# Dual economies, Kaldorian underemployment and the big push

Giovanni Valensisi*

October 2007

*This paper develops a two-sector model with specific factors of production, featuring diminishing returns and standard wages in agriculture, while increasing returns and efficiency wage mechanisms in industry. The asymmetric interaction of the two sectors is such that, under plausible parametrization, the model may display multiple equilibra and a low-development trap. Additionally, parametric increases of sectoral TFP may reduce the basin of attraction of the low-equilibrium and increase the steady state capital stock (and wage level) for the stable equilibrium of full industrialization.*

## I. *Introduction*

The concept of poverty trap has been used fruitfully since the very dawn of development economics, and implicitly it can be traced back even to Adam Smith's "Early draft of part of the Wealth of Nations"[1]. Starting with the seminal paper of Rosenstein Rodan (1943), the idea that underdevelopment could constitute a state of equilibrium thrived with Nurkse's *vicious circle of poverty* (1953) and Nelson's *low-level equilibrium trap* (1956). Several mechanisms, essentially concerning increasing returns pecuniary externalities or demographic traps, were from time to time held responsible for creating a multiplicity of equilibra, and possibly preventing the spontaneous development of certain economies[2].

*Dipartimento di Economia Politica e Metodi Quantitativi; University of Pavia; (giovanni.valensisi@eco.unipv.it).
I am gratefully indebted to Gianni Vaggi, Marco Missaglia, Amit Bhaduri, Jaime Ros, Paolo Bertoletti, Carluccio Bianchi and Mario Cimoli for their helpful comments. I also benefitted from the suggestions of Alberto Botta, Francesco Bogliacino, Luca Mantovan, Lorenza Salvatori, Chiara Valensisi, Francesco Magalini and Sara Baroud. The responsibility of remaining errors is of course mine.

[1] In Smith (1763) page 579 the author argues:

"That is easier for a nation, in the same manner as for an individual, to raise itself from a moderate degree of wealth to the highest opulence, than to acquire this moderate degree of wealth."

[2] Authors such as Young (1928), Rosenstein-Rodan (1943) and Nurkse (1953) emphasized the importance of increasing returns, while Nelson (1956), Jorgenson (1964) and - later - Dixit (1970) focused on the role of demographic dynamics in creating poverty traps, in which economic growth in absolute terms is balanced by the counteracting dynamics of population, so that GDP per capita remains at a

Despite the deep interest enjoyed by the so-called "high development theory" in the fifties, its predominantly discursive argumentation jointly with the difficulty to reconcile increasing returns with competitive market structures[3] contributed to its decline in favor of the more rigorous paradigm, described by Solow (1956) and Swan (1956). Notwithstanding many important contributions on the role of increasing returns and learning by doing, during the 60's the mainstream approach to growth became that of the neoclassical convex economy converging to a stable and unique steady state.

Additionally, attention shifted from the "developmental perspective" - emphasizing the interactions between "sectoral balances" (in terms of labor, goods and saving flows) along the process of industrialization, as well as the dualistic nature of the economies of developing countries - to an aggregate growth perspective - focusing more on reproducible factors' accumulation, and on the determinants of the steady state. In this respect, the use of a linearly homogeneous aggregate production function requires great caution, because of the subtle but delicate implications of such choice in terms of positive theory[4]. More importantly, the choice of an aggregate model overlooks the empirically-founded recognition that economic growth goes hand in hand with structural change, meaning that "the permanent changes in the absolute levels of basic macro-economic magnitudes are invariably associated with changes in their composition, that is, with the *dynamics* of their *structure*"[5]. Obviously, the importance of structural dynamics is reinforced *a fortiori*, when referring to developing economies that are undergoing a process of industrialization.

Regardless of the possible limits of aggregate models, also the mainstream approach has played a key role in bringing back to the center of the attention the issue of increasing returns, along with their crucial implications for multiple equilibra. The twist away from the traditional paradigm of conditional convergence occurred in the mid 80's, when endogenous growth theory stressed the role of knowledge and human capital, be it a sort of separate product (different from the composite good) or simply the stock of experience and learning by doing. The assumption of increasing returns to reproducible factors, where the latter typically include knowledge, in addition to physical capital, responded to the need to rationalize two elements of industrial economies that were basically assumed exogenously in Solow's conceptual framework: the persistence of growth even after the capital labor ratio has reached fairly high levels, and the continuity (or possibly even the acceleration) of technical change. In light of this, endogenous growth theory was mostly concerned with issues other than explaining the take off of initially poor countries, consequently it focused more on the properties of the steady

low level.

[3]Notably during the 50's and 60's only demographic traps had been analyzed in mathematical form, while poverty traps based on increasing returns, specialization and pecuniary externalities were treated only in narrative contributions.

[4]Solow himself recognized the difficulty to apply a linearly homogeneous aggregate production function to both agriculture and industry. The point is raised in two different articles: Solow (1956) page 67 cautions about applying an aggregate production function, which is linearly homogeneous, to the case in which production depends on a "nonaugmentable resource like land"; Solow (1957) page 314 states the need to net out agricultural contribution to GDP when applying the aggregate production function to the analysis of real economies.

[5]The quotation is taken by Pasinetti (1993), italics in the original.

state path, rather than on the possible obstacles to industrialization. This different *raison d'être* also explains why early endogenous growth models featured predominantly a one sector or "quasi-one-sector" set-up[6], omitting by definition any possible role for labor reallocation and structural change.

In any case, in the mid 90's concepts like poverty traps, structural change and multiplicity of equilibra recovered a central role in the debate about economic growth, leading to what has been called a "counter-counterrevolution in development theory"[7]. On the one hand, it had been shown that even in the standard neoclassical set-up (one sector with convex technologies operating under perfect competition), multiple equilibra cannot be excluded *a priori* once empirically significant elements such as heterogeneity in saving behavior, low elasticity of technical substitution, or capital market imperfections are taken into account[8]. On the other hand, advances in the theoretical analysis of non-perfectly competitive market structure, jointly with a new stream of literature on structural change, caused a revival of categories characterizing the "high development theory"[9].

The renewed interest in poverty traps came also under the pressure of empirical literature, which increasingly questioned the validity of the neoclassical paradigm of conditional beta-convergence across countries, in favor of more complex dynamics able to generate convergence clubs and twin peaked distributions. In the literature two approaches to growth empirics have confronted each other, differing in the instruments used as much as in the results obtained. Cross-country regressions - widely employed to support the validity of the neoclassical hypotheses - seem to confirm that economies tend to converge to their own steady state at a rate consistent with the "augmented versions" of the Solow model, once controlling for the determinants of the steady state itself: typically the saving rate, the initial level of human capital, political stability and degree of price distortion[10]. On the other hand, different studies based on inference about the ergodic distribution of stochastic Markovian processes of growth tend to reveal the formation of a bimodal distribution of per capita GDP, entailing the creation of

---

[6]The expression "quasi-one-sector" refer to those models where the economy produces both knowledge (a production input) and one composite good that can be both consumed or invested. See Ros (2001) page 10.

[7]See Krugman (1992).

[8]See Galor (1996), and later Azariadis (2005), Easterly (2006), Kraay and Raddatz (2007)

[9]Among the mechanisms proposed to justify the existence of multiple equilibra, and possibly of poverty traps, we may cite: technological non-convexities (see Murphy, Shleifer, Vishny (1989); Azariadis, Drazen (1990); and Ros, Skott (1997)), saving based poverty traps with subsistence consumption (see for instance Ros (2001)), learning by doing, knowledge or search externalities (see Matsuyama (2002), Stokey (1988), Kremer (1993)), credit market imperfections (see Galor, Zeira (1993); Banerjee, Newman (1993) and Aghion, Bolton (1997)), and institutional traps (see Murphy, Shleifer, Vishny (1993)). In addition to these elements, the literature on structural change adds two other factors that may explain important facets of the development process: the role of sector-specific technical change (Matsuyama 1992, Hansen and Prescott 2002), and the introduction of new goods (Stokey 1988 and Matsuyama 2002).

[10]See among others Barro (1991); Mankiw, Romer, Weil (1992); Barro, Sala-i-Martin (1995); Sala-i-Martin (1996); Easterly (2006).

two different *convergence clubs*[11]. While not necessarily incompatible with neoclassical growth models, the existence of convergence clubs seems to come at odds with the traditional paradigm of conditional $\beta$-convergence, while it rationalizes immediately the observed absolute $\sigma$-divergence across countries[12].

In light of the long standing debate summarized above, in this paper we aim at reconciling the "developmental perspective" (with its emphasis on structural change entailed by industrialization) and the neoclassical theory of growth (highlighting the role of reproducible factors' accumulation). In particular, while taking advantage of recent contributions on structural change and endogenous growth, we retain from the early development literature the dualistic set-up with its asymmetric treatment of agriculture and industry, in order to highlight the role played by factors' reallocation in the early phases of industrialization[13]. We do so by developing a specific-factor macromodel à la Ricardo-Viner-Jones, which under plausible parametrization may display multiple equilibra and poverty trap. For several aspects our set-up resembles the "Rosenstein-Rodan / Leibenstein model" formulated in Ros (2001); however we depart from it in adopting a sociological theory of efficiency wage and eliminating the recourse to the Lewisian labor surplus. These choices allow us to generalize Ros's results while adopting a fully neoclassical formalization - with flexible prices, perfect competition and under the marginal theory of distribution.

The paper is organized as follows: section II outlines the macromodel and the determination of the equilibra, section III explains the effect of exogenous technical progress (here intended as a parametric increase of sectoral TFP)in each of the two sectors, section IV concludes and draws policy implications.

# II. *The model*

### PREFERENCES

The economy consists of two sectors, agriculture and industry, producing respectively food - a consumption good - and manufactures, which can be used alternatively for personal consumption or as investment goods. A traditional Cobb Douglas utility function is used to describe consumers' preferences across goods:

$$U = (X_a^c)^{\alpha} \;\; (X_i^c)^{1-\alpha} \, ;$$

[11]See Quah (1993 and 1996); Ros (2001); Azariadis, Stachurski(2005); Azariadis(2005).

[12]At this regard, Azariadis (2005) states:

"If one excludes East and Southeast Asia from the sample, then the group of less developed countries is not catching up to the OECD nations unless one controls for a long, and not altogether meaningful, list of differences in structural features."

[13]The emphasis on the asymmetries - technological as well as organizational - between agriculture and industry is the distinctive feature of dual economy models, among which Lewis (1954 and 1958), Ranis and Fei (1961), Jorgenson (1961), Preobrazensky (1965), Kaldor (1967 and 1968) and Dixit (1970).

where $X_a^c$ and $X_i^c$ represent respectively the amount of food and manufactures consumed, while $\alpha$ is the constant expenditure share for food. Through standard utility maximization under budget constraint, representative consumers' demand can be shown to be:

$$\frac{\alpha}{1-\alpha}\frac{X_i^c}{X_a^c} = \frac{P_a}{P_i}; \tag{1}$$

where $P_a/P_i$ denotes the agricultural terms of trade (the relative price of food with respect to manufactures). Consistently with the above specification of demand, the corresponding price index $\underline{P}$ is

$$\underline{P} = P_a^{\alpha} P_i^{1-\alpha}. \tag{2}$$

## TECHNOLOGIES

The agricultural sector produces food employing a backward technology that uses labor and land, but has no scope for reproducible inputs[14]. Agricultural production function is thus given by

$$X_a^s = A_a L_a^{1-b}; \qquad\qquad 0 \leq b < 1 \tag{3}$$

where $X_a^s$ denotes food output, $L_a$ the labor employed in agriculture, $1-b$ and $A_a$ are two technological parameters describing respectively the degree of returns to labor and the sectoral TFP (which in the case of agriculture summarizes both technological factors but also geographical and climatic conditions). The restriction on the parameter $b$ derives from the hypothesis that land endowment is fixed even in the long-run[15], and implies decreasing returns to labor ($b = 0$ is a limiting case, representing constant return to labor).

For what concerns the industrial sector, firms utilize labor (in efficiency units) and capital in the production of manufactures. The manufacturing sector is assumed to exhibit aggregate increasing returns to scale due to a *positive external effect of capital*[16] captured by a Kaldor-Verdoorn coefficient, which is explained by the presence of "capital-embodied-knowledge". In other words, we assume that the stock of knowledge is proxied by the average economy-wide stock of capital, and that investments in capital goods translate automatically into improvements of the industrial TFP at a constant rate $\mu$ (precisely the Kaldor-Verdoorn coefficient). The present formalization is equivalent to assume a learning by doing process, in which the cumulative gross investment represents the index of experience, and knowledge depreciates at the same

[14] The absence of capital among agricultural inputs is evidently unappropriate for high and middle income countries displaying capital-intensive techniques of cultivation (which is the case, for example, in many Latin American nations), however it represents a suitable approximation for less developed countries (LDC). Such assumption is widely adopted in the literature regarding dual economies; obviously, however, it restricts the relevance of the present model to those countries, where subsistence agriculture is especially widespread and the scarce physical capital is employed in non-agricultural activities: predominantly South Asian and Sub-Saharan African countries.

