}{\alpha}}\underline{w}^{-\frac{1}{\alpha}}(\underline{w}(1-\gamma)-B)$. Therefore, stability requires that $-1<\frac{\phi}{1+n}\psi^{\frac{1}{\alpha}}\underline{w}^{-\frac{1}{\alpha}}(\underline{w}(1-\gamma)-B)<1$.[17] The latter derivative may be positive or negative depending on the sign of the term $\underline{w}(1-\gamma)-B$. If $B<\underline{w}(1-\gamma)$, then $0<\partial k_{t+1}/\partial k_t<1$ holds. In this case, the trajectory is stable and converges monotonically towards the steady-state equilibrium. On the contrary if $B>\underline{w}(1-\gamma)$, then $-1<\partial k_{t+1}/\partial k_t<0$. In the latter case, the trajectory is stable with oscillatory movements. Thus, the model with home technology possesses the noteworthy property to generate dumped economic fluctuations. The following figure shows the locus of dynamic capital accumulation equation $k_{t+1}=f(k_t)$ in both cases with and without home production. The parametric configuration is that of Table 4, with in addition $B=1$ for the home technology case.[18] In this case, we see that, as expected from the analytical considerations above mentioned, the steady-state capital stock is higher in the home production case and that, noteworthy fluctuations may occur in the economy.

[17] This condition is always satisfied but the algebraic demonstration is rather cumbersome and omitted here for economy of space.
[18] The same qualitative shape of the Figure 5 holds even in the parametric case of Table 2 here omitted for brevity.

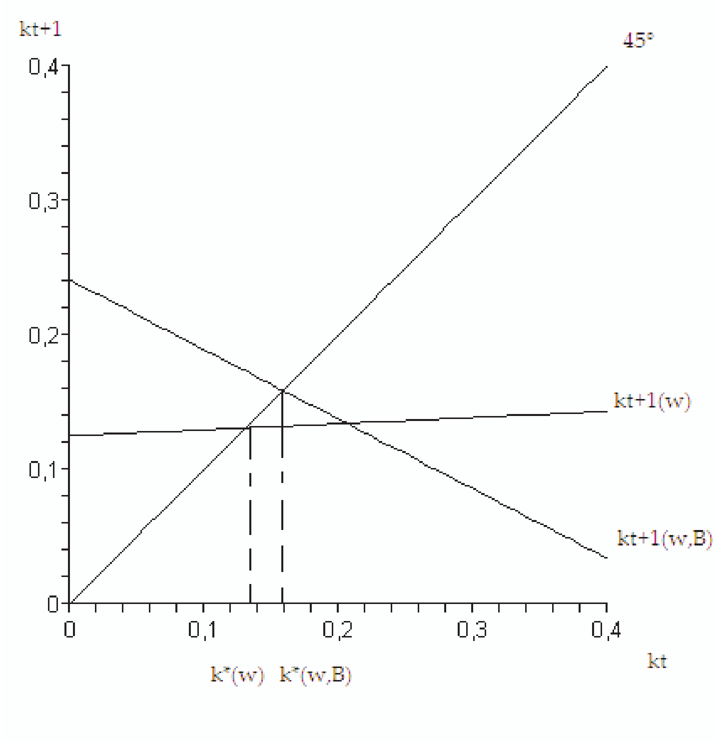

Figure 5. The behaviour of the capital accumulation equations in both cases with and without home production, kt+1(w,B) and kt+1(w) respectively. Parameter set: $A=10$, $\alpha=0.59$, $\phi=0.10$, $\gamma=0.95$, $B=1$ and $n=0$.

The steady-state government constraint is simply given by:

$$\gamma \underline{w} u^*(\underline{w},B)+\overline{g}=\tau_{mw} r(\underline{w})k^*(\underline{w},B) \Rightarrow \tau(\underline{w},B)=\frac{\gamma \underline{w} u^*(\underline{w},B)}{r(\underline{w})k^*(\underline{w},B)}+\frac{\overline{g}}{r(\underline{w})k^*(\underline{w},B)}. \quad (52)$$

The existence of a backyard technology used only by unemployed workers, modifies the steady-state results of the model of the previous section in a clear-cut way. The new results, in comparison with those of the model without such a technology can be formally summarised in the following proposition:

Proposition 1 The existence of a home production technology implies that in steady-state, *ceteris paribus* as regards α, ϕ and $\underline{w}$, we have: 1) $k^*(\underline{w},B)>k^*(\underline{w})$, 2) $u^*(\underline{w};B)<u^*(\underline{w})$, 3) $q^*(\underline{w},B)>y^*(\underline{w})$, 4) $c^y(\underline{w},B)>c^y(\underline{w})$, 5) $\tau(\underline{w};B)<\tau(\underline{w})$, 6) $c^o(\underline{w},B)>c^o(\underline{w})$ and 7) $V(\underline{w},B)>V(\underline{w})$ for any $\underline{w}>w_{pc}$ and $B>0$.

Proof Since $W(\underline{w},B)>W(\underline{w})$ for any $\underline{w}>w_{pc}$ and $B>0$ by definition, then 1) given that[19] $\partial k^*(\underline{w},B)/\partial B>0$ for any $\underline{w}>w_{pc}$, and if $B=0$ then $k^*(\underline{w},B)=k^*(\underline{w})$, it follows that $k^*(\underline{w},B)>k^*(\underline{w})$ for any $\underline{w}>w_{pc}$ and $B>0$; 2) by looking at eq. (22), it can be easily seen that the rate of unemployment is a decreasing function of the per-capita stock of capital. Since

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[19] $\partial k^*(\underline{w},B)/\partial B=\dfrac{\phi \underline{w}^{\frac{1}{\alpha}}\left[\underline{w}^{\frac{1}{\alpha}}(1+n)-\phi \underline{w}\psi^{\frac{1}{\alpha}}\right]}{\left[\underline{w}^{\frac{1}{\alpha}}(1+n)-\phi(\underline{w}(1-\gamma)-B)\psi^{\frac{1}{\alpha}}\right]^2}>0$ for any $\underline{w}>w_{pc}$.

