Distributional effects of capital and labor on economic growth

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Abstract

In the following, we propose a growth model for an economy consisting of firms which are heterogeneous in technologies and input demands. We show that the growth rate in this economy depends not only on changes in the aggregate level of capital and labor, but also on changes in the allocation of these inputs across firms. As the latter effects are neglected in conventional growth models, they are misleadingly captured by the residual TFP measure. In contrast, we are able to quantify the influence of these components. Our empirical analysis, which is based on structural estimation from firm-level data, reveals that changes in allocation of capital and labor have pronounced effects on GDP-growth for most European countries. Further, we take cross-country differences in the distributional effects into account to improve conventional growth accounting exercises. In particular, we find that they explain additionally up to 17% of growth differences among 19 European countries. Consequently, allowing for heterogeneity in firm-level technologies and input demands increases the explanatory power of the inputs.

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1 Introduction

In the following, we propose a growth model for an economy consisting of firms which are heterogeneous in technologies and input demands. We show that the growth rate in this economy depends not only on changes in the aggregate level of capital and labor, but also on changes in the allocation of these inputs across firms. As the latter effects are neglected in conventional growth models, they are misleadingly captured by the residual measure, referred to as total factor productivity (TFP). In contrast, we are able to quantify the influence of these components by structural estimation from firm-level data. Further, we take cross-country differences in the distributional effects into account to improve conventional growth accounting exercises.

Why do some countries grow and others stagnate?[1] This question initiated the growth accounting literature, which assigns cross-country differences in growth or income to differences in physical and human capital as well as the unobservable efficiency with which input factors are combined. The consensus view in this literature is that only approximately one third of the cross-country growth or income differences is explained by differences in input factors. The residual two thirds are left unexplained and attributed to differences in the unobservable efficiency which is referred to as total factor productivity (TFP).[2] In this context, Abramovitz (1956) refers to TFP as the *measure of our ignorance*.

The fact that TFP is unobservable and at the same time explains the major part of cross-country differences triggered tremendous efforts to identify its determinants in recent years.[3] However, we show in this paper that the above growth accounting results have to be revised if one consistently aggregates over heterogeneous firms. In order to illustrate the relevance of aggregation for growth models we briefly discuss fundamental results of the aggregation literature.

The pillar of every macroeconomic growth model is an aggregate production function F, which relates aggregate capital $\bar{K}$ and labor $\bar{L}$ to aggregate output $\bar{Y}$, i.e., $\bar{Y} = F(\bar{K}, \bar{L})$. However, although there exists a well developed microeconomic theory of production for a single firm, there is no corresponding theoretical foundation for the entire economy. In fact, the aggregate production function suffers from two types of aggregation problems. The first,

[1] The Science magazine considers this question as one of the 125 "most compelling puzzles and questions facing scientists today" (Science, 2005).

[2] See, for example, Caselli (2005), Hall and Jones (1999) or Jorgensen (2005).

[3] This issue is best summarized by the title of a recent paper by Prescott (1998) "Needed: A Theory of Total Factor Productivity."

often referred to as the "measurement problem," involves the aggregation of different types of capital, labor, and output within a firm into one capital and labor input and one output. The second is concerned with aggregation of heterogeneous technologies and input demands across firms into their aggregate counterpart. These problems have been dealt with extensively in the aggregation literature. Early works by Nataf (1948), Gorman (1953), and a series of papers by Franklin Fisher (collected in Fisher, 1993)[4] have shown that in the absence of perfect competition and perfect factor mobility the aggregate production function F cannot be linked to microeconomic production functions unless all firms operate according to identical and constant returns to scale technologies.

A frequent short-cut that circumvents the problem of aggregation over heterogeneous technologies is the assumption that the production function of an entire economy complies with the one of a single *representative firm*. Although the above theoretical results show that this link is only possible under very restrictive assumption, it is often applied in theoretical and empirical analysis due to its simplicity. However, from a practical point of view, growth models that ignore consistent aggregation over heterogeneous firms will suffer from serious drawbacks:[5] they neglect growth effects of (i) changes in the allocation of inputs[6] and (ii) changes in the pattern of economic interactions between firms. Yet, it is reasonable to expect that these factors affect growth substantially, since they represent changes in growth due to changes in the market structure. For example, differences in the degree of competition in different industries as well as different incentives to innovate for small, medium, and large firms are found to affect technological change (see, e.g., Aghion and Griffith, 2005). Where are these effects in the growth literature? As they are not assigned to the levels of aggregate capital or labor, they are assigned to the unobserved efficiency. Therefore, they are misleadingly captured by the residual TFP measure.

[4]For a comprehensive survey on aggregation of production functions, see Felipe and Fisher (2003).

[5]Hopenhayn (1992) initiated a literature on the effect of firm heterogeneity on industry dynamics. His approach was extended, e.g., by Melitz (2003) to analyze the impact of trade liberalization on the aggregate productivity of an economy. In these models firms are heterogeneous in productivity which is included in a way such that the impact of the productivity distribution on aggregate demand for inputs is fully determined by the average productivity. Consequently, under this parsimonious aggregation rule, aggregate output depends on average productivity and average input demands but not on the allocation of inputs across firms. That is, once the average productivity level is determined the model yields identical aggregate outcomes as a model based on a representative firm.

[6]Empirical studies document that these changes are substantial in developed and developing countries. For example, Roberts and Tybout (1997) quantify the rate of labor reallocation among manufacturing firms between 25 and 30 percent.

In order to assess the impact of changes in the allocation of capital and labor on growth, we apply the aggregation procedure established by Hildenbrand and Kneip (2005). Our main result is that the growth rate of aggregate output depends on changes in the levels of aggregate capital and labor as well as changes in the distribution of capital and labor in the economy. We quantify the growth effect of each component by means of structural estimation based on firm-level data. These effects are estimated separately for each of 20 European countries. Our main findings are that distributional effects are significant in all countries. Further they are as large as the corresponding level effects in most countries. Finally, we exploit the information on the different distributional changes across countries to conduct a growth accounting exercise. More precisely, we assess the explanatory power of the distributional changes with respect to cross-country growth differences. It turns out that these effects explain additionally up to 17%. Accordingly, an aggregation approach that consistently accounts for firm heterogeneity can help explain the growth path of a single country as well as cross-country growth differences. Hence, the role of capital and labor in explaining the growth path of a single country or growth differences across countries is understated.

In the next section, we present our growth model for an economy consisting of heterogeneous firms. In Section 3, we describe the data, the empirical strategy, and discuss our results. Section 4 presents the growth accounting exercise, whereas the final section concludes.

2 The Model

Assume that in period t each firm j from a heterogeneous population of firms J_t produces according to the firm-specific production function $f_t^j(\cdot)$ defined by

$$Y_t^j = f_t^j(K_t^j, L_t^j),$$

where Y_t^j denotes the output level, K_t^j the capital stock and L_t^j the labor demand.[7] Further, we assume that the heterogeneity in production functions f_t^j, i.e., in technologies and input demands, can be parametrized by a vector of parameters V_t^j. In general, V_t^j is unobservable. Then one can write

$$Y_t^j = f(K_t^j, L_t^j, V_t^j). \quad (1)$$

Hence, technological changes over time translate into changes in the distribution of V_t^j across J_t. The function f can therefore, without loss of generality, be regarded as time-invariant and equal for all individuals. In the simplest scenario, f could be a Cobb-Douglas production function

[7]One can easily extend the model to the case of multiple capital and labor inputs.

with $V_t^j = (V_{1,t}^j, V_{2,t}^j)$ such that $Y_t^j = V_{1,t}^j \cdot {K_t^j}^{V_{2,t}^j} \cdot {L_t^j}^{1-V_{2,t}^j}$. However, in order to establish our main result at the aggregate level, an explicit parametric specification of f is not required.

In the above setup, we define aggregate output $\bar{Y}_t$ in period t as

$$\bar{Y}_t = \int f(K, L, V)\, dG_{t,KLV}, \tag{2}$$

where K, L, and V are generic random variables corresponding to capital, labor, and unobservable productivity parameters of a randomly chosen firm, respectively, and $G_{t,KLV}$ is the joint distribution of (K, L, V) across the population J_t. Thus, G_{KLV} is the explanatory variable for aggregate output. However, we do not need to model G_{KLV} but only its changes over time, since our objective is to determine the growth rate instead of the level of aggregate output.