[15] The fixed argument "land" has been omitted from the production function to lean down the notation.

[16] Concerning technological external economies, see Marshall (1920) and Scitovsky (1954).

rate as physical capital[17].

In accordance with the previous discussion, the industrial technology is described by a Cobb Douglas production function

$$X_i^s = A_i \tilde{K}^\mu K^\beta \left(E_{(w_i,w_a)} L_i\right)^{1-\beta}; \qquad \mu > 0, \quad 0 < \beta < 1;$$

where $X_i^s$, $L_i$ and $K$ denote respectively manufactures output, industrial labor and capital stock, while the function $E_{(w_i,w_a)}$ represents labor efficiency, the parameters $\beta$, $(1-\beta)$ and $A_i$ are respectively the capital and labor shares, and the industrial TFP[18], and finally $\tilde{K}^\mu$ represents the external positive effect of capital accumulation, $\tilde{K}$ being the average capital stock of our economy.

The fact that technological economies are <u>external</u> to each firm derives from assuming, that the non-rival and non-excludable nature of knowledge is such that the experience acquired by one firm spills over <u>completely</u> and <u>immediately</u> to the others, exerting a positive externality on all manufacturing producers[19]. In light of this, we can argue that in equilibrium the average capital stock of the economy will match that of the representative firm; accordingly, the industrial production function can be rewritten as

$$X_i^s = A_i K^{\mu+\beta} \left(E_{(w_i,w_a)} L_i\right)^{1-\beta}; \qquad \mu > 0, \quad 0 < \beta < 1 \tag{4}$$

Clearly, as long as $\mu > 0$ the above production function displays aggregate increasing returns, though not necessarily constant or increasing returns to capital, as typically assumed in endogenous growth models à la Romer or in AK models[20].

Concluding the analysis of technologies, it is straightforward to see that capital accumulation will not trigger a "homotetic growth" for the economy as a whole, precisely because in this set-up reproducible inputs are specific to only one sector: industry. Unlike in aggregate models, here the accumulation of reproducible factors affects asymmetrically the marginal productivity of labor in the two sectors, leaving the burden of

[17]In this respect, the present model differs from both Arrow's original approach (1962), in which experience is also proxied by cumulative gross investment but without knowledge depreciation, as well as from recent models of structural change that disregard the idea of capital embodied knowledge and relate the learning process to cumulative output (for instance Krugman 1987, Stokey 1988, Matsuyama 1992 and 2002).

[18]Note that, because of the algebraic properties of Cobb Douglas production functions, all forms of technical change - unbiased, labor augmenting and capital augmenting (also called Hicks neutral, Harrod neutral and Solow neutral) - translate into variations of the parameter $A$, and are thus essentially indistinguishable from one another.

[19]Despite the caveats about some more realistic refinements of the learning by doing process, the hypothesis of complete knowledge spillovers is quite commonly used in the structural change literature (see Krugman 1987, Matsuyama 1992, 2002, Stokey 1988) for it allows to concentrate on the impact of increasing returns without further analytical complications as regards the market structure.

[20]In this way, the formalization of increasing returns overcomes the problem of excessive sensitivity to restrictive parametrization, unlike the whole class of AK models, which necessarily require constant returns to capital. See Stiglitz (1992) and Solow (1994) for a critique of AK models in this respect. Obviously increasing returns to capital arise here only if $\mu > 1-\beta$, with equality yielding constant returns to capital.

equilibrium adjustment to labor reallocation, capital-labor substitution (in industry) and eventually to price adjustments. At the same time, resource reallocation across sectors determines a change in output composition and employment shares.

## DISTRIBUTION AND LABOR MARKET

In line with the traditional literature on dual economies, distributive issues and "organizational asymmetries" between agriculture and industry play a key role in the present model, especially as concerns the labor market. In this respect, however, our approach here departs from the debated hypothesis that rural wages are determined à la Lewis by the average productivity of labor, giving rise to labor surplus[21]. Instead, we suppose that landlords maximize their rents, hiring all available labor and paying it at a wage rate equal to the marginal revenue product. Analytically we will thus have:

$$W_a = (1-b)\, A_a\, (L_a)^{-b}\, P_a; \tag{5}$$

and

$$R = b\, A_a\, (L_a)^{1-b}\, P_a = \frac{b}{1-b}\, W_a\, L_a; \tag{6}$$

where $W_a$ represents the rural wage in nominal terms and $R$ the rents.

Organizational dualism comes into play as regards wage determination in the industrial sector, where we assume the existence of an efficiency mechanism, linking labor productivity with the wage received. While such mechanism does not seem appropriate for the agricultural sector in LDCs, dominated by casual labor and informal relations, it is indeed much more credible for the formal labor markets of the urban industrial sector[22]. In light of such wage-productivity linkage, the problem faced by industrial entrepreneurs will be

$$\max_{L_i, Wi}[\Pi] = A_i\, K^{\mu+\beta} \left(E_{(w_i,w_a)}\, L_i\right)^{1-\beta} P_i - L_i\, W_i; \quad \textit{subject to} \quad Wi \geq W_a$$

where upper-case $W$ indicates wages in nominal terms (lower-case $w$ are expressed in real terms), and $E_{(w_i,w_a)}$ is a non-decreasing function relating workers' efficiency with the real wage they receive, and with the real wage they could get if working in agriculture. Notably, the problem faced by industrial entrepreneurs is a constrained maximization, since they cannot hire any worker at a wage lower than the reservation wage the latter could get in agriculture.

Consistently with Akerlof's interpretation of labor contracts as partial gift exchanges, the effort function $E_{(w_i,w_a)}$ reflects those sociological considerations (including the real wages paid in the other sector of the economy) that govern the determination of work norms, and hence regulate labor productivity[23]. Suppose additionally that the

[21] The labor surplus assumption is followed also by Ros in his "Rosenstein Rodan-Leibenstein model"; see Ros (2001).

[22] This was already noted by Mazumdar (1959) and is confirmed by Rosenzweig (1988) and Basu (1997).

[23] In Marxian terminology this function may be viewed as governing labor extraction from labor power; see Bowles (1985).

effort function takes the convenient form

$$E(W_i) = \begin{cases} 0; & \text{for } W_i < \omega^{\frac{1}{d}} W_a^{\gamma} \underline{P}^{1-\gamma} \\ \left[\dfrac{W_i/\underline{P}}{(W_a/\underline{P})^{\gamma}}\right]^d - \omega; & \text{for } W_i \geq \omega^{\frac{1}{d}} W_a^{\gamma} \underline{P}^{1-\gamma} \qquad 0 < d, \gamma < 1 \quad \omega > 0 \end{cases} \tag{7}$$

in which the parameter $\omega$ implies a minimum threshold to obtain positive effort (see the piecewise definition of the effort function), $d$ is a positive parameter and is lower than one to ensure the effort function to be well-behaved (meaning increasing and concave), and $\gamma$ represents the elasticity of industrial real wage to agricultural one. This specification is a generalization of the effort function proposed by Akerlof (1982), and opens the additional possibility of having a less that proportional relationship between the wage received by industrial workers, and the wage they would receive if employed in agriculture[24].

Under the above assumptions, and as long as the constraint $Wi \geq W_a$ is not binding, the FOC for their profit maximization problem imply the Solow condition of unitary wage elasticity of effort, which ensures cost minimization

$$W_i = \left(\frac{\omega}{1-d}\right)^{\frac{1}{d}} W_a^{\gamma} \underline{P}^{1-\gamma}; \tag{8}$$

plus the usual labor demand function

$$L_i = (1-\beta)^{\frac{1}{\beta}} A_i^{\frac{1}{\beta}} (E^*)^{\frac{1-\beta}{\beta}} K^{\frac{\mu+\beta}{\beta}} (W_i)^{-\frac{1}{\beta}} P_i^{\frac{1}{\beta}}; \tag{9}$$

where $E^* \equiv d\omega/(1-d)$ is the effort level corresponding to $W_i$. Given that the second order conditions are met for the assumed well-behaving production and effort functions, and that the constraint is satisfied for the assumed values of $d$, the FOC define the solution of the above profit maximization.

Figure 1(a) represents the diagram corresponding to our specification of effort function on the $W_i - E$ space. The payroll cost per efficiency unit of labor corresponding to each point of the effort function is given by the coefficient of the ray from the origin to the same point. Clearly the optimal wage (indicated in the graph as $W_i^*$) corresponds to the point of tangency between the ray and the effort function, since the said coefficient is at its minimum attainable level[25]. Figure 1(b) instead represents the correspondent industrial labor demand on the $W_i - L_i$ space: at $W_i^*$ the labor demand schedule has a kink, because entrepreneurs will resist any wage undercutting and keep the wage at its optimal level, since wages different than $W_i^*$ would not minimize the cost of labor per efficiency unit and consequently will not be profit maximizing.

[24]Note that Akerlof's formalization can be obtained by simply assuming $\gamma = 1$, entailing the perfect proportionality of industrial wages and agricultural ones. Apart from this aspect, the rationality for choosing the above specification is the usual one: the threshold $\omega$ is included to avoid the trivial solution of an optimal zero wage (see Akerlof (1982) for more details), and the restrictions on $d$ are needed to ensure the existence of a unique internal maximum.

[25]It should be noted, however, that the effort function depends on the real agricultural wage ($W_a/\underline{P}$) and on the price index $\underline{P}$, so that the optimal industrial wage itself is increasing in ($W_a/\underline{P}$) and $\underline{P}$.

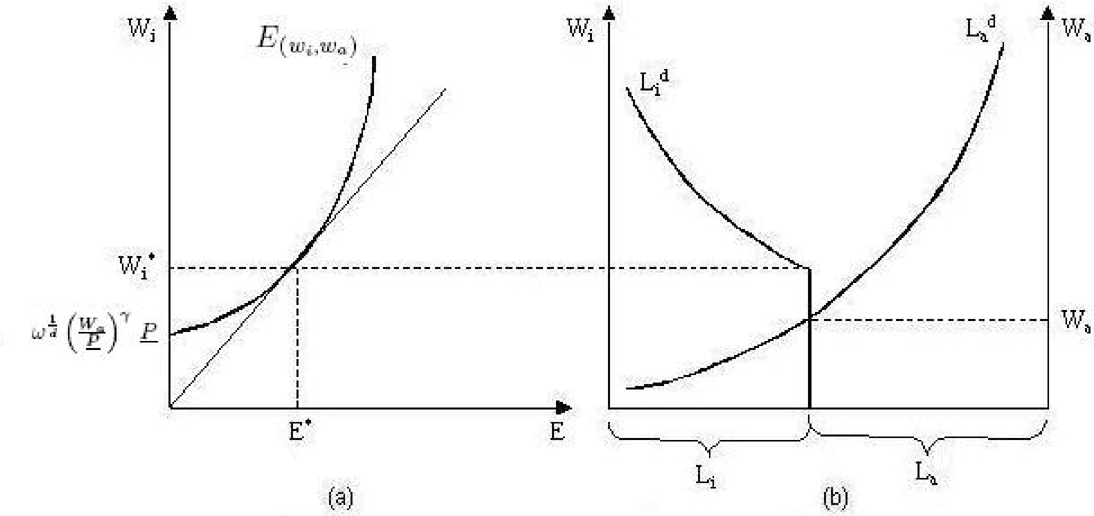


Figure 1: The efficiency wage mechanism

Unless the constraint forces them to act differently, capitalists will hence set the wage at $W_i^*$; as a result of the downward rigidity of the industrial wage, high-earning jobs will be rationed and only $L_i^*$ workers will be hired. The remaining workers will be all employed in the rural sector at the wage that clears the labor market (see the $L_a^d$ curve in figure 1b), so that a wage gap will arise endogenously across sectors. Clearly, the position of the $L_i^d$ curve depends, among other factors, on the existing stock of capital, with a higher $K$ causing, *ceteris paribus*, an outwards shift of the curve and hence an increase in $L_i$.