$k^*(\underline{w},B) > k^*(\underline{w})$, then $u^*(\underline{w};B) < u^*(\underline{w})$ for any $\underline{w} > w_{pc}$ and $B > 0$; 3) since $y^*(\cdot) = f\left(\overbrace{k^*(\cdot)}^{+}, \overbrace{u^*(\cdot)}^{-}\right)$, then given points 1) and 2) we have $y^*(\underline{w},B) > y^*(\underline{w})$ for any $\underline{w} > w_{pc}$ and $B > 0$. Therefore, $q^*(\underline{w},B) := y^*(\underline{w},B) + Bu^*(\underline{w},B) > y^*(\underline{w})$ holds a fortiori; 4) since $c^y(\underline{w},B) = (1-\phi)W(\underline{w},B)$ and $c^y(\underline{w}) = (1-\phi)W(\underline{w})$, and knowing that $W(\underline{w},B) > W(\underline{w})$, it follows that $c^y(\underline{w},B) > c^y(\underline{w})$ for any $\underline{w} > w_{pc}$ and $B > 0$; 5) since the real rate of interest, eq. (21), does not depend on B, and given points 1) and 2), eqs (38) and (52) yield $\tau(\underline{w};B) < \tau(\underline{w})$ for any $\underline{w} > w_{pc}$ and $B > 0$, that is the capital income tax rate balancing the government budget is smaller with home production than in the case in which $B = 0$; 6) since $c^o(\underline{w},B) = \phi[1 + r(\underline{w})(1 - \tau(\underline{w},B))]W(\underline{w},B)$ and $c^o(\underline{w}) = \phi[1 + r(\underline{w})(1 - \tau(\underline{w}))]W(\underline{w})$, knowing that $W(\underline{w},B) > W(\underline{w})$ and given point 5) it easily follows that $c^o(\underline{w},B) > c^o(\underline{w})$ for any $\underline{w} > w_{pc}$ and $B > 0$; 7) given points 4) and 6), it follows directly that $V(\underline{w},B) > V(\underline{w})$ for any $\underline{w} > w_{pc}$ and $B > 0$.

The following figures 6-9 illustrate the comparison between the results of two models (with and without home technology) for the same parameters above used for Italy. Figures 6 and 7 show that in both cases of capital weight in technology estimated in the recent decade, and for the current approximated propensity to save, for Italy (and assuming a high replacement ratio, that is $\gamma = 0.95$, which it would be in line with the suggestion of some current political debates), the increase in the lifetime long-run welfare is very significant, even with a value of the home productivity sufficiently low (i.e. marginal productivity in the "backyard" sector, $B = 1$, versus a marginal productivity in the "firms" sector of about 1.69). Figures 8 and 9 show a very interesting fact. While in the case without home production the consumption of the representative individual when old is lower than competitive one, in presence of home production also the consumption of the second period may be higher than in the competitive-wage case. Finally, in Figures 10 and 11 we may compare, when the tax rate is fixed at 0.20, the lifetime welfare in the two cases with and without home technology. When $\alpha = 0.55$, the index of utility raises from 0.51 without home production to 0.64 with home production. When $\alpha = 0.59$, the increase in the utility is even higher: the index of utility raises, from 0.16 without home production to 0.35 with home production. Note, as shown by Figures 6 and 7, that a level of minimum wage maximising the lifetime welfare does always exist, and this maximising value is much higher to that corresponding to the tax rate of 20 per-cent: this means that in order to maximise long-run welfare, a tax rate higher than 20 per-cent should be fixed (for example, for the case without home technology (B=0), Figure 6 shows that the welfare-maximising value of the minimum wage is about 3.7 at which corresponds a tax rate about 0.90, which is surely much higher than the 20 per cent value introduced with the reform and above investigated; in any case, these observations are beyond the scope of the present exercise).
To sum up the role of the existence of a home technology, the following remark holds: *in presence of a minimum wage, the higher the productivity of the home technology is, the more likely or higher the welfare gain is.*