In order to impose a structure on the evolution of the unobservable distribution of V, we introduce a set of observable firm specific attributes A_t^j with the corresponding random variable A, which are expected to be correlated with V: the age of a firm, the region or industry in which it operates, its ownership structure, and its legal form.

Further, we use A to decompose $G_{t,KLV}$ into the distributions $G_{t,V|KLA}$, $G_{t,A|KL}$, and $G_{t,KL}$. The first is the conditional distribution of V given (K, L, A), the second is the conditional distribution of A given (K, L), the third is the joint distribution of (K, L). We write

$$\bar{Y}_t = \int \left[\int \left(\int f(K, L, V)\, dG_{t,V|KLA} \right) dG_{t,A|KL} \right] dG_{t,KL} = \int \left(\int \bar{f}_t(K, L, A)\, dG_{t,A|KL} \right) dG_{t,KL}, \tag{3}$$

where $\bar{f}_t(K, L, A)$ is the conditional mean of output Y given (K, L, A) in period t. Thus, it is a regression function of Y on (K, L, A), which can be estimated from a cross-section of firms in period t.

From (3) we infer that assumptions on changes in $G_{V|KLA}$, $G_{A|KL}$, and G_{KL} are required in order to model output growth. It is easier to model the evolution of a distribution if it is symmetric, because a symmetric distribution can be well-described by its first few moments, like its mean and variance. Since the distributions of capital and labor are right-skewed in all countries, we formulate the model assumptions in terms of log capital $k_t^j := \log K_t^j$ and log labor $l_t^j := \log L_t^j$ with the corresponding random variables k and l. Further, we define $\bar{k}_t$ and $\bar{l}_t$ as the mean of k and l across J_t, respectively, and σ_t^k and σ_t^l as the corresponding standard deviations. In addition, by analogy to $G_{V|KLA}$, $G_{A|KL}$, and G_{KL}, we define $G_{V|klA}$, $G_{A|kl}$, and G_{kl}, respectively. In addition, G_k and G_l represent marginal distributions of log capital and log labor, respectively. Finally, let $G_{\tilde{k}\tilde{l}}$ denote a component-wise standardized joint distribution of (k, l), which is defined as a joint distribution of $(\tilde{k}, \tilde{l})$, where $\tilde{k} := \frac{k-\bar{k}}{\sigma^k}$ and $\tilde{l} := \frac{l-\bar{l}}{\sigma^l}$.

In line with the aggregation approach of Hildenbrand and Kneip (2005), we impose following assumptions.

Assumption 1: ("Structural stability"[8] of G_{kl}) *The component-wise standardized joint distribution of log capital and log labor $G_{\tilde{k}\tilde{l}}$ is approximately equal for two consecutive periods t and $t-1$, i.e., $G_{t,\tilde{k}\tilde{l}} \approx G_{t-1,\tilde{k}\tilde{l}}$.*

It is important to note that $G_{\tilde{k}\tilde{l}}$ refers to a standardized distribution. That is, if Assumption 1 holds, the entire change in G_{kl} over two consecutive periods is fully captured by the changes in means and the variances of k_t^j and l_t^j.[9]

In order to impose the assumption on the evolution of $G_{A|kl}$ we define $k_{t,\tau}$ as the τ-quantile of the distribution $G_{t,k}$ and $l_{t,\eta}$ as the η-quantile of the distribution $G_{t,l}$.

Assumption 2: *The conditional distribution of A given $k = k_\tau$ and $l = l_\eta$ denoted by $G_{A|k_\tau l_\eta}$ is approximately equal for two consecutive periods t and $t-1$, i.e., $G_{t,A|k_\tau l_\eta} \approx G_{t-1,A|k_\tau l_\eta}$.*

Assumption 2 refers to the distribution of A across firms with log capital and log labor in the *same quantile position* (τ, η) of G_{kl} in period t and $t-1$, instead of firms with the *same values* of k and l. We employ the former specification since it increases the likelihood that we condition on the same group of firms in both periods. That is, if G_{kl} shifts over time due to a common trend, we refer to the same group of firms in both periods by conditioning on the quantile position as opposed to conditioning on the same values of k and l.

Note that one is able to verify Assumptions 1 and 2, since G_{kl} and $G_{A|kl}$ are observable in firm-level data. We document in the Appendix A that both assumptions are supported by our

[8]The concept of structural stability of a distribution relies on an empirical regularity that distributions of individual variables across large populations of economic agents change very slowly over time. It has been first noticed by Pareto (1896) and introduced into macroeconomic models by Malinvaud (1993). More precisely, for a distribution of a certain parametric form, for example, the normal distribution, structural stability holds, if its normal structure prevails and its entire evolution is captured by changes in its mean and its variance. However, this concept of structural stability cannot be applied to distributions which are poorly approximated by a parametric form. In this context, Hildenbrand and Kneip (1999) proposed a nonparametric counterpart of Malinvaud's idea. Instead of keeping the parametric structure constant and allowing for changes over time in few parameters, one can keep these parameters constant and allow the shape of the distribution to vary over time. This can be achieved by simple transformations of the distribution like centering (constant mean) or standardizing (constant mean and variance). Accordingly, structural stability as defined by Hildenbrand and Kneip (1999) holds, if a centered or standardized distribution does not change over two consecutive periods.

[9]To be more precise, Hildenbrand and Kneip (2005) model the evolution of G_{kl} in terms of a distribution which is standardized by a full covariance matrix $\Sigma_t := \begin{pmatrix} (\sigma_t^k)^2 & \sigma_t^{kl} \\ \sigma_t^{kl} & (\sigma_t^l)^2 \end{pmatrix}$, instead of a component-wise standardized one, which uses the matrix $\tilde{\Sigma}_t = \begin{pmatrix} (\sigma_t^k)^2 & 0 \\ 0 & (\sigma_t^l)^2 \end{pmatrix}$. Our version of the assumption is more stringent, as it requires that the correlation between log capital and log labor is does not change significantly over two consecutive periods. The main advantage of our formulation (see Proposition and Appendix C) is the possibility to separate growth effects of changes in σ^k from growth effects of changes in σ^l.

data. In contrast, one is not able to falsify the following two assumptions on $G_{V|klA}$ as they concern a distribution of unobservable variables.

Let $J_t(k,l,A)$ denote the subpopulation of firms with capital k, labor l and attributes A and $\bar{V}_t(k,l,A)$ denote the mean of V across $J_t(k,l,A)$. Further, $G_{\tilde{V}|klA}$ denotes the centered distribution of V across $J_t(k,l,A)$, whereby $\tilde{V}$ corresponds to the centered variable $\tilde{V} := V - \bar{V}_t(k,l,A)$.

Assumption 3: *The distribution $G_{\tilde{V}|klA}$ is approximately equal for two periods t and $t-1$, i.e., $G_{t,\tilde{V}|klA} \approx G_{t-1,\tilde{V}|klA}$.*

Note that Assumption 3 is a very mild assumption since we allow for any form of heterogeneity in V across firms with different capital stocks, labor stocks, or firm characteristics. Furthermore, we even allow for heterogeneity in V across firms with the same capital stock, labor stock, and firm characteristics, as long as changes in $G_{V|klA}$ are captured by changes in $\bar{V}(k,l,A)$. In this case, we assume that $\bar{V}_t(k,l,A)$ is additively separable in (k,l) and t. More precisely,

Assumption 4: *$\bar{V}_t(k,l,A)$, can be additively factorized by $\bar{V}_t(k,l,A) = \varphi(k,l,A) + \psi(t,A)$, where the function φ is continuously differentiable in k and l.*

Proposition: (Hildenbrand and Kneip, 2005) *If Assumptions 1-4 hold, the growth rate of aggregate output in the economy, $g_t := \frac{\bar{Y}_t - \bar{Y}_{t-1}}{\bar{Y}_{t-1}}$, is given by*

$$\begin{aligned} g_t &= \beta^k_{t-1}(\log \bar{K}_t - \log \bar{K}_{t-1}) + \beta^l_{t-1}(\log \bar{L}_t - \log \bar{L}_{t-1}) && (4) \\ &+ \gamma^k_{t-1}\Big(\frac{\sigma^k_t - \sigma^k_{t-1}}{\sigma^k_{t-1}}\Big) + \gamma^l_{t-1}\Big(\frac{\sigma^l_t - \sigma^l_{t-1}}{\sigma^l_{t-1}}\Big) && (5) \\ &+ \text{(effects due to changes in } \bar{V}_{t-1}(k,l,A)) \\ &+ \text{(second order terms of the Taylor expansion).} \end{aligned}$$