The adjustment process described so far, follows Kaldor's insights according to which employment creation in the manufacturing sector of typical developing countries is constrained by demand and not by supply factors[26]. For this reason, the phase in which $W_a < W_i$ and wage gaps arise across economic sectors, will be called hereafter *Kaldorian underemployment*[27]. Moreover, Kaldorian underemployment refers to a situation in which

"... a faster rate of increase in the demand for labour in the high-productivity sectors induces a faster rate of labour- transference even when it is attended by *a reduction,*

[26]Quoting Kaldor's own words: "... the supply of labour in the high-productivity, high-earning sector is continually in excess of demand, so that the rate of labour-transference from the low to the high-productivity sectors is governed only by the rate of growth of demand for labor in the latter."(1968) See also Kaldor (1967).

[27]Kaldor actually calls this situation "labor surplus", but we preferred a different definition, in order to avoid confusion between the notion applied here, and Lewis's concept of surplus labor. Clearly, the notion of Kaldorian underemployment is logically tied to that of disguised unemployment, but in the present case the mismatch between the shadow wage (that is the opportunity cost of labor outside the modern sector) and the market wage in the industrial sector occurs without any breach of the marginal theory of distribution.

*and not an increase, in the earnings-differential between the different sectors.*"[28].

The complete analytical description of the inputs market during the Kaldorian underemployment phase requires to derive, in addition to equations 5, 6, 7, 8 and 9, the profit rate and the labor market clearing, which are respectively given by

$$r = \frac{\beta}{1-\beta} \frac{W_i L_i}{K}; \tag{10}$$

and

$$L_i + L_a = 1. \tag{11}$$

Note that in the last equation we have normalized the labor force to 1, so that $L_a$ and $L_i$ respectively represent the employment share of the traditional and of the modern sector; this simplifying normalization, however, comes at the cost of eliminating the effect of demographic variables on our economy.

It should be clear at this point, that Kaldorian underemployment persists only as long as the solution implied by the FOC is admissible, that is as long as $W_a < W_i$. Given the hypothesis of diminishing returns to labor in agriculture, however, the withdrawal of labor from the rural sector is bound to increase $W_a$; moreover, since the elasticity of industrial wages to rural ones is lower than one, eventually the latter will reach $W_i$ and the constraint will become binding. With reference to figure 1b, the expansion of the industrial sector (a shift of the $L_i^d$ curve toward north-east) tends to close the wage gap, until eventually one uniform wage prevails. Indeed, capitalists are then compelled to pay workers a wage equal to the agricultural one, and the Kaldorian underemployment phase gives way to the *economic maturity*: "a state of affairs where real income per head had reached broadly the same level in the different sectors of the economy"[29]. During the maturity phase employees will be indifferent between working in industry or in agriculture, and thus lack any incentive to increase their effort beyond $E^*$, despite any possible increase in the uniform real wage rate.

In light of this reasoning, during the maturity phase wages will be set at

$$W_i = W_a; \tag{12}$$

while industrial labor demand and profit rate will still be determined by the same equations holding during Kaldorian underemployment (equations 9 and 10), with the only caveat that now the uniform wage rate replaces the value of $W_i$ determined according to efficiency considerations. Obviously, the rural wage and rents determination, and the labor market clearing will hold also during maturity, so equation 5, 6 and 11 complement the description of the labor market.

## MARKET CLEARING

The complete characterization of the economy in the short-run involves two more equations related to the market clearing for final goods: assuming that the economy

[28]The quotation is taken from Kaldor (1968) page 386, italics in the original.

[29]The quotation is Kaldor's own definition of economic maturity, which he also defined as "the end of the dual economy" (1968).

is closed to international trade, such conditions are stated directly for food output, and by mean of the consumption expenditure flow identity as concerns manufactures. In determining the proportion of income devoted to personal consumption, we assume that wage income as well as rents are entirely consumed, while profit-earners save a constant proportion $s$ of their total income $\Pi \equiv r\,K$. Our system will therefore be completed by the following two equations:

$$X_a^c = X_a^s; \tag{13}$$

for the food market (with the $d$ and $s$ suffixes meaning respectively demanded and supplied), and

$$P_a\,X_a^c + P_i\,X_i^c = W_a\,L_a + R + W_i\,L_i + (1-s)\,\Pi; \tag{14}$$

for manufactures[30]. We note by passing that Walras law can be used to take manufactures as the numeraire, in order to have the industrial product wage equal to its nominal value, so that

$$P_i = 1; \quad \underline{P} = P_a^{\alpha}. \tag{15}$$

## DYNAMIC OF CAPITAL STOCK

As concerns the dynamic of the state variable $K$, we follow the usual assumption that savings are automatically reinvested into increases of the capital stock. Combining this hypothesis with those underlying equation 14 we can describe the dynamic the capital stock as

$$\dot{K} = s\Pi - \delta K;$$

where $\dot{K}$ is the time derivative of the capital stock, and $\delta$ expresses the depreciation rate of capital. Denoting by $\hat{K}$ the capital growth rate, the dynamic of the capital stock may be rewritten as

$$\hat{K} = s\frac{\Pi}{K} - \delta = sr - \delta. \tag{16}$$

This equation represents the fundamental differential equation of our model, and corresponds to the well-know Solow-Swan equation.

Stated as it is, the present model belongs to the category of the "supply-limited models of industrial growth" - using Taylor's jargon - with market-clearing prices and flexible capital labor ratio, as opposed to the fix prices and technological coefficients characterizing the structuralist literature. In any case, it is important to emphasize that the choice of a supply-limited model in this context is not meant to undervalue the importance of keynesian arguments concerning the level of effective demand, but

[30] We can clarify the reason for closing the model using the consumption flow identity, by making use of some relations explained above: equations 5 and 6, together with food market clearing (equation 13) imply that $W_a L_a + R = P_a\,X_a^s = Pa\,X_a^c$; while equation 10 implies that during Kaldorian underemployment $\Pi = W_i L_i \beta/(1-\beta)$. Analogous implications hold during maturity, with the only difference that the wage rate is then common across the two sectors for equation 12. Regardless of the economic phase, hence, equation 14 can be rewritten as $X_i^s - X_i^c = s\Pi$, which shows that in equilibrium the excess supply of manufactures shall equate the total amount of savings of the profit-earners.

only to focus our attention on the *potential* growth path of an economy. Apart from the presence of increasing returns in industry, the distinctive feature of this model is the peculiar characterization of the labor market; it is this aspect that permits us to rationalize one crucial insight of the "dual economy literature": the mismatch, which exists at low level of development, between the labor productivity in the modern sector and the correspondent opportunity cost of labor in the traditional agricultural sector.

## THE EQUILIBRIUM CONFIGURATION

Analytically, the economy in the short-run is described by a system of twelve independent equations (remember that Walras law was used to define the numeraire in equation 15), with 11 endogenous variables ($X_a$, $X_i$, $L_a$, $L_i$, $P_a$, $W_a$, $R$, $W_i$, $E$, $\Pi$, $r$) plus the capital stock $K$ which is pre-determined in the short-run, and whose long-run dynamic is given by equation 16.

Instead of directly solving the whole system of equations and determine the steady states, we prefer to proceed in a slightly different way to highlight the different economic mechanisms at work in the development process. We will first determine the nominal industrial wage consistent with the clearing of the goods'market for each given level of capital stock (hereafter the correspondent locus of short-run equilibra in the $\log W_i - \log K$ space is called **real wage schedule** and indicated as RW); secondly, we obtain from the dynamic equation 16 the **locus of stationary capital stock**, which gives the value of the nominal industrial wage ($W_i$) corresponding to the break-even situation with null <u>net</u> investment. Finally confronting the relative position of the two loci, we will determine the steady state equilibra and their stability properties. Clearly, because of the dicothomic working of the labor market before and after the maturity threshold $W_a = W_i$, the equilibra shall be derived separately for the two phases.

As emphasized by classical authors (Malthus, Marx and Ricardo above all) and by development economists of the 50's and 60's (Lewis, Ranis and Fei, Nurkse, Jorgenson), the elasticity of industrial labor supply is the pivotal magnitude summarizing the economic mechanisms at work. Its crucial role is evident once we note that in two-sectors macromodels - unlike in aggregate models - this elasticity depends on the interaction between technological conditions (namely the evolution of labor productivity across sectors), demographic variables, and movements in relative prices, while it concurs to determine the speed of labor reallocation across sector, and the effect of such reallocation in terms of profitability.

During the Kaldorian underemployment phase, the twelve independent equations describing the system in the short-run are 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14 (plus equation 15 which defines the numeraire). After some algebraic manipulations (see the Mathematical Appendix I.A) it can be shown, that the elasticity of labor supply faced by industrial entrepreneurs during Kaldorian underemployment is equal to

$$\epsilon^{LS} \equiv \frac{\partial \log L_i}{\partial \log W_i} = \frac{(1-\alpha)\,(1-\gamma)\,(1-L_i)}{\gamma\, +\, \alpha(1-\gamma)(1-bL_i)}; \tag{17}$$

Two observations are straightforward: labor supply elasticity is non negative for the assumed parametrization (as expected), and is a decreasing function of the food expenditure share $\alpha$, of the elasticity of industrial wage to that of the agricultural sector $\gamma$, and of the industrial employment share $L_i$. The negative dependency of $\epsilon^{LS}$ on $L_i$ (on $\alpha$) arises because *ceteris paribus* a higher industrial labor share (a higher food expenditure share) turns relative prices in favor of agriculture, hence the nominal wage $W_i$ will have to grow proportionally more to attract additional workers to industry. The economic reason behind the negative dependency of $\epsilon^{LS}$ on $\gamma$ lies instead in the fact that *ceteris paribus*, a higher $\gamma$ makes industrial wages more sensitive to agricultural ones, so that an increase in $L_i$ triggering a correspondent raise in $W_a$, will in turn increase industrial nominal wages even faster.

Continuing with a bit of algebra (see the Mathematical Appendix I.B), it can be demonstrated that the equation of the short-run equilibrium in log terms is given by

$$\frac{\gamma+\alpha(1-\gamma)(1-b)}{\gamma+\alpha(1-\gamma)} \log \left[1 - A_i^{\frac{1}{\beta}} (1-\beta)^{\frac{1}{\beta}} (E^*)^{\frac{1-\beta}{\beta}} \exp\left(-\frac{1}{\beta}\log W_i + \frac{\mu+\beta}{\beta}\log K\right)\right] +$$
$$+ \log\,[Q\, A_a^{\frac{\alpha(1-\gamma)}{\gamma+\alpha(1-\gamma)}}\, A_i^{-\frac{1}{\beta}}] - \frac{\mu+\beta}{\beta}\log K + \left[\frac{(1-\alpha)(1-\gamma)}{\gamma+\alpha(1-\gamma)} + \frac{1}{\beta}\right]\log W_i = 0; \quad (18)$$

where $Q$ is a constant defined as

$$Q = \frac{1-\alpha}{\alpha}\frac{(1-\beta)^{-\frac{1-\beta}{\beta}}}{1-s\beta} \left[\frac{(1-d)^{\frac{1}{d}}}{\omega^{\frac{1}{d}}(1-b)^{\gamma}}\right]^{\frac{1}{\gamma+\alpha(1-\gamma)}} (E^*)^{-\frac{1-\beta}{\beta}}.$$

Total differentiation of equation 18 yields the coefficient of the real wage schedule, which is equal to

$$\frac{\partial \log W_i}{\partial \log K} = \frac{\mu+\beta}{1+\beta\epsilon^{LS}} = \frac{(\mu+\beta)\,[\gamma+\alpha(1-\gamma)(1-bL_i)]}{\gamma+\alpha(1-\gamma)(1-bL_i)+\beta(1-\gamma)(1-\alpha)(1-L_i)}. \quad (19)$$

This coefficient is surely positive, given that the labor supply elasticity is non-negative, and furthermore it is decreasing in $\epsilon^{LS}$. Indeed, a given increase in the capital stock will trigger an outflow of labor from agriculture[31], and the higher the elasticity of industrial labor supply the smaller - *ceteris paribus* - the adjustment in nominal industrial wages required by the expansion $L_i$. Besides, since a raise in industrial employment reduces $\epsilon^{LS}$, the real wage schedule will be flatter for low levels of industrial labor share, and get gradually steeper as the industry expands its employment basin. On the other hand, the higher $\mu$, the stronger the external capital effects prompted by the given augment in the capital stock, and the higher the industrial wage in equilibrium; hence the greater the coefficient of the real wage schedule.

In plain words during Kaldorian underemployment higher values of the capital stock trigger the expansion of the industrial labor share and of industrial output, leading the agricultural terms of trade to augment; this relative price movement, summed to the withdrawal of labor from agriculture, causes a sharp raise of the rural wage. Both these forces drive the upwards adjustment of industrial wages to satisfy the Solow condition. As shown in Mathematical Appendix I.C, the adjustment process required to get the equilibrium in the goods' market is such that higher levels of $K$ entail a reduction in the wage (and productivity) gap between manufacturing and agricultural activities, to the

[31] Industrial labor demand depends positively on the capital stock $K$ (see equation 9).

extent that for sufficiently high capital stock a unique uniform (and labor productivity) will prevail in the economy.