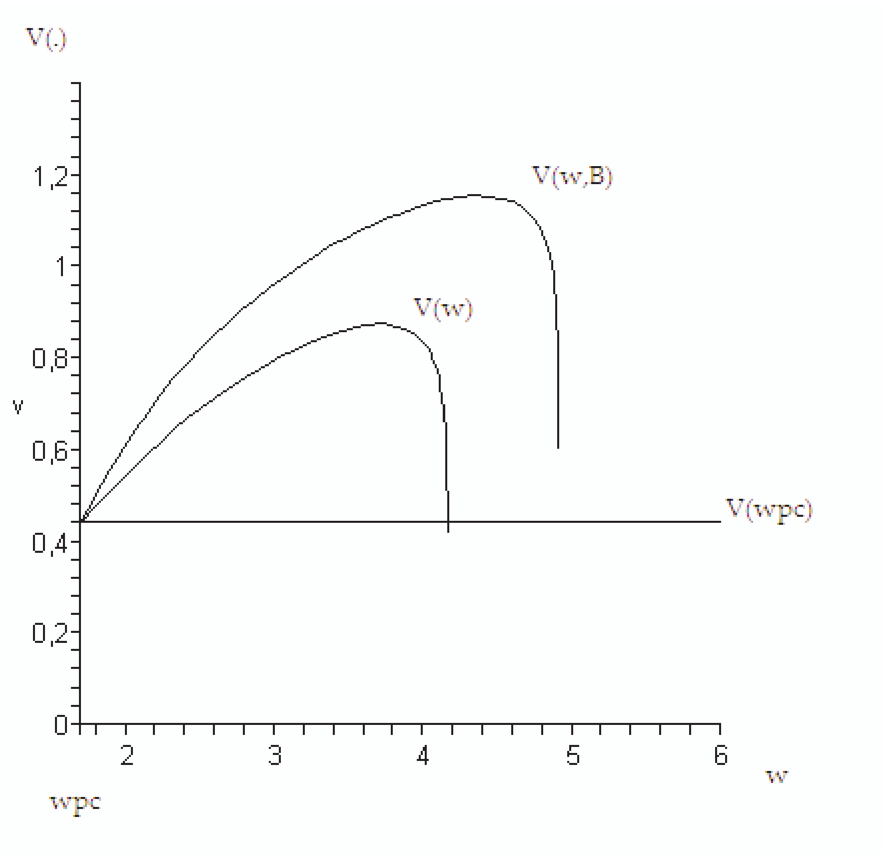


Figure 6. The comparison between the levels of lifetime utility in the case with and without home technology as a function of the minimum wage. Parameter set: $A=10$, $\alpha=0.55$, $\phi=0.10$, $B=1$, $\gamma=0.95$ and $n=0$.

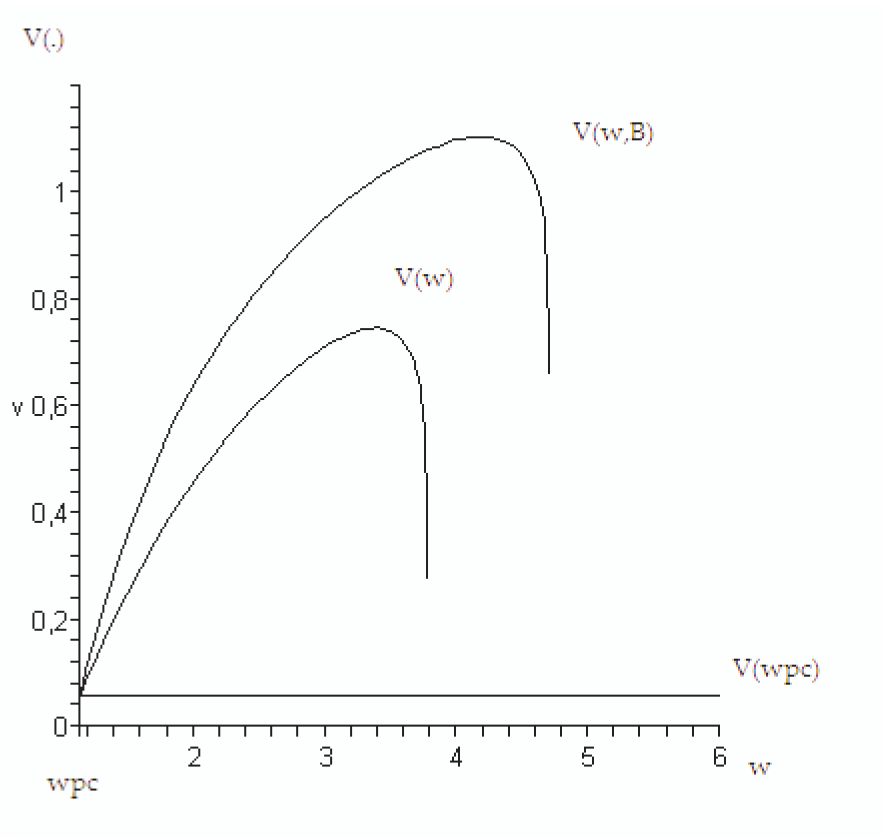


Figure 7. The comparison between the levels of lifetime utility in the case with and without home technology as a function of the minimum wage. Parameter set: $A = 10$, $\alpha = 0.59$, $\phi = 0.10$, $B = 1$, $\gamma = 0.95$ and $n = 0$.