The coefficients β^k_{t-1}, β^l_{t-1}, γ^k_{t-1}, and γ^l_{t-1} are defined in terms of partial derivatives of the regression function $\bar{f}_{t-1}(k,l,A)$. For $s = \{k,l\}$ and $S = \{K,L\}$, β^s_{t-1}, γ^s_{t-1} are defined by

$$\beta^s_{t-1} = \frac{1}{\bar{Y}_{t-1}} \int \partial_s \bar{f}_{t-1}(k,l,A)\, dG_{t-1,klA}, \tag{6}$$

$$\gamma^s_{t-1} = \frac{1}{\bar{Y}_{t-1}} \int (s - \bar{s}_{t-1}) \partial_s \bar{f}_{t-1}(k,l,A)\, dG_{t-1,klA} - \frac{\beta^s_{t-1}}{\bar{S}_{t-1}} \int (s - \bar{s}_{t-1}) \exp(s) dG_{t-1,s} \tag{7}$$

Remark 1: The proof is given in Hildenbrand and Kneip (2005). However, the above Proposition differs from the one in Hildenbrand and Kneip (2005) in two aspects. First, our Assumption

1 relies on a component-wise standardization which makes it possible to separate growth effects of changes in σ^k from growth effects of changes in σ^l. Second, we model the aggregate relation in terms of the logarithm of aggregate variables, i.e., $\log \bar{K}$ and $\log \bar{L}$ and not the aggregates of the logarithms of individual variables, i.e., $\bar{k}$ and $\bar{l}$. This distinction yields different definitions of γ^k_{t-1} and γ^l_{t-1} and is essential to compare our model with conventional growth models, which are based on (the logarithm of) aggregate variables. See Appendix C for the derivations.

From the above representation we infer that the growth rate g of aggregate output does not only depend on changes in aggregate capital and aggregate labor (term (4)). It also depends on changes in the allocation of inputs (term (5)) measured by the standard deviation of log capital and log labor across firms.

The aggregate coefficients $(\beta^k_{t-1}, \gamma^k_{t-1})$ and $(\beta^l_{t-1}, \gamma^l_{t-1})$ depend on the derivatives of the regression function $\bar{f}_{t-1}$ with respect to k and l, respectively. All other variables in (7) are observable. The derivatives $\partial_k \bar{f}_{t-1}(k, l, A)$ and $\partial_l \bar{f}_{t-1}(k, l, A)$ can be estimated using a cross-section of firms in period $t-1$. Hence, they can be estimated independently of each other in each period. It is important to note that in the estimation of our representation of the growth rate *no time-series model* fitting takes place, which would require to include all potential growth determinants. Our estimation procedure does not require the information on the growth rate of aggregate capital and aggregate labor nor the corresponding standard deviations since the computation of the aggregate coefficients is based on the estimation of a single cross section of firms. In contrast, we are able to quantify the growth effect of changes in the distribution of inputs without specifying an exhaustive model for the aggregate growth rate. We describe the estimation methodology for these coefficients in more detail in Section 3.2.

Remark 2: Under Assumption 1 coefficients β^k_{t-1} and β^l_{t-1} can be interpreted as elasticities of aggregate output with respect to aggregate capital and aggregate labor, respectively. Accordingly, γ^k_{t-1} and γ^l_{t-1} are elasticities of aggregate output with respect to σ^k and σ^l, respectively.[10] One expects β^k_{t-1} and β^l_{t-1} to be positive. However, to draw conclusions on the expected sign of γ^k_{t-1} and γ^l_{t-1} one needs to impose additional assumptions on the impact of changes in the market structure on the standard deviation of inputs. For example, if a higher degree of product market competition leads to more similarity in firm size, negative γ^k_{t-1} and γ^l_{t-1} indicate a positive relationship between growth and competition. Alternatively, we outlined above that changes in the standard deviation represent changes in the pattern of economic interactions between firms. These interactions comprise, for instance, technology spill-overs between firms. If technology diffusion is stronger among more similar firms, we expect a negative relation

[10] See Hildenbrand and Kneip (2005) for a detailed discussion on the interpretation of the coefficients.

between spill-overs and the standard deviation of inputs and, hence, negative γ^k_{t-1} and γ^l_{t-1}.

Our theoretical result has an important implication for growth accounting. To illustrate this point, let us hypothetically claim that all variables in our model other than capital and labor do not change over time. Then, in a classical growth model, changes in $\bar{Y}$ would be in part attributed to changes in $\bar{K}$ and $\bar{L}$. However, a part of the change in $\bar{Y}$, which is not captured by the effect of changes in $\bar{K}$ and $\bar{L}$, would be attributed to changes in aggregate TFP. Such a conclusion, however, would be misleading, since we assumed that TFP did not change. From the Proposition we know that it is the effect of changes in the distribution of inputs, which is erroneously attributed to changes in TFP. Obviously, such a correct conclusion is only possible in models which allow for input heterogeneity of firms.

3 Empirical Analysis

In the following, we structurally estimate the effects of changes in the level and allocation of capital and labor on growth separately for each of 20 European countries in our sample.

3.1 Data

The analysis is based on European firm-level data from 2002 until 2004.[11] The data stem from the Bureau van Dijk's AMADEUS data base. It contains information from firm balance sheets and covers all firms in each country. We measure output as real[12] value added. Capital and labor are measured as real fixed tangible assets and the real total cost of of employees,[13] respectively. Our procedure requires that the firms have non-missing observations in 2003. Moreover, we only include countries in which data for at least 200 firms are available.

Furthermore, we include firm's age and other variables to control for differences in economic environment across firms. In particular, we account for industry-specific and region-specific

[11]We estimate the corresponding coefficients exclusively for 2003. Yet, we need additional observations in 2002 for the Olley and Pakes (1996) estimation procedure and in 2004 for the growth accounting exercise.

[12]Real variables are obtained by deflating by the national output price deflators. Unfortunately, price deflators were not available at the industry level for most of the 20 European countries.

[13]We define labor in this way in order to account, to a certain extent, for differences in the quality of employees, i.e., human capital, across firms. These differences are captured by the total cost of employees, as long as firms that are characterized by the same capital stock, number of employees and the same attribute profile A, (that is, the same industry, region, age, ownership structure, etc.) but a higher human capital stock pay higher wages. We emphasize that the qualitative results do not change if we define labor as the number of employees. These results are available from authors upon request.

fixed effects, in that we distinguish sectors by means of two digit NACE codes and include regional dummies. Moreover, we incorporate dummy variables that capture the ownership status of a firm: (i) `quoted` takes value 1 if a firm is publicly quoted and 0 if not, while (ii) `indep1- indep9` correspond to independence indicators (defined in the AMADEUS data base) which represent different shareholder structures. Finally, we include gross investment, measured by the change in the capital stock plus depreciation, which is included as an instrument for the unobservable technology shock in the estimation procedure of Olley and Pakes (1996).

The descriptive statistics of the variables for each country in 2003 and 2004 are listed in Table 1. The first column indicates that the number of observations used for estimation varies substantially across countries in our sample. These differences can be attributed to different filing regulations of individual countries. For example, German companies are not legally obliged to reveal the information from their balance sheets. Hence, although the full sample for Germany covers over 800,000 firms in 2003, joint information on value added, fixed tangible assets and the number of employees is available for only roughly 6,000 German firms. In contrast, the corresponding information is available for most companies in the Spanish or Italian sample which contain about 360,000 and 117,000 observations in 2003, respectively. Analogously, mean and variances of the variables differ noticeably across countries. We observe relatively large firms in Germany, the Netherlands, Austria, Great Britain and Portugal, whereas the sample covers relatively many small firms in Romania, Spain, Italy, and Sweden. Accordingly, we also observe analog differences in the standard deviations.

Summing up, the data reveals a substantial amount of heterogeneity both across firms within a country as well as across countries.