Once this happen, and the constraint $W_a = W_i$ becomes binding, the system enters the maturity phase and the above equilibrium configuration ceases to hold. The mature economy in the short-run is still described by equations 1, 3, 4, 5, 6,, 7, 9, 10 11, 13, 14, 15, but now equations 12 replaces equation 8. As shown formally in Mathematical Appendix II.A, the prevalence of one uniform wage alters significantly the dynamic in the labor market: sectoral labor shares stabilize at the constant level

$$\bar{L_i} = \frac{(1-\alpha)(1-\beta)}{(1-\alpha)(1-\beta)+\alpha(1-b)(1-s\beta)}; \qquad \bar{L_a} = 1-\bar{L_i}; \tag{20}$$

regardless of the capital stock, while the labor supply elasticity turns to zero[32].

The null elasticity of industrial labor supply during maturity modifies also the real wage schedule, whose equation is

$$\log\bar{L_i} - \frac{1}{\beta}\log(1-\beta) - \frac{1}{\beta}\log A_i + \frac{1}{\beta}\log W_i - \frac{\mu+\beta}{\beta}\log K - \frac{1-\beta}{\beta}\log E^* = 0; \tag{21}$$

from which we see that the RW curve on the usual $\log W_i - \log K$ plane degenerates into a half-line sloped

$$\frac{\partial \log W_i}{\partial \log K} = \mu + \beta; \tag{22}$$

as formally proved in the Mathematical Appendix II.B. The slope of the short-run equilibrium locus is now steeper that during the Kaldorian underemployment phase, since the tendency of wages to grow along with capital accumulation (captured by the term $\mu+\beta$) is not mitigated by the effect of elastic labor supply. Considering the whole trend of the RW schedule on the $\log W_i - \log K$ space, it is first increasing and convex as long as Kaldorian underemployment persists, while after the corner point at the maturity threshold it turns into an upward-sloping half line[33].

To determine the long-run equilibrium of the system, it is necessary to consider the fundamental differential equation of capital accumulation. The equation of the stationary capital locus as a function of $\log W_i$ and $\log K$ can be obtained by simply replacing $L_i$ in equation 16 with its short-run equilibrium value from equation 9 [34]. This operation yields

$$\log\left[\frac{s\beta}{\delta}(1-\beta)^{\frac{1-\beta}{\beta}}(E^*)^{\frac{1-\beta}{\beta}}\right] + \frac{1}{\beta}\log A_i - \frac{1-\beta}{\beta}\log W_i^{**} + \frac{\mu}{\beta}\log K = 0; \tag{23}$$

[32]The economic reason behind this result is the movement of the agricultural terms of trade: under the above preference specification the agricultural terms of trade adjusts in order to maintain the constancy of the expenditure shares; but since the uniform wage is also a linear function of $P_a$ (see equation 5 and 12), in equilibrium the price adjustment will balance out other factors (including capital accumulation) and maintain a stable employment structure.

[33]While being a piece-wise function, the short-run equilibrium locus is continuous over its whole domain, and continuously differentiable but with the exception of the corner point.

[34]Note that equation 9 holds in both Kaldorian underemployment and maturity.

where we used the notation $W_i^{**}$ in order to distinguish the wage compatible with break-even investment from the short-run equilibrium wage. A close inspection of equation 23 shows that in the $\log W_i - \log K$ space it represents a straight line sloped

$$\frac{\partial \log W_i^{**}}{\partial \log K} = \frac{\mu}{1-\beta}. \tag{24}$$

Given the parametrization, the coefficient is positive and increasing in $\mu$: the higher the external capital effect, the stronger the positive impact of capital accumulation on the industrial TFP, the higher total profits and the higher the nominal wage compatible with the break-even level of investment. On the other hand, the stationary capital locus is also steeper the greater capital share $\beta$, because a higher $\beta$ means, *ceteris paribus*, a higher level of total profits for the same increase in capital stock[35], so a higher level of reinvestment.

Superimposing the short-run and long-run equilibrium loci on the same graph we can determine the equilibra, at the interception points, and their stability properties, according to the relative position of the two curves. Ideally, the economy moves along the real wage diagram, with the capital stock growing as long as the short-run equilibrium wage lies below the $\hat{K} = 0$ locus, and shrinking if the opposite happens. The reason for this is the behavior of total profits, and hence of investment: when the short-run equilibrium wage lies below that compatible with null net investment, reinvested profits will exceed depreciation costs and fuel capital accumulation, while in the opposite situation net investment will be negative and capital stock will fall. More precisely, it can be shown that

$$\hat{K} = s\,\beta\,[E^*(1-\beta)]^{\frac{1-\beta}{\beta}}\,A_i^{\frac{1}{\beta}}\left(\frac{K^\mu}{W_i}\right)^{\frac{1}{\beta}}(W_i - W_i^{**}).$$

Figure 2 presents two possible configurations of the system characterized by different

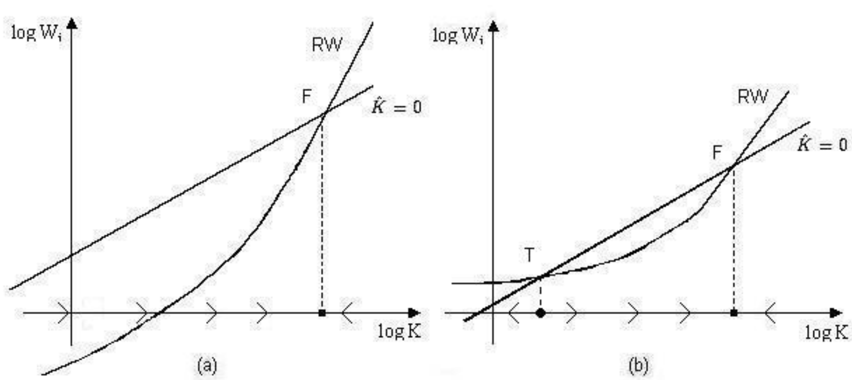


Figure 2: The model

parametrizations (a third possible configuration is one with no interception between the two schedules, but we disregard this trivial case[36]).

[35] Recall that $\Pi = W_i L_i \beta/(1-\beta)$.

[36] With no interception between the short-run and long-run equilibrium loci, the system either diverges toward an infinite capital stock or to a pure subsistence-agricultural economy, depending on whether the real wage schedule lies entirely above or below the stationary capital locus.

Considering at first the case of figure 2a, there are two possible equilibra: an unstable equilibrium of pure subsistence at zero capital stock, and an asymptotically stable equilibrium of full-industrialization F, where both sectors coexist. In the convex interval of the RW curve the economy witnesses a dramatic change in output and employment composition, undergoing a process of industrialization (Rostow's well-known take-off) fostered by the relatively elastic supply of industrial labor, which moderates the upward tendency of industrial wages[37]. Moreover, the transference of labor from the low-productive to the high-productive sector entails a double gain in terms of growth: on the one hand, marginal labor productivity in agriculture grows because of diminishing returns to labor (remember that the amount of land is fixed), on the other, increasing returns accelerate the growth of productivity in industry, fueling the expansion of the capital stock and of the whole manufacturing sector[38].

The economic dynamic described by the Kaldorian Underemployment phase explains several stylized facts often cited in the literature concerning developing countries[39]:

- The "agriculture-industry shift", meaning the declining importance of agriculture in terms of both employment share and percentage contribution to GDP in the course of economic development;
- The persistence of wide productivity gaps across economic sectors in developing countries, with agriculture featuring a much higher employment share than its correspondent GDP share, and hence having a lower average labor productivity than the rest of the economy. Such productivity gaps are mirrored by urban-rural wage gaps, which act as a stimulus to labor reallocation toward city-based industrial employment;
- The progressive reduction of the intersectoral differences in productivity (and wages), as labor reallocation toward industry raises agricultural labor productivity relative to the rest of the economy
- The S-shaped dynamic of saving and investment ratios as GDP grows, with a strong acceleration at low-middle income levels[40].

[37]Population growth, which was omitted from our analysis, would basically prolong the Kaldorian underemployment phase by increasing the number of agricultural workers (since industrial jobs are rationed), and reinforcing the tendency of the labor supply to be elastic.

[38]Again Kaldor (1968) expresses this idea very clearly:

"... the growth of productivity is accelerated as a result of the transfer at both hands - both at the gaining end and at the losing end; in the first, because, as a result of increasing returns productivity in industry will increase faster, the faster output expands; in the second because when the surplus-sectors lose labour, the productivity of the remainder of the working population is bound to rise."

[39]For a more detailed exposition of these stylized facts see among others Kuznets (1966), Chenery and Syrquin (1975), Syrquin (1989), Taylor (1989) and Bhaduri (1993 and 2003); as regards sectoral wage differential, evidence is often cited in the migration literature, especially for the so-called Todarian models.

[40]Note that the S-shaped dynamic of the investment share of GDP may also shed some light on why capital accumulation is a particularly important engine of growth at low- and middle-income levels, while TFP growth becomes dominant at high income levels.

Gradually, however, labor supply turns more and more inelastic counterbalancing the effect of increasing returns, so that the system eventually enters in the maturity phase and stabilizes its employment structure (see equation 20) with the coexistence of both sectors. From that point onwards, capital accumulation proceeds at an even slower pace, while the combined effect of relative price movements and wage adjustment tends to reduce total profits bringing the system to the stable equilibrium F[41]. Clearly the maturity stage describes the situation of more developed countries, in which the "agriculture-industry shift" has already taken place and structural dynamics typically involve the further expansion of the service sector.

In the case depicted in figure 2a, the stability of the F equilibrium requires the coefficient of the real wage schedule, for the maturity phase, to be greater than the correspondent coefficient of the $\hat{K} = 0$ locus at the interception point. Taking the relevant expressions from equations 19 and 24, stability requires

$$\mu < 1 - \beta;$$

meaning decreasing returns to capital. Had the industrial production function been an AK technology (for which $\mu = 1 - \beta$), the RW schedule and the $\hat{K} = 0$ would have been parallel in the maturity phase, so that capital accumulation could have proceed indefinitely, with RW below the stationary capital locus.

Alternatively, consider the case illustrated in figure 2b, where the system displays two interceptions between the short-run and long-run equilibrium schedules. Three equilibra are now possible in the system: (i) a locally stable equilibrium of pure subsistence with zero capital stock, (ii) an unstable low development equilibrium at point T, and (iii) a stable equilibrium of full industrialization at F. Because in T the real wage schedule cuts the $\hat{K} = 0$ locus from above, for capital stocks lower than $K_T$ reinvested profits are insufficient to cover entirely the depreciation, the short-run equilibrium wage being given by the correspondent value of the real wage schedule. There is hence an unstable poverty trap causing capital stock to shrink over time until the economy goes back to the state of pure agricultural subsistence. On the other hand, when $K > K_T$ the effect of increasing returns raises profitability sufficiently to trigger a self-fulfilling process of capital accumulation, driving the system to the equilibrium of full industrialization F.

The situation depicted in figure 2b may call for a big push à la Rosenstein Rodan[42], that is a concerted investment capable to bring the capital stock beyond $K_T$, breaking the poverty trap and making the industrialization process feasible. The relevance of the big push argument is further reinforces if we consider the role of the "social overhead capital", of infrastructures, and of all sorts of capital characterized by large comple-

[41] These findings seem to confirm the empirical evidence which suggests growth accelerations occurring at middle income level, when capital accumulation is faster and the economy enjoys a double gain from industrialization. See Chenery and Syrquin (1975), who find investment following an S-shaped dynamic and Syrquin 1988.

[42] A quotation from Skott and Ros (1997) summarizes clearly the concept of the big push
"the essence of a big-push argument is a model with multiple equilibra in which, under certain initial conditions, the economy gets stuck in a poverty trap that can only be overcome trough a "big push": No individual firm may have an incentive to expand on its own, even though the coordinated expansion by all firms will be profitable and welfare enhancing."

mentarities, and thus capable to crowd in private investments and stimulate significant supply responses[43].

Likewise in other poverty trap models, given the lack of an explicit formulation for the real wage schedule, it is impossible to determine sufficient conditions for the existence of the poverty trap; nevertheless we can derive the necessary conditions, which essentially require the short-run equilibrium curve to be flatter than the stationary capital locus. Taking the relevant expressions for the Kaldorian underemployment phase respectively from equation 19 and 24, the presence of a poverty trap requires

$$\mu > \frac{1-\beta}{1+\epsilon^{LS}}; \tag{25}$$

where $\epsilon^{LS}$ and its determinant $L_i$ are valued in the neighborhood of the point of interception.