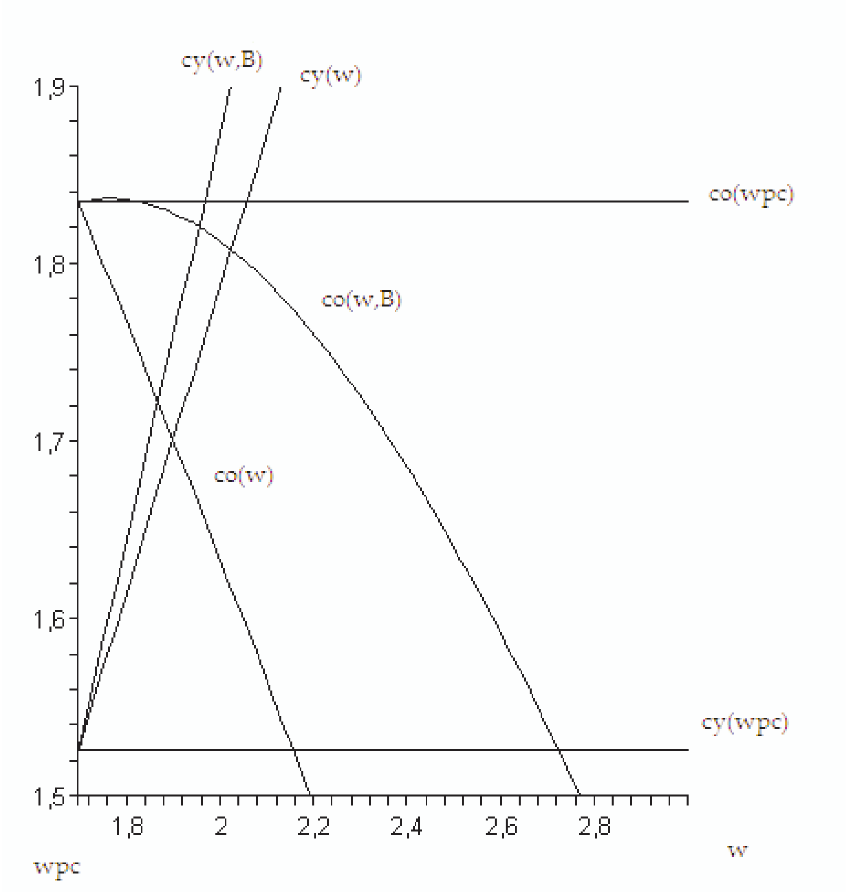


Figure 8. The comparison between the levels of young and old age consumption in the case with and without home technology as a function of the minimum wage. Parameter set: $A = 10$, $\alpha = 0.55$, $\phi = 0.10$, $B = 1$, $\gamma = 0.95$ and $n = 0$.

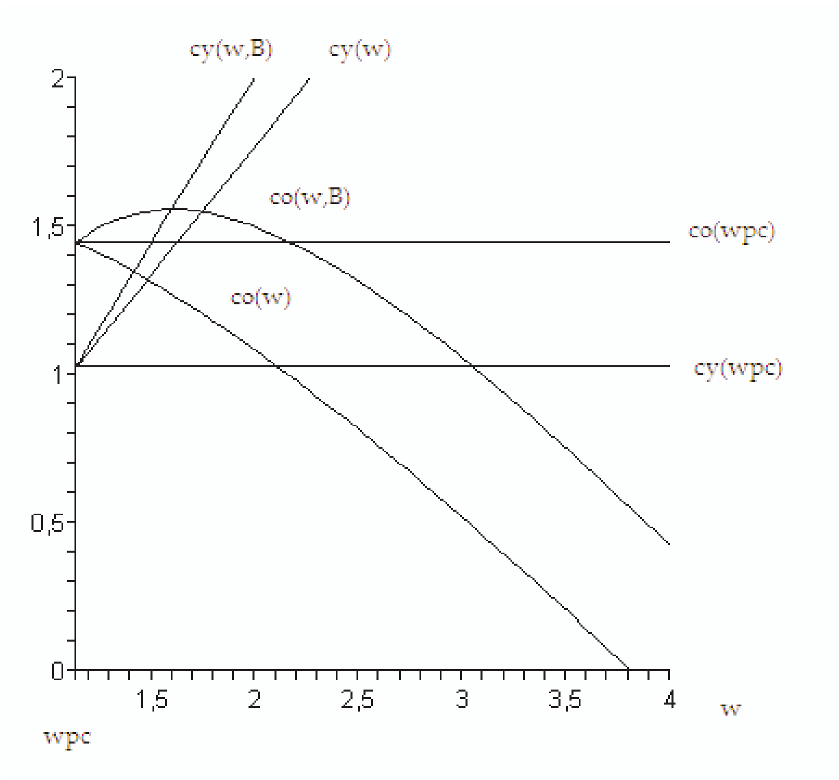
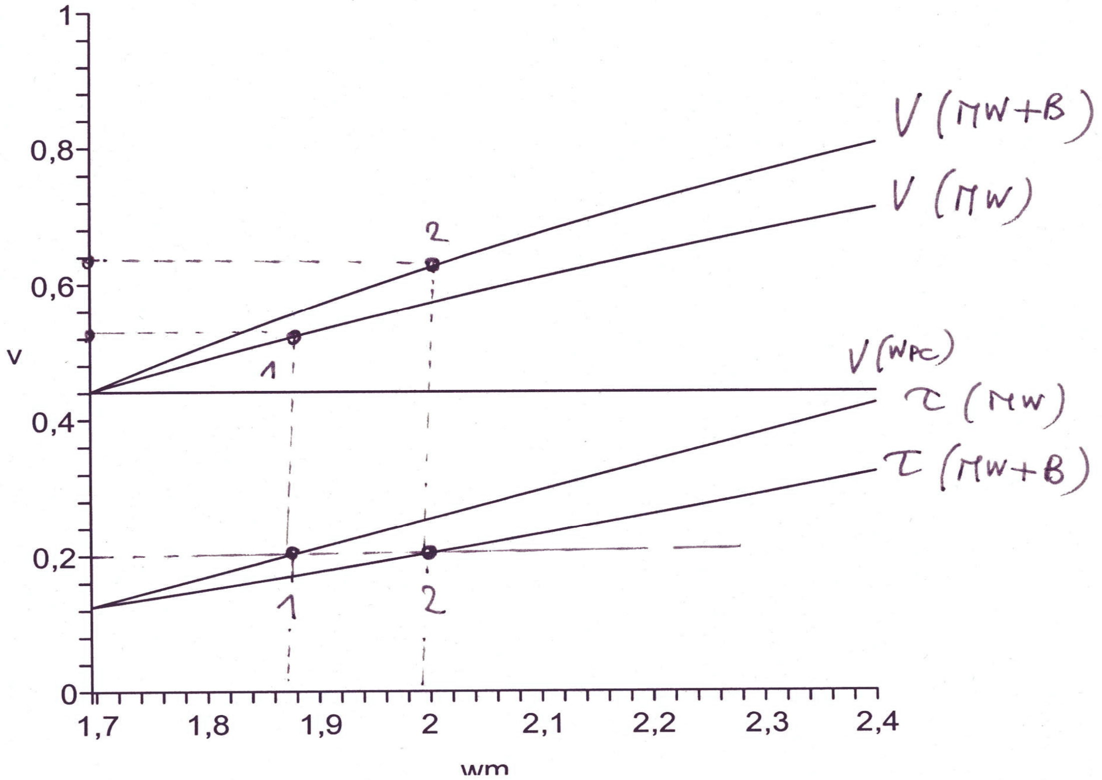


Figure 9. The comparison between the levels of young and old age consumption in the case with and without home technology as a function of the minimum wage. Parameter set: $A=10$, $\alpha=0.59$, $\phi=0.10$, $B=1$, $\gamma=0.95$ and $n=0$.