3.2 Estimation strategy

The aggregate coefficients β_t^s and γ_t^s, $s \in \{k, l\}$ can be estimated as (suitably weighted) average derivatives in the regression of value added Y_t^j on log capital k_t^j, log labor l_t^j, and a vector of firm specific attributes A_t^j, i.e., in the model

$$Y_t^j = \bar{f}_t(k_t^j, l_t^j, A_t^j; \zeta) + u_t^j, \tag{8}$$

where ζ is the vector of parameters to be estimated and u_t^j is the error term with $E(u_t^j) = 0$. Hence, according to (6) and (7), once consistent estimates $\partial_s \widehat{\bar{f}_t(k, l, A}; \zeta)$ of $\partial_s \bar{f}_t(k, l, A; \zeta)$, $s \in \{k, l\}$, are obtained, one can estimate aggregate coefficients by

$$\hat{\beta}_t^k = \frac{\sum_{j \in J_t} \partial_k \bar{f}_t\widehat{(k_t^j, l_t^j}, A_t^j)}{\sum_{j \in J_t} Y_t^j}, \quad \hat{\beta}_t^l = \frac{\sum_{j \in J_t} \partial_l \bar{f}_t\widehat{(k_t^j, l_t^j}, A_t^j)}{\sum_{j \in J_t} Y_t^j}, \tag{9}$$

$$\hat{\gamma}_t^k = \frac{\sum_{j\in J_t}(k_t^j - \hat{\bar{k}}_t)\partial_k \bar{f}_t\widehat{(k_t^j, l_t^j, A_t^j)}}{\sum_{j\in J_t} Y_t^j} - \frac{\hat{\beta}_t^k}{\bar{K}_t}\sum_{j\in J_t}(k_t^j - \hat{\bar{k}}_t)K_t^j, \text{ and} \tag{10}$$

$$\hat{\gamma}_t^l = \frac{\sum_{j\in J_t}(l_t^j - \hat{\bar{l}}_t)\partial_l \bar{f}_t\widehat{(k_t^j, l_t^j, A_t^j)}}{\sum_{j\in J_t} Y_t^j} - \frac{\hat{\beta}_t^l}{\bar{L}_t}\sum_{j\in J_t}(l_t^j - \hat{\bar{l}}_t)L_t^j. \tag{11}$$

Our empirical strategy is focused on the model specification and estimation for $\bar{f}_t$. However, our analysis revealed that a regression of $y_t^j := \log Y_t^j$ on (k_t^j, l_t^j, A_t^j) provides a significantly better model fit and stability of results, as compared to the regression of Y_t^j on (k_t^j, l_t^j, A_t^j). Consequently, we estimate derivatives of $\bar{f}_t$ from the model

$$y_t^j = \bar{h}_t(k_t^j, l_t^j, A_t^j; \theta) + \varepsilon_t^j, \tag{12}$$

where θ is the vector of parameters to be estimated and ε_t^j is the error term with $E(\varepsilon_t^j) = 0$. In doing so, we use the fact that $\partial_s \bar{f}_t(k_t^j, l_t^j, A_t^j; \hat{\zeta}) = Y_t^j \partial_s \bar{h}_t(k_t^j, l_t^j, A_t^j; \hat{\theta})$, if $\hat{\zeta}$ and $\hat{\theta}$ are consistent estimates of ζ and θ, respectively.Our basic specification for $\bar{h}_t$ is linear in (k, l, A) and can be estimated using OLS. Further, we analyze the robustness of our results in two ways. First, we control for possible simultaneity between ε_t^j and (k, l) using the Olley and Pakes (1996) method. Second, we extend our analysis to a partially linear specification of $\bar{h}_t$, in which the relationship between y and (k, l) is modeled nonparametrically. Doing this, we avoid a parametric misspecification of $\bar{h}_t$.

The loglinear model

Our basic specification for $\bar{h}_t$ is the loglinear model, i.e.,

$$y_t^j = \theta_0 + \theta^k k_t^j + \theta^l l_t^j + \theta'_A A_t^j + \varepsilon_t^j, \tag{13}$$

which implies that $\partial_k \bar{f}_t\widehat{(k_t^j, l_t^j}, A_t^j) = \hat{\theta}^k Y_t^j$ and $\partial_l \bar{f}_t\widehat{(k_t^j, l_t^j}, A_t^j) = \hat{\theta}^l Y_t^j$.[14] These quantities are then imputed into (9) - (11), in order to calculate aggregate parameters.

In the simplest case, (13) can be estimated by the OLS method from a single cross-section in 2003. However, the vast literature on estimation of production functions from plant-level data points out that OLS may suffer from a simultaneity problem. This problem arises if there is a contemporaneous correlation between the demand for inputs k_t^j, l_t^j and the realization of the unobservable technology shock contained in ε_t^j. In such a case, estimates $\hat{\theta}^k$ and $\hat{\theta}^l$, and, hence, $\hat{\beta}^k$ and $\hat{\beta}^l$ would be biased. There are several approaches to correct for simultaneity between

[14]Note that in this model, $\hat{\beta}^k = \hat{\theta}^k$ and $\hat{\beta}^l = \hat{\theta}^l$.

(k_t^j, l_t^j) and ε_t^j and all of them put additional restrictions on the data. For instance, Olley and Pakes (1996) propose a method, which uses changes in firm's investment decision as a proxy for the productivity shock. However, only firms with non-missing data for 2002 and 2003 on value added, capital, labor, and investment can be used for estimation. Depending on the country, this requirement involves an elimination of up to 70% of the companies from our sample of firms with non missing data on value added, capital, and labor in 2003. Moreover, the above method may introduce a sample selection bias, if dropping out of the sample between 2002 and 2003 is non-random. Following the same idea, Levinsohn and Petrin (2003) suggest the use of intermediate inputs instead of the investment variable as a proxy.[15] Finally, as described in Blundell and Bond (2000), the simultaneity problem in estimation of production function can also be bypassed by a GMM system estimator, though it requires a long time-series of cross-sections and is therefore not attractive for our analysis.

Being aware of problems mentioned above, we consistently estimate (13) following Olley and Pakes (1996) in controlling for both simultaneity bias and sample attrition. The method is based on a two-step procedure and requires following assumptions: (i) labor is the only input which contemporaneously responds to a technology shock, (ii) capital stock is predetermined and hence uncorrelated with a contemporary technology shock, (iii) changes in corporate investment decisions depend on the contemporaneous technology shock, the age and the capital stock of a firm, (iv) investments are monotonically increasing in the technology shock for a given value of age and capital. Under these assumptions, the technology shock can be instrumented as a function of capital, age, and investment. The estimation of this function is carried out by a series estimator. A detailed description of the method is given in the Appendix B.

Semiparametric model

In order avoid a misspecification of the relationship between y and (k, l, A) we model $\bar{h}_t$ semi-parametrically and include an interaction term, i.e.,

$$y_t^j = \theta_0 + \bar{h}_t^k(k_t^j) + \bar{h}_t^l(l_t^j) + \theta^{kl} k_t^j l_t^j + \theta_A' A_t^j + \varepsilon_t^j, \tag{14}$$

where $\bar{h}_t^k$ and $\bar{h}_t^l$ are differentiable in k and l, respectively. We model $\bar{h}_t^k$ as a quadratic splines function with D^k knots $d_1^k < d_2^k < \cdots < d_{D^k}^k$. Defining basis functions $b_i^k(k) = \max\{0, k - d_i^k\}^2$, we obtain $\bar{h}_t^k(k) = \theta_1^k k + \theta_2^k k^2 + \sum_{i=1}^{D^k} \theta_{3,i}^k b_i^k(k)$. Analogously, we model $\bar{h}_t^l$ as $\bar{h}_t^l(l) = \theta_1^l l +$

[15] They motivate their choice by weaker data requirements and argue that an adjustment in intermediate inputs is likely to have better properties as an instrument for a technology shock than an adjustment in investment. Interestingly, the approach of Levinsohn and Petrin (2003) requires even more firms to be eliminated from our sample due to the very large number of firms with missing data on the use of materials.