In light of the recent wave of criticism against the idea of poverty traps[44], few words should be spent commenting the situation described in Figure2b. First of all, it should be pointed out that the poverty trap discussed here is not driven by lack of savings, but by insufficient profitability. Increases in the saving propensity do not alter the necessary condition, but only act as a parametric shift of the two curves, and as such may change the basins of attraction, but not alter the necessary conditions (see Section III for more details). As a consequence, the poverty trap may hold even in presence of international flows of capital, regardless of whether capital markets work perfectly or not. If anything, international capital markets would rather attract resources away from low-yielding national assets, thereby exacerbating the situation. Secondly, the unstable equilibrium of pure agrarian economy does not necessarily entail a zero growth: the analysis so far has taken sectoral TFP as parameters, however exogenous technical progress acts also on a completely agricultural economy, and certainly spurs its growth performances (in addition to modifying the whole equilibrium configuration, as will be shown later). Finally, it is worth noting that the degree of increasing returns necessary to make the poverty trap a relevant case in our set-up is far lower than in aggregate models[45]; even a value of $\mu$ around 0.2 (hence within the estimates cited by Kraay and Raddatz) may be sufficient. The reason is that the effect of increasing returns is amplified here by the elasticity of industrial labor supply, a factor rather disregarded in aggregate models of growth, although crucial for classical authors.

[43] In the recent literature, the importance of big push considerations in presence of non-tradeable inputs as infrastructures (and more generally of social overhead capital) is emphasized also by Ros an Skott (1997) and Sachs (2005). Nevertheless, the relevance of possible coordination failures requiring a minimum critical effort should be carefully distinguished from the so-called "classical aid narrative" (see Easterly 2006) which claims that a sufficient amount of aid would be capable of lifting countries out of the poverty trap and let them enter the take off. In our model this last statement would hold only in as much as the only effect of aid would be an increase in capital stock, without adverse effects on relative prices and/or crowding-out of national producers. Experience shows, however, that this is a rather simplistic view, since too often international aid ends up increasing government consumption, fueling corruption and rent-seeking, or at best being too volatile to be used for long-term concerted investment project, etc.

[44] See Kraay and Raddatz 2007 and Easterly 2006.

[45] See for instance equation 11 and page 339 of Kraay and Raddatz 2007.

We note by passing that the above model suggests a theoretical mechanism able to link the multiplicity of equilibra in terms of different income levels, with the structural characteristics of the economy, and the extent of the so-called agriculture-industry shift. While this statement holds directly as regards the level-variables (notably GDP per capita), it may be extended also to the growth rates, under the plausible assumption that industrial TFP grows faster than agricultural one.

## III. *The effect of technical progress*

### THE CASE OF PARAMETRIC INCREASE IN AGRICULTURAL TFP

So far, the analysis of the two-sectors economy has been concerned with the determination of the short- and long-run equilibra abstracting from technical progress, and treating the sector-specific TFPs as exogenous parameters. This approach may be convenient from an analytical point of view, but overlooks one of the main forces - if not the main force - behind the long-term increases in income: technical change.

Needless to say, increases in TFP, be it agricultural or industrial, have an unambiguous positive welfare effect, for they allow a greater supply of goods by using more efficiently the given amount of resources. More complex, however, are the effects of technical progress on the equilibrium configuration for the whole dynamic system. Precisely to grasp these effects, in this section we carry out some comparative statics exercises, with regard to sectoral TFPs.

As seen before, any long-run equilibrium, whether stable or unstable, is basically defined by the system between the stationary capital locus (equation 23) and of the relevant expression for the real wage schedule (equation 18 for Kaldorian Underemployment and equation 21 for the maturity phase). To lean down the notation let us rewrite the system as

$$\begin{cases} RW(\log W_i, \log K, A_a, A_i) = 0; \\ G(\log W_i, \log K, A_a, A_i) = 0; \end{cases} \tag{26}$$

where the implicit function G(.) indicates the stationary capital locus, and RW (.) as usual is the short-run equilibrium schedule.

Besides, recall that the real wage schedule is continuously differentiable with respect to its four arguments but with the exception of the corner point corresponding to the maturity threshold, while the $\hat{K} = 0$ locus is continuously differentiable on its whole domain, with respect to the four arguments. In light of this, and provided that the Jacobian of system 26 is non singular, the hypotheses underlying the implicit function theorem are satisfied over the whole domain, excluding the neighborhood of the corner point. With such exception, the implicit function theorem can therefore be applied in the neighborhood of an equilibrium (call it point Z) to rewrite system 26 as

$$\begin{cases} RW(\log W_i^Z(A_a, A_i), \log K^Z(A_a, A_i), A_a, A_i) = 0; \\ G(\log W_i^Z(A_a, A_i), \log K^Z(A_a, A_i), A_a, A_i) = 0; \end{cases} \tag{27}$$

in which ($\log W_i^Z$,$\log K^Z$) are the coordinates of the equilibrium point. This formulation of system 26 represents the starting point for all comparative statics regarding changes in the sectoral TFPs.

As concerns changes in the agricultural total factor productivity, the chain rule theorem can be used to compute the total derivative of each function in system 27 with respect to $A_a$, obtaining:

$$\begin{cases} \left.\dfrac{\partial RW}{\partial \log W_i}\right|_Z \dfrac{\partial \log W_i^Z}{\partial A_a} + \left.\dfrac{\partial RW}{\partial \log K}\right|_Z \dfrac{\partial \log K^Z}{\partial A_a} = -\dfrac{\partial RW}{\partial A_a}; \\ \left.\dfrac{\partial G}{\partial \log W_i}\right|_Z \dfrac{\partial \log W_i^Z}{\partial A_a} + \left.\dfrac{\partial G}{\partial \log K}\right|_Z \dfrac{\partial \log K^Z}{\partial A_a} = -\dfrac{\partial G}{\partial A_a}; \end{cases}$$

Solving this last system for $\partial \log W_i^Z/\partial A_a$ and $\partial \log K^Z/\partial A_a$ permits to obtain, from the sign of these derivatives, the direction in which the new equilibrium value (call it Z') lies as a result of the change in the underlying parameter $A_a$.

Analytically, it can be shown that[46]

$$\frac{\partial \log W_i^Z}{\partial A_a} = \frac{\begin{vmatrix} -\frac{\partial RW}{\partial A_a} & \frac{\partial RW}{\partial \log K} \\ -\frac{\partial G}{\partial A_a} & \frac{\partial G}{\partial \log K} \end{vmatrix}}{|J|}; \qquad \frac{\partial \log K^Z}{\partial A_a} = \frac{\begin{vmatrix} \frac{\partial RW}{\partial \log W_i} & -\frac{\partial RW}{\partial A_a} \\ \frac{\partial G}{\partial \log W_i} & -\frac{\partial G}{\partial A_a} \end{vmatrix}}{|J|}; \tag{28}$$

While these two expressions hold in general over the whole domain (except in the neighborhood of the corner point), the piecewise nature of the real wage schedule implies that comparative statics should be carried out separately for each phase: Kaldorian underemployment and maturity.

Proceeding with a taxonomic logic, suppose first that the Z equilibrium occurs during the Kaldorian underemployment phase. In such a case, the partial derivatives in 28 should be replaced with their actual values computed from equations 18 and 23. Indicating with $J^{KU}$ the Jacobian corresponding to the Kaldorian underemployment phase, this operation yields:

$$\frac{\partial \log W_i^Z}{\partial A_a} = -\frac{\mu\alpha(1-\gamma)}{[\gamma+\alpha(1-\gamma)]\beta}\frac{1}{A_a}\frac{1}{|J^{KU}|}; \qquad \frac{\partial \log K^Z}{\partial A_a} = -\frac{\alpha(1-\gamma)(1-\beta)}{[\gamma+\alpha(1-\gamma)]\beta}\frac{1}{A_a}\frac{1}{|J^{KU}|}; \tag{29}$$

Under the assumed parametrization, 29 implies that the derivatives $\partial \log W_i^Z/\partial A_a$ and $\partial \log K^Z/\partial A_a$ assume the opposite sign of $\left|J^{KU}\right|$ (see Mathematical Appendix III.A for more details).

Furthermore, Samuelson's "correspondence principle between statics and dynamics"[47] can be utilized to prove that

$$\left|J^{KU}\right| > 0 \quad \Leftrightarrow \quad \mu > \frac{1-\beta}{1+\epsilon^{LS}};$$

[46] Of course in the following section all partial derivatives should be valued at Z, that is at the value corresponding to the equilibrium; for simplicity we omit this detail from the notation of the text.

[47] The principle was analyzed by Samuelson in 1941 and 1947; for a recent treatment of the principle see Gandolfo (1997).

meaning that $|J^{KU}|$ is positive when the corresponding equilibrium point is unstable, and negative in the opposite case[48].

Moving to the maturity phase, the same procedure should actually be followed to carry out the comparative statics, replacing the partial derivatives of equation 28 with their actual values calculated from equations 21 and 23. However, recalling that during maturity both $\partial RW/\partial A_a$ and $\partial G/\partial A_a$ are zero, it can directly be argued that shifts in the TFP of the agricultural sector leave the equilibrium of the mature economy unchanged, regardless of its stability (see Mathematical Appendix III.A for more details).

It is hence demonstrated that

- parametric increases in agricultural TFP reduce the basin of attraction of the locally stable equilibrium of pure subsistence, under the condition that there exists an unstable equilibrium in the Kaldorian underemployment phase ($\partial \log W_i^Z/\partial A_a$ and $\partial \log K^Z/\partial A_a$ from 29 are both negative);
- alternatively, increases in $A_a$ move the stable equilibrium occurring in the Kaldorian underemployment phase (if any) towards North-East, increasing the steady state value of $\log W_i$ and $\log K$ ($\partial \log W_i^Z/\partial A_a$ and $\partial \log K^Z/\partial A_a$ from 29 are both positive);
- finally, parametric modification of $A_a$ leave unchanged all equilibra occurring in the maturity phase (if any), since $\partial \log W_i^Z/\partial A_a$ and $\partial \log K^Z/\partial A_a$ from 29 are both zero.

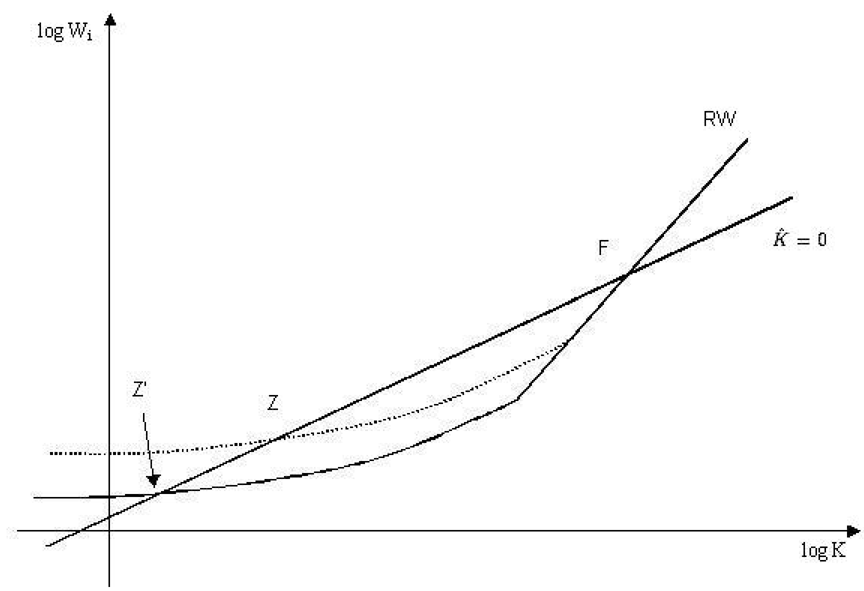


Figure 3: The effect of an increase in agriculture TFP

These comparative statics results are shown diagrammatically in figure 3, representing the case in which a poverty trap occurs during Kaldorian underemployment (dashed

[48]Recall that for the implicit function theorem to hold, $|J^{KU}|$ must be different from zero.

schedules represent the equilibrium loci before the TFP increase) [49]. The shift of the real wage schedule (in the Kaldorian underemployment interval), vis à vis the invariance of the stationary capital locus, reduces "the hold of the poverty trap" - more precisely the basin of attraction of the low-level equilibrium - from $(-\infty, \log K^Z)$ to $(-\infty, \log K^{Z'})$, correspondingly lowering the minimum critical level of capital beyond which increasing returns make industry profitable and capital accumulation self-sustaining. Intuitively, the increase in $A_a$ leads to a larger availability of food for given agricultural employment share and capital stock; this fact lowers the agricultural terms of trade and in turn raises *ceteris paribus* the real wages of both sectors, thus allowing a higher profitability to capitalist entrepreneurs in industry.