PARAMETER SET: $\alpha = 0.55$, $\varphi = 0.10$, $B = 1$, $\gamma = 0.95$, $\tau_{PC} = 0.125$, $n = 0$

POINT 1: $\underline{w} = 1.872$, $V(MW) = 0.520$, $\tau^{(MW)} = 0.20$

POINT 2: $\underline{w} = 1.987$, $V(MW+B) = 0.617$, $\tau(MW+B) = 0.20$

Figure 10. The comparison between the levels of lifetime utility in the case with and without home technology when the capital income tax rate is 20%.

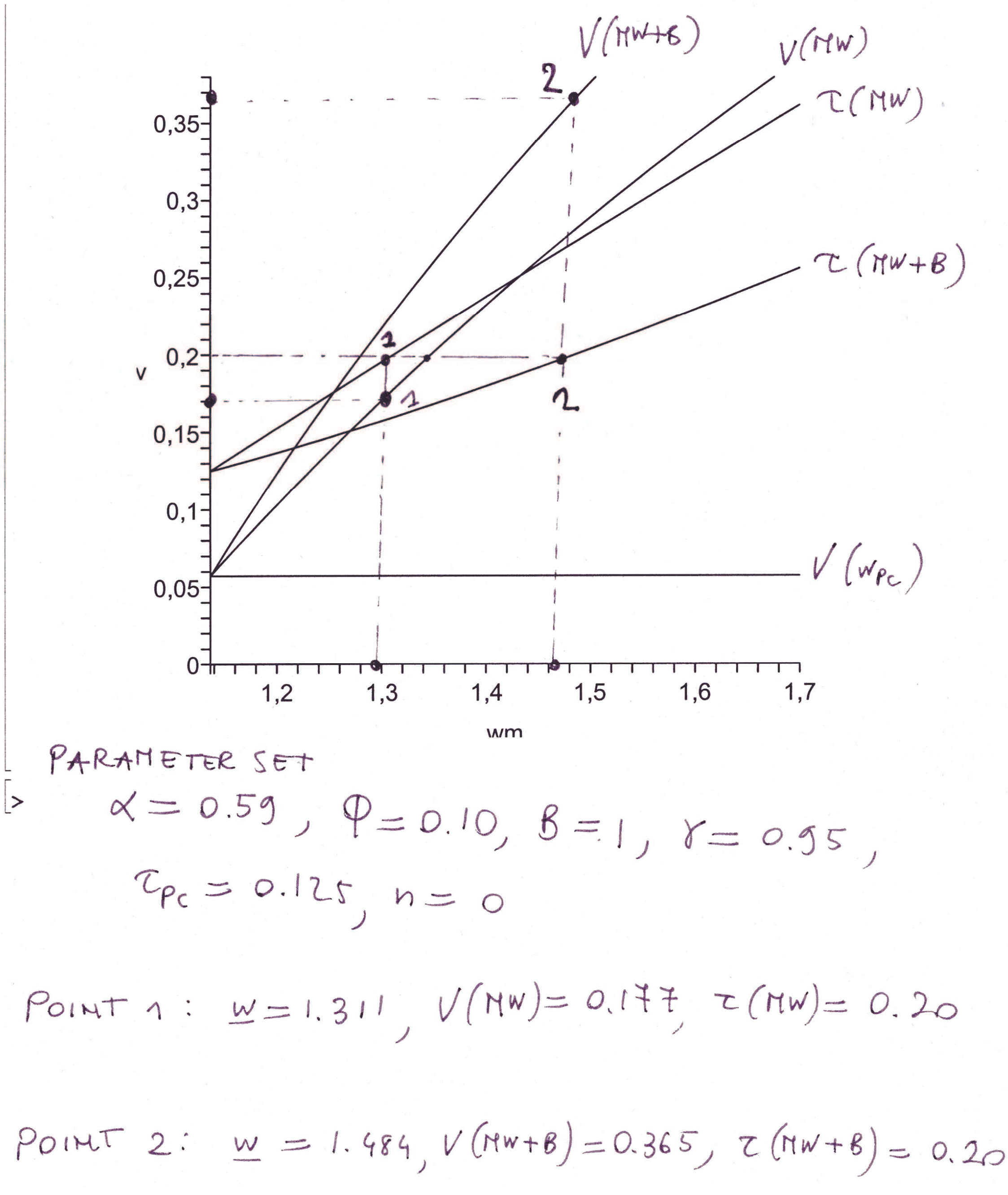


Figure 11. The comparison between the levels of lifetime utility in the case with and without home technology when the capital income tax rate is 20%.

6 Conclusions

The debate on the macroeconomic effects of the regulation of wages and of the introduction of unemployment benefits - which has also occurred in the recent years in Italy - is long lasting. Conventional wisdom holds that both may be harmful for efficiency, although often advocated for equity reasons. Another controversial debate concerned the taxation of capital income. In this paper we have shown that the introduction of a binding minimum wage although on the one hand it generates market inefficiencies and unemployment, on the other hand it may also, surprisingly, generate production as well as welfare gains in the long run. Moreover, it may generate welfare gains even when it generates a production loss. From a policy point of view, motivated by the recent political debate in

Italy about the increase from 12.5% to 20% in the capital income tax rate, the aim of this paper was to study the impact on the long run lifetime welfare of a proposal for reform, consisting in the introduction of a minimum wage and of an unemployment insurance system, with the increase of the tax burden on capital income used for preserving the balanced budget instead of financing other public expenditures. This impact depends crucially on the values of the capital weight in technology, of the propensity to save and of the replacement ratio. As a matter of fact, our simulation exercises showed that, in contrast with the prevailing wisdom, positive effects always appear with a "calibration" of parameters largely corresponding to the current Italian situation.