$\theta_2^l l^2 + \sum_{i=1}^{D^l} \theta_{3,i}^l b_i^l(l)$. All coefficients in (14) can be estimated by the OLS method. Accordingly, $\partial_k \bar{f}_t(k_t^j, l_t^j, A_t^j)$ can be estimated as

$$\partial_k \bar{f}_t\widehat{(k_t^j, l_t^j, A_t^j)} = \Big(\hat{\theta}_1^k + 2\hat{\theta}_2^k k_t^j + \hat{\theta}^{kl} l_t^j + 2\sum_{i=1}^{D^k} \hat{\theta}_{3,i}^k \max\{0, k_t^j - d_i^k\}\Big) Y_t^j.$$

Similarly, one obtains $\partial_l \bar{f}_t\widehat{(k_t^j, l_t^j, A_t^j)} = (\hat{\theta}_1^l + 2\hat{\theta}_2^l l_t^j + \hat{\theta}^{kl} k_t^j + 2\sum_{i=1}^{D^l} \hat{\theta}_{3,i}^l \max\{0, l_t^j - d_i^l\}) Y_t^j$. The optimal number of knots and their position is obtained by the minimization of the Mallows' C_p criterion (see Mallows, 1973) using the knot deletion method as described by Fan and Gijbels (1996, p. 42).[16]

Statistical significance of the aggregate coefficients

Condifence intervals for the aggregate coefficients as well as standard errors of the estimates are determined by bootstrap. For i.i.d. bootstrap resamples $(Y_t^{j*}, k_t^{j*}, l_t^{j*}, A_t^{j*})$ the distribution of $(\hat{\beta}_t^k - \beta_t^k)$ is approximated by the conditional distribution of $(\hat{\beta}_t^{k*} - \hat{\beta}_t^k)$ given $(Y_t^j, k_t^j, l_t^j, A_t^j)$, where $\hat{\beta}_t^{k*}$ is the estimate of β_t^k based on the bootstrap sample. We asses the significance of β_t^k on the basis of the 95% confidence interval, $[\hat{\beta}_t^k - q_{0.975}^*, \hat{\beta}_t^k - q_{0.025}^*]$, where q_α^* is the α-quantile of the distribution of $(\hat{\beta}_t^{k*} - \hat{\beta}_t^k)$. Analogously, we compute confidence intervals for β_t^l, γ_t^k, and γ_t^l. Distributional effects are statistically significant, if the condifence interval for γ_t^k or γ_t^l does not include zero. The consistency proof of such a naive bootstrap in the context of average derivative estimation can be found in Härdle and Hart (1992).

3.3 Empirical results

In the following, we present the results for the estimation of β^k, β^l, γ^k, and γ^l. We report results based on the OLS estimation of (13) in Table 2. The first two columns of the table reveal that, as expected, changes in the levels of aggregate capital and labor have a positive significant effect on growth in all countries. Further, the capital coefficient appears to be higher

[16] Knot deletion is an iterative procedure. We start with a large number $\bar{D}^k$ of initial knots for k, i.e., $d_1^k < d_2^k < \cdots < d_{\bar{D}^k}^k$, which divide the domain of k into intervals $[d_i^k, d_{i+1}^k]$ with approximately equal number of observations. Similarly, we determine the corresponding $\bar{D}^l$ initial knots for l. In step 0, we estimate (14) by the OLS method and obtain $\bar{D} = \bar{D}^k + \bar{D}^l$ estimated spline coefficients $\hat{\theta}_{3,1}^k, \ldots, \hat{\theta}_{3,\bar{D}^k}^k, \hat{\theta}_{3,1}^l, \ldots, \hat{\theta}_{3,\bar{D}^l}^l$ with the corresponding t-values, $t := \hat{\theta}/SE(\hat{\theta})$. At step 1, we delete the knot with the lowest absolute t-value at step 0 and reestimate (14) using $\bar{D} - 1$ knots. We repeat this process $\bar{D}$ times until no knots are left. At each step r, $0 \leqslant r \leqslant \bar{D}$, we compute the *residual sum of squares* $RRS_r = \sum_{j=1}^n (\hat{\varepsilon}_t^j)^2$. Finally, we choose the model with the lowest value for Mallows' C_p defined by $C_r := RSS_r + 3(\bar{D} + 6 + n_A - r)\hat{\sigma}_0^2$, where n_A is the number of attributes in A_t^j and $\hat{\sigma}_0$ is the estimated standard deviation of ε_t^j at the 0^{th} model.

for transition than for developed countries. Overall, the estimated aggregate output elasticities with respect to aggregate capital and labor, i.e., $\hat{\beta}^k$ and $\hat{\beta}^l$, are comparable with those obtained by other studies.[17] More interestingly, we find that distributional effects of capital or labor, associated with γ^k and γ^l, are significant at 1% level in all countries. These coefficients are displayed in the last two columns of Table 2. Further, the distributional effects of capital are negative and higher (in absolute value) than the corresponding level effects associated with β^k. As for distributional effects of labor, they turn out to be negative and significant at 1% level for all countries except from Austria, Czech Republic, Portugal and Slovakia. For Portugal they are positive and significant at the 5% level. Summing up, distributional effects of capital and labor, which have been overlooked in the growth literature so far, are statistically and economically significant.

We investigate the robustness of this finding, in that we control for potential simultaneity and misspecification of the functional form. Table 3 reports the estimation results according to the Olley and Pakes (1996) method. Overall, the estimates are similar to the OLS estimates but exhibit higher standard errors. We infer that the simultaneity problem is of less importance in our sample. In particular, γ^k is still negative and significant for all countries. Moreover, apart from Germany and Romania, the distributional effects of capital are again stronger (in absolute value) than the corresponding level effect. The distributional effects of labor are negative and significant in 13 out of 20 countries. The results for the semiparametric estimation are reported in Table 4. We observe that the estimates of β^k exceed the corresponding OLS estimates in most countries. In contrast, $\hat{\beta}^l$ are comparable to the OLS counterparts. At least one of the distributional effects, i.e., γ^k or γ^l, is significant in all countries apart from the Czech Republic and Slovakia. Interestingly, accounting for a more flexible functional form yields positive significant distributional effect of capital in Denmark, Italy and Norway. In contrast, γ^k is negative significant for eleven countries. Besides, the distributional effects of capital are lower than the ones resulting from the loglinear model. As opposed to previous models, they are also lower than the corresponding level effects. As for distributional effects of labor, they are negative significant in ten countries and positive significant in Portugal. Summing up, the importance of the distributional effects, which are the main focus of this paper, is robust to simultaneity and parametric misspecification.

The negative impact of changes in the standard deviation of inputs in most countries sup-

[17] Recall that under this specification $\hat{\beta}^k = \hat{\theta}^k$ and $\hat{\beta}^l = \hat{\theta}^l$. Hence, we can compare our estimates with those obtained in studies on production function estimation from the firm-level data, e.g., Olley and Pakes (1996), Levinsohn and Petrin (2003), and Blundell and Bond (2000).

ports the intuition outlined in Remark 2. First, under the assumption that a higher degree of product market competition among firms is associated with more similarity in firm size, i.e., smaller standard deviations of capital and labor, we find a positive relationship between competition and economic growth. This positive relation is also found in the literature, for instance, by Nicoletti and Scarpetta (2003).

Second, changes in the distribution of inputs capture changes in the pattern of economic interactions between firms. In particular, the literature on economic growth emphasizes the importance of technology spill-overs among firms in developed economies. A standard assumption in the literature is that technology spill-overs are more likely between firms that are more similar in terms of the inputs they use in the production process.[18] Accordingly, an increase in the standard deviation of capital or labor corresponds to less intensive technology spill-overs and, hence, to lower growth rates.

4 Growth Accounting

We exploit the economic signficance of the distributional effects outlined above to refine conventional growth accounting exercises. That is, we explore whether cross-country growth differences can be explained by differences in changes in the allocation of capital and labor. Their explanatory power depends on the cross-country heterogeneity in γ^k and γ^l as well as in the growth rates of the standard deviations of the inputs.