In line with the above results, the importance of the primary sector in the early phases of industrialization is confirmed by the comparison between two emblematic historical cases: Russia in the 20s and China in the late 70s and 80s[50]. Russian land reform of 1917 distributed more equally land tenure, but was unable to stimulate decisive productivity improvements in agriculture. As a result of the de-kulakization, grain supply collapsed, leading to sharp increases in food prices and to great social unrest between urban and rural classes, tensions that culminated at a political level in the Trotskyan view that industry could only expand *at expenses* of peasantry[51]. In line with our theoretical results on the potential poverty trap, industrialization in USSR implied a deep conflict between city and countryside, and the impressive capital accumulation could take place only forcibly.

In contrast with the Russian experience, China under Deng-Xiao-Ping embarked in a large program of agrarian reforms[52], which managed to stimulate large productivity improvements in agriculture, with output growing by over 40% between 1978 and 1984. While the price liberalization lead initially to food price increases - starting however from a highly repressed base - later the dramatic raise of grain supplies spurred by the reforms helped maintaining real wages at a competitive level, favoring a rapid capital accumulation and fuelling industrial growth. At the same time, the boom in agriculture was a decisive factor in lifting millions of Chinese from absolute poverty, while the rural sector maintained a reservoir of cheap labor for the high-yielding industrial areas on the coast. In this case, it could be argued that agrarian reforms may have reduced or even eliminated the potential bottleneck of the poverty trap, fostering proactive conditions

[49] Clearly, in absence of the poverty trap the only significant effects of agricultural technical progress concern the possibility that the full industrialization equilibrium - if occurring in the Kaldorian underemployment phase - is pushed towards North-East, with higher steady state levels of capital stock.

[50] At this regard more recent evidence concerns of the contrasting experience of Asian and African countries as regards the impact of the Green Revolution in raising agricultural yields (see Sachs 2005). Whereas in the former countries agricultural productivity rose steadily along the 70s paving the way for the successive industrialization, in Sub-Saharan Africa food production per capita actually fell. Though interesting, the picture is however blurred by other factors such as demographic changes, soil depletion, desertification etc.

[51] Preobrazhensky for instance criticized the New Economic Policy suggesting that the terms of trade should be turned against agriculture in order to extract from the peasantry that surplus necessary for the growth of heavy capital goods industry.

[52] The post-78 reforms included notably the so-called two-tier pricing and the household responsibility system.

for the structural change.

It is important to mention at this stage that the positive link between agricultural productivity growth and industrialization would be further reinforced when including Engel effects in consumers' demand. Non-homotetic preferences were not used here for lack of explicit solution in the determination of the RW schedule; nevertheless, it is well accepted that Engel effect can play a mayor role in reinforcing structural change, as shown for instance in Murphy Shleifer and Vishny (1989), Stokey (1988), Matsuyama (1992), but also in Pasinetti (1993).

Besides, an interesting parallel could be drawn between the role of agriculture in the present model of industrialization and the role of agriculture in the Kaleckian and the structuralist interpretation of inflation in developing countries. In the Kaleckian literature[53], the inability of agricultural productivity to keep the pace with the growing industrial sector leads to the so-called "wage-good-constraint": the increase in food prices exerts upward pressure on nominal wages (which are set in real terms), thus triggering an inflation spiral[54]. In the present set-up, it could be argued that the efficiency wage mechanism during the Kaldorian underemployment phase acts in a way that turns the "wage-good-constraint" into a potential profitability constraint possibly giving raise to a poverty trap: unless food is available at a sufficiently low price, given the capital stock, capital accumulation is simply not self-sustaining, and the system falls back towards a purely agrarian economy.

## THE CASE OF PARAMETRIC INCREASE IN INDUSTRIAL TFP

Applying the same procedure used for parametric changes in the agricultural TFP, we can shed some light also on the comparative statics regarding increases in $A_i$. Total derivation of system 27 with respect to $A_i$ yields

$$\begin{cases} \left.\dfrac{\partial RW}{\partial \log W_i}\right|_Z \dfrac{\partial \log W_i^Z}{\partial A_i} + \left.\dfrac{\partial RW}{\partial \log K}\right|_Z \dfrac{\partial \log K^Z}{\partial A_i} = -\dfrac{\partial RW}{\partial A_i}; \\ \left.\dfrac{\partial G}{\partial \log W_i}\right|_Z \dfrac{\partial \log W_i^Z}{\partial A_i} + \left.\dfrac{\partial G}{\partial \log K}\right|_Z \dfrac{\partial \log K^Z}{\partial A_i} = -\dfrac{\partial G}{\partial A_i}; \end{cases} \tag{30}$$

while solving the above system for $\partial \log W_i^Z / \partial A_i$ and $\partial \log K^Z / \partial A_i$ obtains

$$\frac{\partial \log W_i^Z}{\partial A_i} = \frac{\begin{vmatrix} -\frac{\partial RW}{\partial A_i} & \frac{\partial RW}{\partial \log K} \\ -\frac{\partial G}{\partial A_i} & \frac{\partial G}{\partial \log K} \end{vmatrix}}{|J|}; \qquad \frac{\partial \log K^Z}{\partial A_i} = \frac{\begin{vmatrix} \frac{\partial RW}{\partial \log W_i} & -\frac{\partial RW}{\partial A_i} \\ \frac{\partial G}{\partial \log W_i} & -\frac{\partial G}{\partial A_i} \end{vmatrix}}{|J|}; \tag{31}$$

Here again all partial derivatives should be values at the equilibrium point, and need to be considered separately for Kaldorian underemployment and for maturity, because of the piecewise nature of the real wage schedule.

[53]Note however that this idea was already present in Kaldor 1954, and 1967, with special reference to the burst of inflation crisis in Latin America.

[54]See for instance Kalecky 1976, Dutta 1988 and Basu 1997.

Following a conditional line of reasoning, let us suppose first that the equilibrium Z occurs during the Kaldorian underemployment phase; accordingly, the relevant expressions for the partial derivatives should be computed from equations 18 and 23. After some algebra (shown with some more detail in Mathematical Appendix III.B) the above formulas reduce to

$$\frac{\partial \log W_i^Z}{\partial A_i} = -\frac{\gamma + \alpha(1-\gamma)(1-bL_i)}{\beta A_i[\gamma + \alpha(1-\gamma)]L_a}\frac{1}{|J^{KU}|}; \frac{\partial \log K^Z}{\partial A_i} = -\frac{1-(1-\gamma)[1-\alpha(1-b)]L_i}{\beta A_i[\gamma + \alpha(1-\gamma)]L_a}\frac{1}{|J^{KU}|}; \tag{32}$$

which imply, under the assumed parametrization, that the derivatives $\partial \log W_i^Z/\partial A_i$ and $\partial \log K^Z/\partial A_i$ take the opposite sign of $|J^{KU}|$. Like in the previous case, the correspondence principle ensures that $|J^{KU}|$ is positive when Z is an unstable equilibrium, so that the direction in which the new equilibrium lies can be univocally determined.

To complete the conditional analysis, suppose instead that the equilibrium point Z belongs to the maturity interval; in such case, the relevant partial derivatives in expression 31 should be computed from equations 21 and 23. After some algebraic manipulation this operation obtains:

$$\frac{\partial \log W_i^Z}{\partial A_i} = -\frac{1}{\beta}\frac{1}{A_i}\frac{1}{|J^{MA}|}; \qquad \frac{\partial \log K^Z}{\partial A_i} = -\frac{1}{\beta}\frac{1}{A_i}\frac{1}{|J^{MA}|}; \tag{33}$$

in which $J^{MA}$ indicates the Jacobian corresponding to the maturity interval. Equation 33 implies that the derivatives $\partial \log W_i^Z/\partial A_i$ and $\partial \log K^Z/\partial A_i$ take the opposite sign of $|J^{MA}|$.

In light of the correspondence principle, it can be shown (see Mathematical Appendix III.B) that

$$|J^{MA}| > 0 \quad \Rightarrow \quad \mu > (1-\beta);$$

so that the sign of $\partial \log W_i^Z/\partial A_i$ and $\partial \log K^Z/\partial A_i$ can be univocally determined.

At this stage, it is therefore possible to summarize the comparative statics results as follows. A parametric increase in $A_i$

- shifts the unstable equilibrium (if any) towards the South-West, more precisely it reduces the basin of attraction of the equilibrium of pure subsistence with zero capital stock; this statement holds in both Kaldorian underemployment and maturity, since $\partial \log W_i^Z/\partial A_i$ and $\partial \log K^Z/\partial A_i$ are negative in both cases;
- moves the stable equilibrium (if any) towards north-east, increasing the steady state level of capital and wages, since $\partial \log W_i^Z/\partial A_i$ and $\partial \log K^Z/\partial A_i$ in this case are both positive.

These results are illustrated graphically in figure 4, which considers the case in which a poverty trap exists (dashed schedules represent the equilibrium loci before the productivity increase)[55]. The economic explanation goes as follows, regardless of which phase the economy goes through. The increase in $A_i$ raises, *ceteris paribus*, the

[55] In the absence of a poverty trap, the only impact of the industrial productivity increase would be to increase the steady state level of capital and wages for the stable interception.

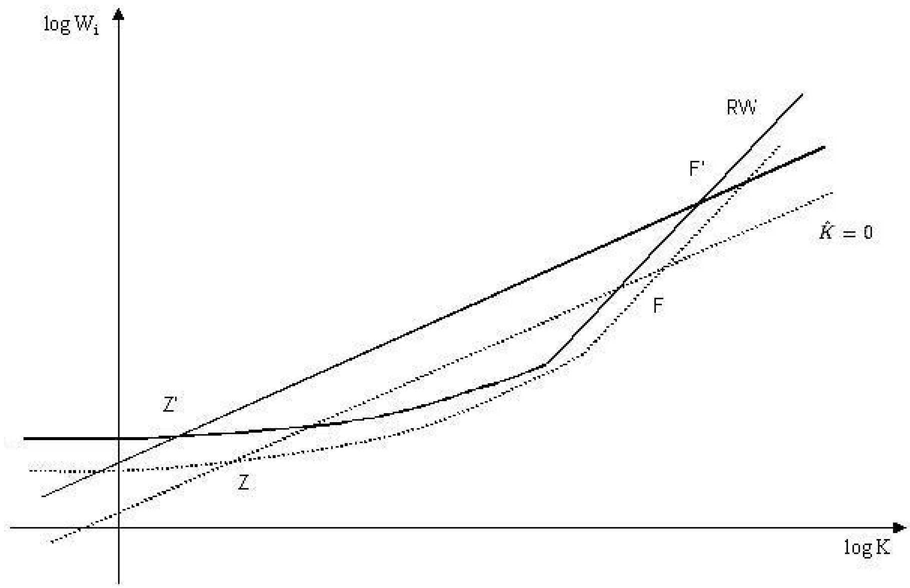


Figure 4: The effect of an increase in industrial TFP

supply of manufactures, leading to a moderate increase in the agricultural terms of trade and in agricultural wages, which in turn trigger an upwards adjustment of the nominal industrial wages. These factors explain the upwards move of the real wage schedule. The gains in terms of industrial productivity bring, however, a much larger gain to entrepreneurs, boosting their profits, and allowing a faster capital accumulation; this is reflected in the upwards shift of the stationary capital locus. Since the vertical movement of the $\hat{K}=0$ locus outweighs that of the real wage schedule[56], the unstable low-development equilibrium (if any) will occur for a lower level of capital stock (in the figure $K_Z > K_{Z'}$). Similarly to what happened for the agricultural TFP, technical progress in industry directly boosts the profitability of entrepreneurs, so that a self-sustaining accumulation of capital becomes viable even for lower capital stocks. For exactly the same reasons, the equilibrium of full industrialization will always be pushed towards higher levels of capital stock by improvements in industrial TFP, regardless of the phase of the economy.

## VI. *CONCLUSIONS*

In line with our main objective, we have combined in this two-sector macro-model several aspects emphasized by the neoclassical theory of growth and structural change, with other insights drawn from the more dated literature about dual economies and big push. Interestingly, the adoption of an efficiency wage mechanism in the urban labor market (unlike in the rural one), combined with technological external economies

[56]This can be verified by directly computing $\partial \log W_i / \partial \log A_i$ for the real wage schedule and for the stationary capital locus: this derivative in the latter case outweighs the correspondent derivative for RW.

in industry, are sufficient to rationalize a view of the agriculture industry shift à la Kaldor, and to originate possible poverty traps that may justify policies of concerted investment to bring capital stock up to a minimum critical level.