Moreover we have remarked that the hours of unemployment should be considered as an additional resource instead of a damage, since there are important leisure values associated with unemployment (for instance leisure time, self-enrichment activities, education, home production and so on). For exploring the further effects of the our proposal for reform in presence of an economic use of the leisure associated with the unemployment, we assumed that a home production technology with constant productivity does exist and then we have shown that in this case the higher the productivity of the home technology is, either the more likely or higher the welfare gain is.

Noteworthy, our conclusions are reached within a standard dynamic general equilibrium overlapping generations model where agents live two periods and the only departure from the textbook OLG model is the assumption that a minimum wage may be imposed by a government. Therefore our paper offer some new results having some important policy implications so far not explored.

While we have made in this paper much progress in understanding the effects of the regulation of wages in a standard OLG dynamic context, there remain many other open interesting research questions. The first is "How these findings regarding a closed economy are modified when the economy is open?". A second research question is "What occurs when in the labour market an efficiency-wage mechanism is present?". Finally a policy related question: "What are the more efficient taxation systems depending on different values of technology and preferences?". Such research questions should be considered in future papers.

Appendix 1

In this appendix we show that taxation systems burdening on the income of the young people as well as burdening on the firms always result in a reduction of output and welfare. In particular we investigates the following three cases: 1) a wage tax; 2) a lump-sum tax on the young people; 3) a contribution paid by the firms which is proportional to the net wage. Firstly, as regard the firms' behaviour, we note that for the first two cases eqs. (18)-(22) in the main text still hold.

Tax on labour income

Individuals. The individual maximisation problem for agents of generation t is:

$$\max_{\{c_t^y, c_{t+1}^o\}} U_t\left(c_t^y, c_{t+1}^o\right) = \left(c_t^y\right)^{1-\phi}\left(c_{t+1}^o\right)^{\phi},$$

subject to

$$c_t^y + s_t = \underline{w}\left(1-\tau_t^w\right)\left(1-u_t\right) + \gamma \underline{w} u_t,$$

$$c_{t+1}^o = \left(1+r_{t+1}\right)s_t,$$

$$c_t^y, c_{t+1}^o \geq 0.$$

The optimal young and old age consumption functions become:

$$c_t^y\left(\underline{w}\right) = \left(1-\phi\right)\left(\underline{w}\left(1-\tau_t^w\right)\left(1-u_t\right) + \gamma \underline{w} u_t\right), \tag{A1}$$

$$c_t^y\left(\underline{w}\right) = \phi\left(1+r_{t+1}\right)\left(\underline{w}\left(1-\tau_t^w\right)\left(1-u_t\right) + \gamma \underline{w} u_t\right). \tag{A2}$$

The savings function, instead, is the following:

$$s_t\left(\underline{w}\right) = \phi\left(\underline{w}\left(1-\tau_t^w\right)\left(1-u_t\right) + \gamma \underline{w} u_t\right). \tag{A3}$$

Government. The government strategy is to adjust the labour income tax rate such as to balance out unemployment benefit expenditures with tax receipts in each period. Thus, the per-capita time-t government constraint is the following:

$$\gamma \underline{w} u_t = \tau_t^w \underline{w}(1-u_t). \tag{A4}$$

The long-run equilibrium. Given eq. (A4), the market clearing condition in goods as well as in capital markets is simply given by:

$$(1+n)k_{t+1} = s_t(\underline{w}), \tag{A5}$$

and combining (A5) with (A3) we find:

$$(1+n)k_{t+1} = \phi\left(\underline{w}(1-\tau_t^w)(1-u_t) + \gamma \underline{w} u_t\right). \tag{A6}$$

Solving eq. (A4) for τ_t^w and substituting out into (11) we get:

$$(1+n)k_{t+1} = \phi \underline{w}(1-u_t). \tag{A7}$$

Substituting out for $u_t(k_t, \underline{w})$ from (22) into (A7), capital evolves over time according to the following first order linear difference equation:

$$k_{t+1} = \frac{\phi}{1+n} \psi^{\frac{1}{\alpha}} \underline{w}^{-\frac{1-\alpha}{\alpha}} k_t. \tag{A8}$$

Eq. (A8) implies there is no steady-state. The long-run rate of growth of the economy is given by:

$$1 + g(\underline{w}) := \frac{k_{t+1}}{k_t} = \frac{\phi}{1+n} \psi^{\frac{1}{\alpha}} \underline{w}^{-\frac{1-\alpha}{\alpha}}, \tag{A9}$$

where $g(\underline{w})$ is the growth rate and k_{t+1}/k_t the growth factor.
Differentiating (A9) with respect to the minimum wage yields:

$$\frac{\partial g(\underline{w})}{\partial \underline{w}} = -\frac{1-\alpha}{\alpha} \frac{\phi}{1+n} \psi^{\frac{1}{\alpha}} \underline{w}^{-\frac{1}{\alpha}}. \tag{A10}$$

By looking at eq. (A10), it can be easily seen that increasing the minimum wage always depresses the growth rate of the economy. Substituting out the market-clearing wage into (A9) we obtain $g(\underline{w}) = 0$. This implies that introducing a minimum wage plus an unemployment insurance benefit financed with a tax rate on labour income creates endogenously a negative rate of growth of the economy.