To measure the success of a model in explaining cross-country growth differences we follow the tradition of variance decomposition. That is, analog to Caselli (2005), we compute the explanatory power of the changes in the aggregate input levels as

$$\texttt{S1} = \frac{var(\hat{g}_{1,t})}{var(g_t)} \tag{15}$$

where

$$\hat{g}_{1,t} = \hat{\beta}^k_{t-1}(\log \bar{K}_t - \log \bar{K}_{t-1}) + \hat{\beta}^l_{t-1}(\log \bar{L}_t - \log \bar{L}_{t-1}).$$

The residual of this indicator, $1 - \texttt{S1}$, is the explanatory power of changes in TFP. However, we know from the Proposition that part of the residual changes should not be associated to changes in the production technology (TFP), but instead, to changes in the higher moments of the distribution of capital and labor across firms. Accordingly, our approach which takes

[18] Theoretical models by Basu and Weil (1998) and Acemoglu and Zilibotti (2000) show that international technology diffusion is stronger if firms employ more similar capital-labor ratios in production. An empirical evidence in favor of this result is provided by Keller (2004).

firm-level heterogeneity in the inputs into account leads to a different growth accounting model:

$$\texttt{S2} = \frac{var(\hat{g}_{2,t})}{var(g_t)}, \tag{16}$$

where

$$\hat{g}_{2,t} = \hat{\beta}^k_{t-1}(\log \bar{K}_t - \log \bar{K}_{t-1}) + \hat{\beta}^l_{t-1}(\log \bar{L}_t - \log \bar{L}_{t-1}) + \hat{\gamma}^k_{t-1}\Big(\frac{\sigma^k_t - \sigma^k_{t-1}}{\sigma^k_{t-1}}\Big) + \hat{\gamma}^l_{t-1}\Big(\frac{\sigma^l_t - \sigma^l_{t-1}}{\sigma^l_{t-1}}\Big).$$

In addition to the estimated aggregate coefficients growth accounting requires data on the growth rate of aggregate output, aggregate capital, aggregate labor and the standard deviations of log capital and log labor. Since the estimation of coefficients relies on data in 2003 (corresponding to $t-1$) we focus on growth rates from 2003 to 2004. All of the required information is available in the AMADEUS data base. However, the computation of aggregate output and inputs from the cross-section of firms yields implausibly high growth rates in some countries as is displayed in Table 1. Therefore, we employ information on aggregate growth rates from the standard cross-country data sets. In particular, we employ Penn World Tables and follow Caselli (2005) in measuring output as real GDP per capita in PPP and computing the aggregate capital stock from the corresponding investment series using the perpetual inventory method and by assuming yearly depreciation rate of 6%. Since aggregate labor in 2004 is not available in Penn World Tables, we measure aggregate labor as total number of employees from the Eurostat data base. Obviously, the information on the standard deviations of log capital and log labor has to be obtained from the firm-level data base. Unfortunately, required aggregate data for Bosnia and Herzegovina are not available and we are forced to omit this country in our analysis. The growth rates of the variables employed in the growth accounting exercise are reported in Table 5.

We derive S1 and S2 based on the three different estimators outlined in the last section. In particular, we find that the aggregate capital and labor explain 28% of the cross-country growth differences based on the OLS estimates ($\texttt{S1}_{OLS} = 0.28$), 29% based on the Olley and Pakes (1996) method ($\texttt{S1}_{OP} = 0.29$), and 40% based on the semiparametric model ($\texttt{S1}_{SP} = 0.40$). These results are consistent with the corresponding findings in the conventional growth accounting literature. If we additionally take the distributional effects into consideration, we are able to explain an additional 17%, 13%, and 6% of the growth differences across countries, respectively ($\texttt{S2}_{OLS} = 0.45$, $\texttt{S2}_{OP} = 0.42$, $\texttt{S2}_{SP} = 0.46$). Recall that our aggregate coefficients are not estimated by fitting changes in aggregate levels and standard deviations to output growth rates, but are computed from a structural estimation based on firm-level data. Hence, in contrast to standard goodness-of-fit measures, the explanatory power could may drop if we additionally account for distributional effects. This would be the case if the changes in σ^k and σ^l were negatively correlated with omitted factors that explain GDP-growth. Consequently, distributional effects of capital and labor across firms help to explain a significant part of variation in growth across the 19 European countries.

We analyze the robustness of the above result in two different ways. First, we redo the growth accounting exercise by excluding one country at a time. We repeat this procedure for all countries. Doing this, we obtain very similar results as in the unrestricted sample. Second, we extend the sample period to 2002-2004, which virtually does not change our results. In all, the growth accounting results are robust to variations in the cross-section as well as in the time series dimension.

Overall, we conclude that accounting for distributional effects of capital and labor helps explain an additional 6-17% of the cross-country variation in output growth among the 19 European countries. Thus, a growth accounting model which is based on the correct treatment of firm heterogeneity improves the explanatory power of the production inputs and reduces the relevance of the residual TFP measure.

5 Conclusion

In this paper, we propose a growth model to examine the effect of distributional changes of capital and labor on economic growth. We show that the growth rate of an economy depends not only on changes in the aggregate level of capital and labor, but also on changes in the allocation of these inputs across firms, which we measure by standard deviations of capital and labor. Our empirical analysis, based on European firm-level data, reveals that changes in the allocation of capital and labor have pronounced effects on GDP-growth in almost all of the 20 European countries. This striking result revises the rather unimportant role of capital and labor distributions in explaining income and growth differences across countries as documented, for instance, by Caselli (2005). Moreover, it suggests that conventional TFP measures misleadingly capture growth effects stemming from changes in the standard deviations of capital and labor. In fact, our framework allows to assess the explanatory power of higher moments of the input distributions and, therefore, reassess the explanatory power of TFP. In this regard, we refine conventional growth accounting exercises by controlling for cross-country differences in aggregate input levels *and* input allocations.

Our empirical results reveal that distributional effects from firm-level heterogeneity in the inputs are statistically and economically significant in almost all countries. In particular, we find that a higher standard deviation in labor and capital have negative effects on output growth. This finding is consistent with a positive relationship between competition and growth if more competition is associated with more similarity in firm size and, hence, lower standard deviations in capital and labor among firms. Our findings are also consistent with the fact that if firms are getting similar, the technology spill-overs are more intensive, which promotes economic growth.

Finally, in a growth accounting exercises we show that distributional effects of capital and labor help explain an additional 6-17% of cross-country growth differences among the 19 European countries.

country	n_{2003}	$\bar{Y}_{2003}$	$\bar{K}_{2003}$	$\bar{L}_{2003}$	n_{2004}	$\bar{V}_{2004}$	$\bar{K}_{2004}$	$\bar{L}_{2004}$
Austria	1071	23.766 (112.158)	27.059 (127.479)	18.098 (62.713)	1364	23.883 (113.935)	27.296 (130.298)	14.039 (43.775)
Belgium	10980	12.159 (123.637)	6.708 (32.035)	4.652 (13.922)	11036	12.146 (123.736)	6.720 (31.395)	5.156 (15.683)
Bosnia & H.	2573	0.420 (2.806)	1.358 (9.766)	0.118 (0.353)	2862	0.399 (2.643)	1.215 (7.586)	0.132 (0.380)
Bulgaria	5818	0.311 (2.734)	0.755 (4.083)	0.156 (0.591)	5955	0.308 (2.738)	0.776 (4.029)	0.175 (0.658)
Czech R.	11494	1.258 (14.420)	1.995 (9.655)	0.622 (1.614)	15799	1.270 (13.455)	2.003 (9.833)	0.615 (1.671)
Denmark	20426	2.915 (78.919)	1.359 (7.839)	1.173 (4.930)	21782	2.981 (77.818)	1.370 (7.804)	1.181 (4.778)
Estonia	7666	0.232 (1.799)	0.239 (0.966)	0.097 (0.243)	8083	0.235 (1.811)	0.257 (1.060)	0.112 (0.299)
Finland	32401	1.695 (39.112)	0.795 (5.318)	0.673 (2.848)	30328	1.700 (39.318)	0.730 (4.813)	0.785 (3.215)
France	157141	1.914 (49.716)	0.739 (4.914)	1.154 (4.081)	168079	2.045 (52.413)	0.731 (4.788)	1.214 (4.354)
Germany	6076	71.486 (840.303)	61.754 (273.698)	45.140 (188.750)	7623	68.272 (778.787)	62.813 (296.033)	34.938 (130.982)
Great Britain	41649	18.927 (263.407)	13.435 (88.357)	8.671 (34.791)	37666	19.163 (269.775)	14.751 (98.518)	11.578 (46.104)
Italy	117111	2.385 (86.508)	1.462 (6.618)	1.059 (3.622)	75392	1.976 (24.330)	1.561 (7.379)	1.984 (6.541)
Netherlands	7365	24.505 (329.132)	19.989 (109.564)	16.695 (71.049)	7375	25.337 (347.447)	20.302 (115.710)	17.977 (78.165)
Norway	12051	1.416 (45.918)	1.792 (9.647)	0.540 (1.295)	14299	1.432 (46.209)	1.747 (9.624)	0.679 (1.981)
Poland	10571	2.612 (26.125)	3.338 (13.321)	0.920 (2.119)	11188	2.551 (25.342)	3.823 (14.851)	1.101 (2.535)
Portugal	1451	9.958 (84.895)	19.793 (147.204)	6.114 (25.616)	1487	9.325 (84.477)	21.399 (154.907)	6.003 (25.167)
Romania	49018	0.102 (2.354)	0.102 (0.446)	0.046 (0.164)	66230	0.102 (2.403)	0.120 (0.505)	0.042 (0.141)
Slovakia	2042	1.626 (11.354)	4.157 (30.283)	0.842 (2.302)	2557	2.413 (22.984)	3.231 (23.581)	0.828 (2.489)
Spain	357410	0.956 (34.631)	0.492 (2.250)	0.313 (1.071)	360517	1.003 (37.231)	0.519 (2.374)	0.340 (1.175)
Sweden	123058	1.555 (42.467)	0.731 (5.522)	0.401 (1.776)	125725	1.474 (38.704)	0.735 (5.553)	0.437 (1.906)