Of course, Kaldor's structure of causality pivots around the central role of effective demand, while we retain a supply-driven framework, resembling rather closer Lewis's model of unlimited supply of labor. Nevertheless, the complex interactions between agriculture and industry, the importance of labor reallocation to the more dynamic sector, and the asymmetric working of the labor market represent common aspects that link the present work to the "Strategic factors in economic development", highlighting the crucial role of industrialization and increasing returns in the process of development.

As concerns instead the long debate on the big push argument, the above analysis has shown how - in presence of a labor market characterized by the mismatch between the wage and productivity levels in agriculture and industry - even moderate degrees of increasing returns in industry may be sufficient to give raise to poverty traps, since the effect of increasing returns if reinforced by the elastic supply of labor for the more dynamic industrial sector. While this result is encouraging for the plausibility of the mechanisms outlined here, our model is surely very sensitive to the parametrization adopted, and, likewise the majority of poverty trap models, it "tends to be lacking in testable quantitative implications"[57]. Nevertheless, we believe the mechanisms analyzed here may well be relevant for LDCs, and above all of todays Sub-Saharan Africa, the region with the closest conditions to our theoretical framework: extremely capital-poor agricultural sector, with widespread areas of subsistence agriculture.

Besides, this work has shown the peculiar relation between sectoral TFP, and the possible bottlenecks to capital accumulation, explaining how increases in the TFP of any of the two sectors, may help making the poverty trap if not less likely at least less stringent. Interestingly, in the event of being stuck by a poverty traps, our model suggests two strategies, which also seem to be at the cornerstone of the Chinese economic success in the last twenty years: raising agricultural productivity and accumulating physical capital, with special attention to the "social overhead capital". Now that these strategies are also becoming the pillar of Chinese growing economic interventions in Sub-Saharan Africa, we may have the chance to see these policies at work in those economies with the closest resemblance to our conceptual set-up.

## *Mathematical Appendix*

### I. THE KALDORIAN UNDEREMPLOYMENT PHASE

#### *A. DETERMINATION OF THE LABOR SUPPLY ELASTICITY*

Substituting in the consumption expenditure flow identity (equation 14), the value of agricultural wages and rents (equations 5 and 6), and using the numeraire (equation

[57] The quotation is taken from Azariadis and Stachurski (2005), recognizing a limit which is common to most models of poverty trap.

15) and the fact that $\Pi = \frac{\beta}{1-\beta} W_i L_i$; yields

$$P\ \ X_a^c + X_i^c = P\ \ X_a^s + \frac{1-s\beta}{1-\beta}\, W_i L_i;$$

which combined with equation 13 obtains

$$X_i^c = \frac{1-s\beta}{1-\beta}\, W_i L_i;$$

Substituting $X_i^c$ from the demand function (equation 1), using equations 8 and 5 to express the agricultural terms of trade as a function of the nominal industrial wage, and applying the labor market clearing relation and the food production function (equations 11 and 3) yields

$$\frac{1-\alpha}{\alpha}\,\frac{1-\beta}{1-s\beta}\left[\frac{(1-d)^{\frac{1}{d}}}{\omega^{\frac{1}{d}}(1-b)^{\gamma}}\right]^{\frac{1}{\gamma+\alpha(1-\gamma)}} A_a^{\frac{\alpha(1-\gamma)}{\gamma+\alpha(1-\gamma)}}\,(1-L_i)^{\frac{\gamma+\alpha(1-\gamma)(1-b)}{\gamma+\alpha(1-\gamma)}} = W_i^{-\frac{(1-\alpha)(1-\gamma)}{\gamma+\alpha(1-\gamma)}}\ L_i; \tag{34}$$

Taking logs obtains

$$\log\left\{\frac{1-\alpha}{\alpha}\,\frac{1-\beta}{1-s\beta}\left[\frac{(1-d)^{\frac{1}{d}}}{\omega^{\frac{1}{d}}(1-b)^{\gamma}}\right]^{\frac{1}{\gamma+\alpha(1-\gamma)}}\right\} + \frac{\alpha(1-\gamma)}{\gamma+\alpha(1-\gamma)}\log A_a +$$
$$+\frac{(1-\alpha)(1-\gamma)}{\gamma+\alpha(1-\gamma)}\log W_i = -\frac{\gamma+\alpha(1-\gamma)(1-b)}{\gamma+\alpha(1-\gamma)}\log\left[1-\exp\left(\log L_i\right)\right] + \log L_i;$$

from which total differentiation yields the expression for the industrial labor supply elasticity $\epsilon^{LS}$ mentioned in equation 17:

$$\epsilon^{LS} = \frac{(1-\alpha)\,(1-\gamma)\,(1-L_i)}{\gamma\,+\,\alpha(1-\gamma)(1-bL_i)};$$

### B. DETERMINATION OF THE REAL WAGE SCHEDULE

To determine the real wage schedule, replace $L_i$ in equation 34 with its value from 9, which obtains

$$Q\ A_a^{\frac{\alpha(1-\gamma)}{\gamma+\alpha(1-\gamma)}}\ A_i^{-\frac{1}{\beta}}\ K^{-\frac{\mu+\beta}{\beta}}\ W_i^{\left[\frac{1}{\beta}+\frac{(1-\alpha)(1-\gamma)}{\gamma+\alpha(1-\gamma)}\right]} =$$
$$= \left[1 - A_i^{\frac{1}{\beta}}\,(1-\beta)^{\frac{1}{\beta}}\,(E^*)^{\frac{1-\beta}{\beta}}\ W_i^{-\frac{1}{\beta}}\ K^{\frac{\mu+\beta}{\beta}}\right]^{-\frac{\gamma+\alpha(1-\gamma)(1-b)}{\gamma+\alpha(1-\gamma)}};$$

where $Q$ is a constant defined as

$$Q = \frac{1-\alpha}{\alpha}\,\frac{(1-\beta)^{-\frac{1-\beta}{\beta}}}{1-s\beta}\ \ (E^*)^{-\frac{1-\beta}{\beta}}\ \left[\frac{(1-d)^{\frac{1}{d}}}{\omega^{\frac{1}{d}}(1-b)^{\gamma}}\right]^{\frac{1}{\gamma+\alpha(1-\gamma)}}.$$

Expressed in log terms, this equation is exactly the RW schedule mentioned in the text (equation 18).

### C. THE EVOLUTION OF THE WAGE GAP

Consider the expression for the agricultural wage relatively to the industrial one[58]

$$\frac{W_a}{W_i} = \frac{(1-b)\,A_a\,(L_a)^{-b}\,P_a}{W_i};$$

using equations 8 and 5 to express $P_a$ as a function of $W_i$, and then making use of equation 11 and 9, the wage ratio can be rewritten as

$$\frac{W_a}{W_i} = \left[\left(\frac{1-d}{\omega}\right)^{\frac{1}{d}} (1-b)^{\alpha(1-\gamma)} A_a^{\alpha(1-\gamma)}\right]^{\frac{1}{\gamma+\alpha(1-\gamma)}} W_i^{\frac{(1-\alpha)(1-\gamma)}{\gamma+\alpha(1-\gamma)}}$$
$$\left[1 - A_i^{\frac{1}{\beta}} (1-\beta)^{\frac{1}{\beta}} (E^*)^{\frac{1-\beta}{\beta}} W_i^{-\frac{1}{\beta}} K^{\frac{\mu+\beta}{\beta}}\right]^{\frac{-b\alpha(1-\gamma)}{\gamma+\alpha(1-\gamma)}}.$$

Taking logs, the wage ratio becomes

$$\log\frac{W_a}{W_i} = \frac{1}{\gamma+\alpha(1-\gamma)}\log\left[\left(\frac{1-d}{\omega}\right)^{\frac{1}{d}} (1-b)^{\alpha(1-\gamma)} A_a^{\alpha(1-\gamma)}\right] + \frac{(1-\alpha)(1-\gamma)}{\gamma+\alpha(1-\gamma)}\log W_i +$$
$$-\frac{b\alpha(1-\gamma)}{\gamma+\alpha(1-\gamma)}\log\left[1 - A_i^{\frac{1}{\beta}} (1-\beta)^{\frac{1}{\beta}} (E^*)^{\frac{1-\beta}{\beta}} \exp\left(\frac{\mu+\beta}{\beta}\log K - \frac{1}{\beta}\log W_i\right)\right];$$

from which total differentiation, yields the following relation:

$$\frac{\partial\log\frac{W_a}{W_i}}{\partial\log K} = \frac{\alpha b(\mu+\beta)(1-\gamma)L_i}{\beta L_a[\gamma+\alpha(1-\gamma)]} + \frac{(1-\gamma)[\beta\,(1-\alpha)L_a - \alpha b L_i]}{\beta L_a[\gamma+\alpha(1-\gamma)]}\frac{\partial\log W_i}{\partial\log K}.$$

Substituting $\frac{\partial\log W_i}{\partial\log K}$ in this expression with the coefficient of the real wage schedule obtained in equation 19, yields

$$\frac{\partial\log\frac{W_a}{W_i}}{\partial\log K} = \frac{(\mu+\beta)(1-\gamma)(1-\alpha)}{[\gamma+\alpha(1-\gamma)]+(1-\gamma)[\beta(1-\alpha)L_a - \alpha b L_i]} > 0.$$

Since this derivative is strictly positive for the parametrization assumed above, the wage ratio tends to grow along with increases in the capital stock, from values lower than one (by construction) ultimately reaching one when the system enters the maturity phase and wage gap disappear. To see this, note that the logarithm is a monotonically increasing transformation of the wage ratio and of the capital stock, hence the sign of the log-derivative $\partial\log\frac{W_a}{W_i}/\partial\log K$ equals the sign of the simple derivative of the wage ratio to capital stock.

## II. THE MATURITY PHASE

### A. DETERMINATION OF THE LABOR SUPPLY ELASTICITY

As before, combining the consumption expenditure flow identity (equation 14), with wage and rents determination and with food market clearing (equations 5, 12, 6 and 13), and using the fact that $\Pi = \frac{\beta}{1-\beta}W_i L_i$; yields

$$X_i^c = \frac{1-s\beta}{1-\beta}W_i\,L_i.$$

[58] The absolute wage gap is tied to the wage ratio by the following relation $W_i - W_a = (1-\frac{W_a}{W_i})\,W_i$.

This relation, combined with equations 1, 5, 11 and 13, obtains

$$(1 - L_i)\,\frac{1-\alpha}{\alpha}\,\frac{1}{1-b} \;=\; \frac{1-s\beta}{1-\beta}\,L_i;$$

from which we derive

$$\epsilon^{LS} = 0;$$

and

$$\bar{L}_i \;=\; \frac{(1-\alpha)(1-\beta)}{(1-\alpha)(1-\beta)+\alpha(1-b)(1-s\beta)}.$$

### *B. DETERMINATION OF THE REAL WAGE SCHEDULE*

To obtain the equation of the real wage schedule during the maturity phase, combine the labor demand with the wage determination (respectively equation 9, 12), and recall that labor efficiency in the maturity phase will still be given by $E^*$. This yields

$$\bar{L}_i \;=\; A_i^{\frac{1}{\beta}}\,(1-\beta)^{\frac{1}{\beta}}\,(E^*)^{\frac{1-\beta}{\beta}}\,W_i^{-\frac{1}{\beta}}\,K^{\frac{\mu+\beta}{\beta}}.$$

Taking logs, obtains from this expression the real wage schedule as given in the text (equation 21).

## III. THE EFFECT OF TECHNICAL CHANGE

### *A. COMPARATIVE STATICS: AGRICULTURAL TFP*

From the previous analysis it should be clear in system 27 the relevant equations during Kaldorian underemployment are actually 18 for RW and 23 in place of G. Accordingly, the following magnitudes are of interest for comparative statics in the Kaldorian underemployment phase:

$$\mathcal{J}^{KU} \equiv \begin{pmatrix} \frac{\partial RW}{\partial \log W_i} & \frac{\partial RW}{\partial \log K} \\ \frac{\partial G}{\partial \log W_i} & \frac{\partial G}{\partial \log K} \end{pmatrix} = \begin{pmatrix} \frac{\gamma+(1-\gamma)[\beta(1-\alpha)(1-L_i)+\alpha(1-bL_i)]}{\beta[\gamma+\alpha(1-\gamma)]L_a} & -\frac{\mu+\beta}{\beta}\,\frac{\gamma+\alpha(1-\gamma)(1-bL_i)}{[\gamma+\alpha(1-\gamma)]L_a} \\ -\frac{1-\beta}{\beta} & \frac{\mu}{\beta} \end{pmatrix};$$

and

$$\frac{\partial RW}{\partial A_a} = \frac{\alpha(1-\gamma)}{\gamma+\alpha(1-\gamma)}\,\frac{1}{A_a}; \qquad\qquad \frac{\partial G}{\partial A_a} = 0.$$

Replacing the partial derivatives of equation 28 with the corresponding values as determined here, obtains after some manipulation 29.