Lump-Sum Taxation

Here we present the representative individual's optimal choices and the government plan by considering the hypothesis of the introduction of a lump-sum tax to finance the unemployment benefit system at balanced budget.

Individuals. The individual maximisation problem for agents of generation t is:

$$\max_{\{c_t^y, c_{t+1}^o\}} U_t\left(c_t^y, c_{t+1}^o\right) = \left(c_t^y\right)^{1-\phi}\left(c_{t+1}^o\right)^{\phi},$$

subject to

$$c_t^y + s_t = \underline{w}(1-u_t) + \gamma \underline{w} u_t - \tau_t,$$

$$c_{t+1}^o = (1+r_{t+1})s_t,$$

$$c_t^y, c_{t+1}^o \geq 0.$$

The optimal young and old age consumption functions become:

$$c_t^y(\underline{w}) = (1-\phi)(\underline{w}(1-u_t) + \gamma \underline{w} u_t - \tau_t), \tag{A11}$$

$$c_t^y(\underline{w}) = \phi(1+r_{t+1})(\underline{w}(1-u_t) + \gamma \underline{w} u_t - \tau_t). \tag{A12}$$

The savings function, instead, is the following:

$$s_t(\underline{w}) = \phi(\underline{w}(1-u_t) + \gamma \underline{w} u_t - \tau_t). \tag{A13}$$

Government. The government strategy is to adjust the lump-sum tax such as to balance out unemployment benefit expenditures with tax receipts in each period. Thus, the per-capita time-t government constraint is the following:

$$\gamma \underline{w} u_t = \tau_t. \tag{A14}$$

The long-run equilibrium. Given eq. (A14), the market clearing condition in goods as well as in capital markets is simply given by:

$$(1+n)k_{t+1} = s_t(\underline{w}), \tag{A15}$$

and combining (A15) with (A13) we find:

$$(1+n)k_{t+1} = \phi(\underline{w}(1-u_t)+\gamma\underline{w}u_t - \tau_t). \tag{A16}$$

Solving eq. (A14) for τ_t and substituting out into (A16) yields eq. (A7). Therefore, introducing a binding minimum wage in the case of a lump-sum tax on the young endogenously leads to a negative rate of growth of the economy.

Tax Burden on the firms

In this section we consider an unemployment benefit scheme financed with a labour tax paid by the representative firm. The model is outlined as follows:

Individuals. The individual maximisation problem for agents of generation t is:

$$\max_{\{c_t^y, c_{t+1}^o\}} U_t(c_t^y, c_{t+1}^o) = (c_t^y)^{1-\phi}(c_{t+1}^o)^{\phi},$$

subject to

$$c_t^y + s_t = \underline{w}(1-u_t)+\gamma\underline{w}u_t,$$

$$c_{t+1}^o = (1+r_{t+1})s_t,$$

$$c_t^y, c_{t+1}^o \geq 0.$$

The optimal young and old age consumption functions become:

$$c_t^y(\underline{w}) = (1-\phi)(\underline{w}(1-u_t)+\gamma\underline{w}u_t), \tag{A17}$$

$$c_t^y(\underline{w}) = \phi(1+r_{t+1})(\underline{w}(1-u_t)+\gamma\underline{w}u_t). \tag{A18}$$

The savings function, instead, is the following:

$$s_t(\underline{w}) = \phi(\underline{w}(1-u_t)+\gamma\underline{w}u_t). \tag{A19}$$

Firms. Standard profit maximisation

$$\max_{\{K_t, L_t\}} Y_t - \underline{w}(1+\tau_t^w)L_t - (1+r_t)K_t,$$

leads to the following marginal conditions for capital and labour:

$$r_t = \alpha A\left(\frac{k_t}{1-u_t}\right)^{\alpha-1} - 1, \tag{A20}$$

$$\underline{w} = \frac{(1-\alpha)A}{1+\tau_t^w}\left(\frac{k_t}{1-u_t}\right)^{\alpha}. \tag{A21}$$

Once the wage has been fixed the real interest rate is exogenous, and substituting (A21) into (A20) for $k_t/(1-u_t)$ we have:

$$r(\underline{w}) = \alpha A\left(\frac{\psi}{\underline{w}(1+\tau_t^w)}\right)^{\frac{1-\alpha}{\alpha}} - 1, \tag{A22}$$

where $\psi := (1-\alpha)A$. An increase in $\underline{w}$ always reduces the real interest rate. The short-run unemployment rate is endogenous and it is given by:

$$u_t(k_t, \underline{w}) = 1 - \left(\frac{\psi}{\underline{w}(1+\tau_t^w)}\right)^{\frac{1}{\alpha}} \cdot k_t, \tag{A23}$$

Eq. (A23) implies that the rate of unemployment is positively related with the minimum wage and the tax rate on labour income and strictly decreasing in the capital per-capita.

Government. The government strategy is to adjust the labour tax such as to balance out unemployment benefit expenditures with tax receipts in each period. Thus, the per-capita time-t government constraint is the following:

$$\gamma \underline{w} u_t = \tau_t^{\,w} \underline{w}(1-u_t). \tag{A24}$$

The long-run equilibrium. Given eq. (A24), the market clearing condition in goods as well as in capital markets is simply given by:

$$(1+n)k_{t+1} = s_t(\underline{w}), \tag{A25}$$

and combining (A25) with (A19) we find:

$$(1+n)k_{t+1} = \phi(\underline{w}(1-u_t) + \gamma \underline{w} u_t). \tag{A26}$$