Table 1: Descriptive statistics of the AMADEUS data for 20 European countries. All values in millions of EUR.

country	$\hat{\beta}^k$	$\hat{\beta}^l$	$\hat{\gamma}^k$	$\hat{\gamma}^l$
Austria	0.151 (0.016)	0.788 (0.025)	-0.190 (0.034)*	-0.037 (0.054)
Belgium	0.140 (0.006)	0.749 (0.008)	-0.293 (0.020)*	-0.250 (0.030)*
Bosnia & H.	0.212 (0.011)	0.581 (0.015)	-0.351 (0.039)*	-0.166 (0.036)*
Bulgaria	0.234 (0.009)	0.639 (0.010)	-0.268 (0.027)*	-0.190 (0.063)*
Czech R.	0.140 (0.004)	0.811 (0.007)	-0.183 (0.011)*	0.035 (0.026)
Denmark	0.116 (0.004)	0.747 (0.006)	-0.181 (0.012)*	-0.149 (0.024)*
Estonia	0.185 (0.008)	0.715 (0.009)	-0.278 (0.019)*	-0.210 (0.029)*
Finland	0.147 (0.002)	0.778 (0.003)	-0.299 (0.014)*	-0.090 (0.011)*
France	0.111 (0.001)	0.854 (0.002)	-0.232 (0.005)*	-0.038 (0.007)*
Germany	0.136 (0.007)	0.803 (0.011)	-0.130 (0.017)*	-0.107 (0.037)*
Great Britain	0.132 (0.003)	0.783 (0.004)	-0.248 (0.010)*	-0.057 (0.016)*
Italy	0.131 (0.002)	0.732 (0.002)	-0.179 (0.004)*	-0.058 (0.007)*
Netherlands	0.119 (0.007)	0.832 (0.010)	-0.171 (0.017)*	-0.158 (0.035)*
Norway	0.091 (0.003)	0.804 (0.006)	-0.210 (0.011)*	-0.123 (0.018)*
Poland	0.152 (0.006)	0.774 (0.009)	-0.213 (0.012)*	-0.077 (0.021)*
Portugal	0.130 (0.017)	0.818 (0.022)	-0.170 (0.032)*	0.132 (0.060)*
Romania	0.252 (0.003)	0.667 (0.004)	-0.241 (0.008)*	-0.319 (0.010)*
Slovakia	0.156 (0.013)	0.743 (0.020)	-0.193 (0.037)*	0.136 (0.086)
Spain	0.115 (0.001)	0.841 (0.001)	-0.181 (0.003)*	-0.103 (0.006)*
Sweden	0.148 (0.001)	0.766 (0.002)	-0.351 (0.008)*	-0.089 (0.012)*

Table 2: Estimated values of aggregate coefficients based on OLS production function estimation. Bootstrapped standard errors are given in parentheses. Asterisks denote statistical significance of distributional effects at the 5% level.

country	$\hat{\beta}^k$	$\hat{\beta}^l$	$\hat{\gamma}^k$	$\hat{\gamma}^l$
Austria	0.165 (0.067)	0.795 (0.087)	-0.240 (0.127)*	-0.010 (0.062)
Belgium	0.159 (0.029)	0.715 (0.009)	-0.298 (0.057)*	-0.184 (0.037)*
Bosnia & H.	0.266 (0.076)	0.509 (0.020)	-0.195 (0.86)*	-0.260 (0.068)*
Bulgaria	0.286 (0.042)	0.560 (0.017)	-0.304 (0.062)*	-0.089 (0.072)
Czech R.	0.111 (0.045)	0.752 (0.014)	-0.124 (0.051)*	0.029 (0.040)
Denmark	0.121 (0.039)	0.760 (0.008)	-0.166 (0.053)*	-0.095 (0.017)*
Estonia	0.185 (0.020)	0.685 (0.012)	-0.209 (0.025)*	-0.080 (0.034)*
Finland	0.156 (0.017)	0.763 (0.005)	-0.282 (0.035)*	-0.067 (0.013)*
France	0.119 (0.009)	0.829 (0.003)	-0.228 (0.018)*	-0.031 (0.008)*
Germany	0.117 (0.038)	0.744 (0.016)	-0.081 (0.035)*	-0.020 (0.044)
Great Britain	0.155 (0.035)	0.782 (0.005)	-0.285 (0.067)*	-0.038 (0.019)*
Italy	0.163 (0.017)	0.705 (0.003)	-0.173 (0.018)*	-0.061 (0.007)*
Netherlands	0.180 (0.031)	0.758 (0.013)	-0.213 (0.041)*	-0.051 (0.034)
Norway	0.064 (0.007)	0.835 (0.008)	-0.109 (0.012)*	-0.059 (0.006)*
Poland	0.123 (0.046)	0.741 (0.011)	-0.164 (0.065)*	-0.091 (0.032)*
Portugal	0.126 (0.051)	0.832 (0.041)	-0.236 (0.101)*	0.007 (0.062)
Romania	0.147 (0.044)	0.629 (0.006)	-0.101 (0.030)*	-0.252 (0.014)*
Slovakia	0.158 (0.053)	0.682 (0.028)	-0.186 (0.072)*	0.234 (0.135)
Spain	0.121 (0.010)	0.817 (0.002)	-0.173 (0.015)*	-0.063 (0.007)*
Sweden	0.154 (0.007)	0.759 (0.002)	-0.353 (0.018)*	-0.070 (0.012)*

Table 3: Estimated values of aggregate coefficients based on the Olley and Pakes (1996) method. Bootstrapped standard errors are given in parentheses. Asterisks denote statistical significance of distributional effects at the 5% level.

country	$\hat{\beta}^k$		$\hat{\beta}^l$		$\hat{\gamma}^k$		$\hat{\gamma}^l$	
Austria	0.171	(0.030)	0.779	(0.035)	-0.095	(0.045)*	-0.212	(0.061)*
Belgium	0.142	(0.011)	0.813	(0.014)	-0.097	(0.018)*	-0.231	(0.041)*
Bosnia & H.	0.240	(0.047)	0.729	(0.040)	-0.340	(0.057)*	0.109	(0.077)
Bulgaria	0.295	(0.036)	0.725	(0.041)	-0.095	(0.053)*	-0.050	(0.087)
Czech R.	0.257	(0.025)	0.793	(0.020)	-0.024	(0.039)	0.067	(0.038)
Denmark	0.174	(0.015)	0.796	(0.013)	0.038	(0.022)*	-0.220	(0.034)*
Estonia	0.187	(0.016)	0.775	(0.020)	-0.119	(0.025)*	-0.109	(0.043)
Finland	0.160	(0.010)	0.833	(0.010)	-0.095	(0.017)*	-0.090	(0.021)*
France	0.119	(0.003)	0.870	(0.004)	-0.059	(0.006)*	-0.024	(0.011)*
Germany	0.178	(0.013)	0.815	(0.016)	-0.006	(0.020)	-0.100	(0.044)*
Great Britain	0.211	(0.008)	0.797	(0.009)	-0.066	(0.012)*	-0.125	(0.021)*
Italy	0.153	(0.007)	0.820	(0.006)	-0.027	(0.021)	-0.063	(0.013)*
Netherlands	0.170	(0.019)	0.829	(0.022)	-0.002	(0.038)	-0.115	(0.050)*
Norway	0.141	(0.010)	0.856	(0.011)	0.060	(0.016)*	-0.050	(0.027)
Poland	0.156	(0.017)	0.856	(0.017)	-0.130	(0.031)*	-0.024	(0.033)
Portugal	0.231	(0.058)	0.805	(0.074)	-0.045	(0.037)	0.149	(0.084)*
Romania	0.209	(0.009)	0.693	(0.008)	-0.264	(0.018)*	-0.206	(0.014)*
Slovakia	0.309	(0.060)	0.730	(0.053)	-0.082	(0.089)	0.141	(0.103)
Spain	0.164	(0.004)	0.831	(0.003)	-0.001	(0.006)	-0.142	(0.009)*
Sweden	0.173	(0.004)	0.820	(0.005)	-0.095	(0.008)*	-0.047	(0.014)*