As concerns the sign of $\left|J^{KU}\right|$, its direct calculation shows after some manipulations that

$$\left|J^{KU}\right| \,>\, 0 \quad\Leftrightarrow\quad \mu > \frac{(1-\beta)[\gamma+\alpha(1-\gamma)(1-bL_i)]}{[\gamma+\alpha(1-\gamma)(1-bL_i)]+(1-\gamma)(1-\alpha)L_a} \;=\; \frac{(1-\beta)}{1+\epsilon^{LS}};$$

which basically verifies the correspondence principle between statics and dynamics.

During maturity, instead, the relevant equations for system 27 are equation 21 (for RW) and 23 for G. Accordingly, we have

$$\mathcal{J}^{MA} \equiv \begin{pmatrix} \frac{\partial RW}{\partial \log W_i} & \frac{\partial RW}{\partial \log K} \\ \frac{\partial G}{\partial \log W_i} & \frac{\partial G}{\partial \log K} \end{pmatrix} = \begin{pmatrix} \frac{1}{\beta L_a} & -\frac{\mu * \beta}{\beta L_a} \\ -\frac{1-\beta}{\beta} & \frac{\mu}{\beta} \end{pmatrix}.$$

and

$$\frac{\partial RW}{\partial A_a} = 0; \qquad \frac{\partial G}{\partial A_a} = 0.$$

Finally replacing the partial derivatives in equation 28 with the corresponding values determined here for the maturity phase, directly obtains

$$\frac{\partial \log W_i^Z}{\partial A_a} = 0; \quad \frac{\partial \log K^Z}{\partial A_a} = 0;$$

proving that the equilibrium occurring in the maturity phase (if any) is invariant to parametric changes in $A_a$, regardless of its stability properties.

*B. COMPARATIVE STATICS: INDUSTRIAL TFP*

Starting with Kaldorian underemployment, the relevant equations for system 30 are 18 (for RW), and 23 (for G). Hence, in addition to the matrix $J^{KU}$ defined above, the magnitudes of interest for the comparative statics regarding $A_i$ in the Kaldorian underemployment phase are:

$$\frac{\partial RW}{\partial A_i} = -\frac{\gamma + \alpha(1-\gamma)(1-bL_i)}{\beta[\gamma + \alpha(1-\gamma)]L_a}\frac{1}{A_i}; \qquad \frac{\partial G}{\partial A_i} = \frac{1}{\beta}\frac{1}{A_i};$$

Replacing these values for the corresponding partial derivatives in equation 31, obtains after some manipulation 32. Recalling finally the condition for a positive determinant of $J^{KU}$, yields the comparative statics result mentioned in the text.

As concerns the maturity phase, instead, the relevant Jacobian

$$\mathcal{J}^{MA} \equiv \begin{pmatrix} \frac{\partial RW}{\partial \log W_i} & \frac{\partial RW}{\partial \log K} \\ \frac{\partial G}{\partial \log W_i} & \frac{\partial G}{\partial \log K} \end{pmatrix} = \begin{pmatrix} \frac{1}{\beta L_a} & -\frac{\mu * \beta}{\beta L_a} \\ -\frac{1-\beta}{\beta} & \frac{\mu}{\beta} \end{pmatrix};$$

can be derived directly from equations 21 (for RW) and 23 for G. From these same equation, it is possible to compute the magnitudes

$$\frac{\partial RW}{\partial A_i} = -\frac{1}{\beta}\frac{1}{A_i}; \qquad \frac{\partial G}{\partial A_i} = \frac{1}{\beta}\frac{1}{A_i}.$$

Substituting these expressions in equation 31 obtains after a bit of algebra equation 33.

Finally, the direct calculation of $\left|J^{MA}\right|$ verifies the correspondence principle, establishing precisely that

$$\left|J^{MA}\right| > 0 \quad \Leftrightarrow \quad \mu > (1-\beta);$$

and with this last condition the sign of the derivatives $\partial \log W_i^Z / \partial A_i$ and $\partial \log K^Z / \partial A_i$ can be univocally determined.

## References


[1] Aghion, P., and Bolton, P. A theory of trickle-down growth and development. *Review of Economic Studies 64* (1997), 151–172.

[2] Akerlof, G. Labor contracts as partial gift exchange. *Quarterly Journal of Economics 97* (1982), 543–569.

[3] Akerlof, G., and Yellen, J. *Efficiency wage models of the labor market.* Cambridge University Press, 1986.

[4] Arrow, K. The economic implications of learning by doing. *Review of Economic Studies 29* (1962), 155–173.

[5] Azariadis, C. *The theory of poverty traps: what have we learned?* (in "Poverty Traps") Bowles, Durlauf and Hoff (editors) Princeton University Press, 2005.

[6] Azariadis, C., and Drazen, A. Threshold externalities in economic development. *The Quarterly Journal of Economics 105* (1990), 501–526.

[7] Azariadis, C., and J., S. *Poverty traps.* (in "Handbook of economic growth") Aghion and Durlauf (editors) Elsevier Science Publishers, 2005.

[8] Banerjee, A., and Newman, A. Occupational choice and the process of development. *Journal of Political Economy 101* (1993), 274–298.

[9] Barro, R. Economic growth in a cross section of countries. *The Quarterly Journal of Economics 106* (1991), 407–443.

[10] Barro, R., and Sala-i Martin, X. *Economics growth.* McGraw-Hill, 1995.

[11] Basu, K. *Analytical Development Economics: The Less Developed Economy Revisited.* MIT press, 1997.

[12] Bhaduri, A. *Alternative development strategies and the rural sector.* (in "Economic crisis and the Third World agriculture") Singh and Tabatabai (editors) Cambridge University Press, 1993.

[13] Bhaduri, A. *Structural change and economic development: on the relative roles of effective demand and the price mechanism in a dual economy.* (in "Rethinking development economics") Ha Joon Chang (editor) Anthem Press, 2003.

[14] Bowles, S. The production process in a competitive economy: Walrasian, neo-hobbesian and marxian models. *American Economic Review 75* (1985), 16–36.

[15] Chenery, H., and Syrquin, M. *Patterns of development, 1950-1970.* Oxford University Press, 1975.

[16] Dixit, A. *Models of dual economies.* (in "Models of economic growth") Mirlees and Stern (editors) Macmillan, 1989.

[17] Dutta, J. The wage good constraint on a developing economy: Theory and evidence. *Journal of Development Economics 28* (1988), 341–363.

[18] Easterly, W. Reliving in the 50s: the big push, poverty traps, and takeoffs in economic development. *mimeo* (2006).

[19] Fei, J., and Ranis, G. A theory of economic development. *American Economic Review 51* (1961), 533–565.

[20] Galor, O. Convergence? inferences from theoretical models. *The Economic Journal 106* (1996), 1056–1069.

[21] Galor, O., and Zeira, J. Income distribution and macroeconomics. *Review of Economic Studies 60* (1993), 35–52.

[22] Gandolfo, G. *Economic dynamics*. Springer, 1997.

[23] Hansen, G., and Prescott, E. Malthus to Solow. *American Economy Review 92* (2002), 1205–1217.

[24] Jorgenson, D. The development of a dual economy. *Economic Journal 71* (1961), 309–334.

[25] Kaldor, N. *Strategic Factors in Economic Development*. Ithaca: Cornell University, 1967.

[26] Kaldor, N. Productivity and growth in manufacturing industry: a reply. *Economica 35* (1968), 385–391.

[27] Kraay, A., and Raddatz, C. Poverty traps, aid, and growh. *Journal of Development Economics 82* (2007), 315–347.

[28] Kremer, M. The o-ring theory of economic development. *Quarterly Journal of Economics 108* (1993), 551–575.

[29] Krugman, P. The narrow moving band, the dutch disease, and the competitive consequences of mrs. thatcher. *Journal of Development Economics 27* (1987), 41–55.

[30] Krugman, P. Toward a counter-counterrevolution in development theory. *Proceedings of the Annual World Bank Conference 1992 (World Bank Economic Review)* (1992).

[31] Kuznets, S. *Modern economic growth*. Yale University Press, 1966.

[32] Lewis, A. Economic development with unlimited supplies of labor. *Manchester School of Economics and Social Studies 28* (1954), 139–191.

[33] Lewis, A. Unlimited labor: further notes. *Manchester School of Economics and Social Studies 32* (1958), 1–32.

[34] Mankiw, G., Romer, D., and Weil, D. A contribution to the empirics of economic growth. *The Quarterly Journal of Economics 107* (1992), 407–437.

[35] Matsuyama, K. Agricultural productivity, comparative advantage, and economic growth. *Journal of Economic Theory 58* (1992), 317–334.

[36] Murphy, K., Shleifer, A., and Vishny, R. Income distribution, market size and industrialization. *Quarterly Journal of Economics 104* (1989), 537–564.

[37] Murphy, K., Shleifer, A., and Vishny, R. Why is rent-seeking so costly to growth? *American Economic Review 83* (1993), 409–414.

[38] Nelson, R. A theory of the low level equilibrium trap in underdeveloped economies. *American Economic Review 46* (1956), 894–908.

[39] Nurkse, R. *Problems of Capital Formation in Underdeveloped Countries.* Oxford: Basil Blackwell, 1953.

[40] Pasinetti, L. *Structural Economic Dynamics.* Cambridge University Press, 1993.

[41] Preobrazhensky, E. *The new economics.* Oxford University Press, 1965.

[42] Quah, D. Galton's fallacy and tests of the convergence hypothesis. *The Scandinavian Journal of Economics 95* (1993), 427–443.

[43] Quah, D. Twin peaks: growth and convergence in models of distribution dynamics. *The Economic Journal 106* (1996), 1045–1055.

[44] Ray, D. *Development economics.* Princeton University Press, 1998.

[45] Ros, J. *Development theory and the economics of growth.* The University of Michigan Press, 2000.

[46] Ros, J., and Skott, P. The big push in an open economy with non-tradable inputs. *Journal of Post Keynesian Economics 20* (1997), 149–162.

[47] Rosenstein-Rodan, P. Problems of industrialization in Eastern and South-Eastern Europe. *Economic Journal 53* (1943), 202–211.

[48] Rosenzweig, M. R. *Labor Markets in Low Income Countries.* (in "Handbook of development economics") Chenery and Srinivasan (editors) Elsevier Science Publishers, 1988.

[49] Sachs, J. *The end of poverty: how we can make it happen in our lifetime.* Penguin Books, 2005.

[50] Sala-i Martin, X. The classical approach to convergence analysis. *The Economic Journal 106* (1996), 1019–1036.

[51] Scitovsky, T. Two concepts of external economy. *Journal of Political Economy 62* (1954), 143–151.

[52] Smith, A. *Early draft of part of the Wealth of Nations.* (in "Lectures of Jurisprudence") Meek, Raphael and Stein (editors) Claredon Press, 1978.

[53] Solow, R. M. A contribution to the theory of economic growth. *Quarterly Journal of Economics 70* (1956), 65–94.

[54] Solow, R. M. Technical change and the aggregate productio function. *The Review of Economics and Statistics 39* (1957), 312–320.

[55] SOLOW, R. M. Perspectives on growth theory. *Journal of Economic Perspectives 8* (1994), 45–54.

[56] STIGLITZ, J. Comment on toward a counter-counterrevolution in development theory. *Proceedings of the Annual World Bank Conference 1992 (World Bank Economic Review)* (1992).

[57] STOKEY, N. Learning by doing and the introduction of new goods. *Journal of Political Economy 96* (1988), 701–717.

[58] SYRQUIN, M. *Sector proportions and Economic Development: the evidence since 1950.* (in "The balance between agriculture and industry in economic development") Panchamukhi and Williamson(editors) Macmillan, 1989.

[59] TAYLOR, L. *Theories of Sectoral Balances.* (in "The balance between agriculture and industry in economic development") Panchamukhi and Williamson(editors) Macmillan, 1989.

[60] VERDOORN, P. J. Fattori che regolano lo sviluppo della produttività del lavoro. *L'industria 1* (1949), 45–53.

[61] YOUNG, A. Increasing returns and economic progress. *Economic Journal 38* (1928), 527–542.