Using (A24), eq. (A29) may be written as:

$$(1+n)k_{t+1} = \phi \underline{w}(1+\tau_t^{\,w})(1-u_t). \tag{A27}$$

Substituting out for $u_t(k_t, \underline{w})$ from (A23) into (A27), capital evolves over time according to the following first order linear difference equation:

$$k_{t+1} = \frac{\phi}{1+n} \psi^{\frac{1}{\alpha}} \left[\underline{w}(1+\tau_t^{\,w})\right]^{-\frac{1-\alpha}{\alpha}} k_t. \tag{A28}$$

Eq. (A28) implies there is no steady-state. The long-run rate of growth of the economy is given by:

$$1 + g(\underline{w}, \tau_t^{\,w}) := \frac{k_{t+1}}{k_t} = \frac{\phi}{1+n} \psi^{\frac{1}{\alpha}} \left[\underline{w}(1+\tau_t^{\,w})\right]^{-\frac{1-\alpha}{\alpha}}, \tag{A29}$$

where $g(\underline{w}, \tau_t^{\,w})$ is the growth rate and k_{t+1}/k_t the growth factor. It can be easily seen that the long-run rate of growth of the economy depends negatively on both the minimum wage and the labour tax. Thus, in this case, introducing minimum wages, a fortiori, confirms the finding of endogenous reductions in the rate of growth of the economy.

The analysis developed in this appendix can be resumed in the following ***Result***: introducing a binding minimum wage in all cases of 1) a wage tax; 2) a lump-sum tax on the young people; 3) a contribution paid by the firms which is proportional to the net wage, the introduction of a binding minimum wage endogenously leads to a negative rate of growth of the economy, and consequently of the welfare as well.

Appendix 2

Unemployment Insurance System in Italy

In Italy there exist various kinds of unemployment insurance benefits. In particular the *ordinary unemployment benefits* are payable for a maximum of seven months for beneficiaries aged 50 or older. Daily benefits are equal to 50 per cent of the gross average daily wage for the first six months and 40 per cent for the seventh month. Beneficiaries aged 50 or older receive 50 per cent of the gross average daily wage during the first six months, 40 per cent for the next three months, and 30 per cent for the tenth month. The gross average daily wage is based on earnings of the previous three months. *Mobility allowance*: 80 per cent of the insured's last earnings are paid for up to twelve months; thereafter, 64%. The maximum duration of the allowance varies from twelve months to thirty-six months, and it is dependent on the age of the worker and the location of the place of employment. In addition to unemployment benefits, Italy has a state fund for employees in industry whose companies put them on temporary redundancy through no fault of their own (e.g., market crisis, natural disaster, etc.), called *Cassa Integrazione Guadagni* (CIG), which is designed to integrate employees' earnings until work is resumed. There are two types of CIG: *Ordinary CIG* comprises 80 per cent of the wage for hours not worked. This cannot exceed a monthly maximum amount. *Extraordinary CIG* has a benefit equal to 80 per cent of the wage designed to cover special situations, e.g. when a production line is being reorganised or converted, and work must temporarily cease. To sum up in a loose way, we can say that in Italy the "replacement ratio" is comprised between 50 and 80 per cent and is only temporary.

References

[1] Azariadis, C. (1993). *Intertemporal Macroeconomics*. Basil Blackwell, Oxford.

[2] Bank of International Settlement (BIS), 71st annual report, (2001). Available at http://www.bis.org/publ/arpdf/ar2001i2.pdf.

[3] Benhabib, J, R. Rogerson and R. Wright (1991). "Homework in Macroeconomics: Household Production and Aggregate Fluctuations". Journal of Political Economy, 99 (6), pp. 1166-1187.

[4] Cahuc, P. and P. Michel (1996). "Minimum Wage, Unemployment and Growth". *European Economic Review*, Vol. 40, (7), pp. 1463-1482.

[5] De La Croix, D. and P. Michel (2002). *A Theory of Economic Growth. Dynamics and Policy in Overlapping Generations*. Cambridge University Press.

[6] Diamond, P. (1965). "National Debt in a Neoclassical Growth Model". *American Economic Review*, Vol. 55, pp 1126-1150.

[7] Jones, C. I. (2003). "Growth, Capital Shares and a New Perspective on Production Functions". Available at http://elsa.berkeley.edu/~chad.

[8] Jones, C. I. (2005). "The Shape of Production Functions and the Direction of Technical Change". *Quarterly Journal of Economics*. Vol. 120 (2), May, pp. 517-549.

[9] Lang, K. (1987). "Job Signalling and Welfare Improving Minimum Wage Laws". *Economic Inquiry*, Vol. 25, pp 145-158.

[10] Mankiw, N. G., D. Romer and D. N. Weil (1992). "A Contribution to the Empirics of Economic Growth". *Quarterly Journal of Economics*, Vol. 107, No. 2., pp. 407-437.

[11] Ravn, M. O. and J. R. Sorensen (1999). "Schooling, Training, Growth and Minimum Wages". *Scandinavian Journal of Economics*, Vol. 10 (3), pp. 441-457.

[12] Rebitzer, J. and L. Taylor (1995). "The Consequences of Minimum Wage Laws: Some New Theoretical Ideas". *Journal of Public Economics*, February, pp. 245-255.

[13] Samuelson, P. A. (1958). "An Exact-Consumption Loan Model of Interest with or without the Social Contrivance of Money". *Journal of Political Economy*, Vol. 12, pp. 1-26.

[14] Stigler, G. J., (1946). "The Economics of Minimum Wage Legislation". *American Economic Review*, Vol. 36, pp. 358--365.

[15] Swinnerton, K. A. (1996). "Minimum Wages in an Equilibrium Search Model with Diminishing Returns to Labor in Production". *Journal of Labor Economics*, Vol. 14, No. 2, pp. 340-355.

[16] West, E. G. and M. McKee (1980). "Monopsony and "Shock" Arguments for Minimum Wages". *Southern Economic Journal*, Vol. 46, pp. 883-891.