Table 4: Estimated values of aggregate coefficients based on the semiparametric specification. Bootstrapped standard errors are given in parentheses. Asterisks denote statistical significance of distributional effects at the 5% level.

country	g_{04}	$\log \frac{\bar{K}_{04}}{\bar{K}_{03}}$	$\log \frac{\bar{L}_{04}}{\bar{L}_{03}}$	$\frac{\sigma^k_{04}-\sigma^k_{03}}{\sigma^k_{03}}$	$\frac{\sigma^l_{04}-\sigma^l_{03}}{\sigma^l_{03}}$
Austria	2.14	-1.31	0.57	-2.46	-1.89
Belgium	2.46	3.52	0.65	0.61	-0.76
Bosnia & H.	-	-	-	-5.14	-6.20
Bulgaria	5.02	10.02	2.59	-0.62	-1.38
Czech R.	3.10	4.73	-0.28	-0.43	2.33
Denmark	1.71	2.22	0.00	0.79	-1.00
Estonia	7.73	-0.54	0.25	1.24	0.48
Finland	3.47	2.75	0.41	-3.33	-0.22
France	1.97	5.03	0.05	0.46	0.38
Germany	1.66	1.13	0.42	1.22	0.27
Great Britain	2.75	1.93	1.00	1.52	0.56
Italy	1.09	0.28	0.37	3.78	10.14
Netherlands	1.23	2.25	-1.42	-0.21	1.79
Norway	2.20	9.26	0.47	0.83	1.39
Poland	5.31	6.36	1.31	-0.28	0.66
Portugal	0.38	1.26	0.09	0.22	3.50
Romania	8.68	1.64	0.39	-5.42	1.86
Slovakia	3.50	9.25	0.27	-10.04	-4.29
Spain	1.61	1.95	3.42	0.06	-0.92
Sweden	3.58	-1.27	-0.57	1.61	1.04

Table 5: Growth rates in 2004 (in %) used in the growth accounting exercise.

Figure 1a

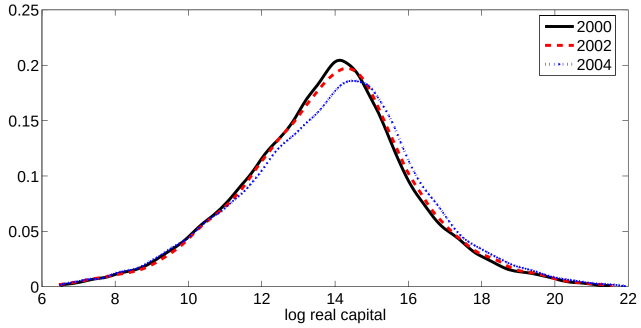


Figure 1b

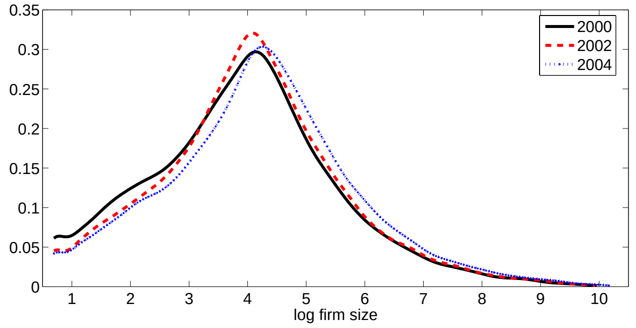


Figure 2a

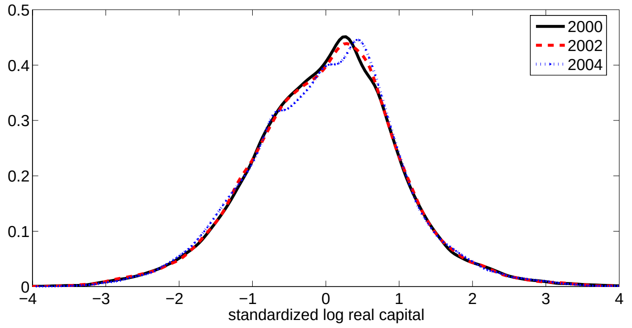


Figure 2b

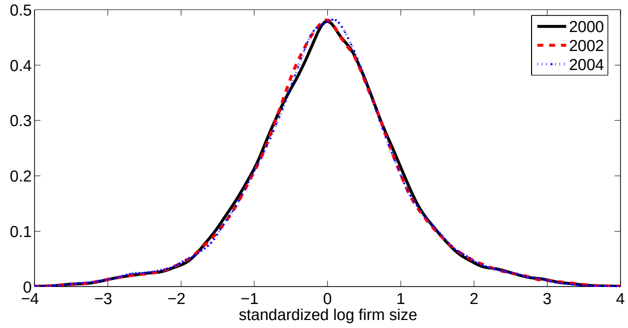


Figure 3: Age distribution for small firms

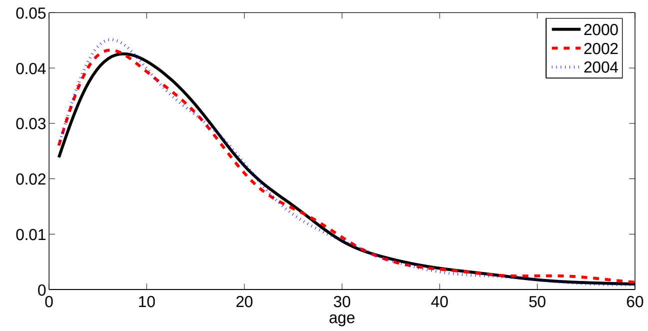


Figure 4: Age distribution for medium firms

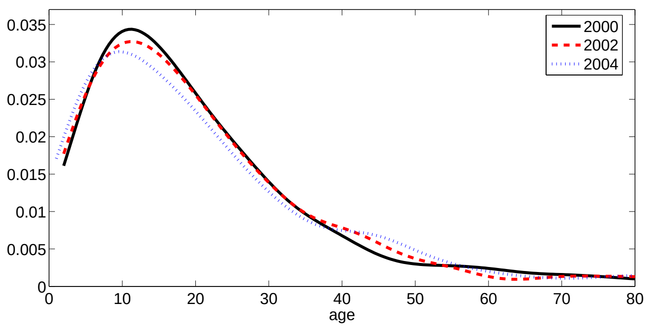
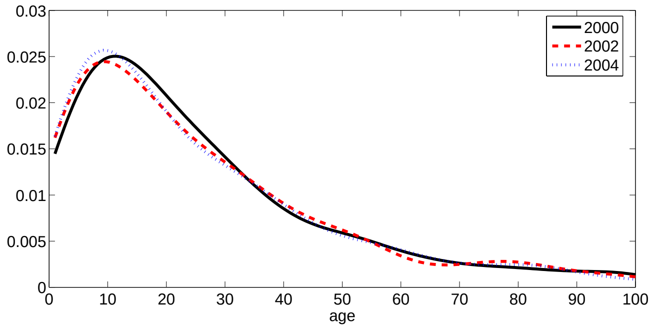


Figure 5: Age distribution for large firms



period	1998	1999	2000	2001	2002	2003	2004	2005
1997	**0.277**	0.004	0.012	0	0	0	0	0
1998		**0.940**	0.235	0.021	0.001	0	0	0
1999			**0.300**	0.144	0.004	0.002	0	0
2000				**0.154**	0.033	0.046	0.014	0
2001					**0.464**	0.310	0.282	0
2002						**0.702**	0.648	0.003
2003							**0.942**	0.049
2004								**0.452**

Table 6: P-values of the Li (1996) test of the local time invariance of the component-wise standardized distribution of log capital and log labor for United Kingdom. P-values smaller than 0.05 indicate that changes in the distribution were statistically significant.

t	1998	1999	2000	2001	2002	2003	2004	2005
1997	0.789	0.206	0.443	0.237	0.061	0.103	0.043	0.525
1998		0.997	0.590	0.655	0.742	0.879	0.484	0.506
1999			0.903	0.173	0.970	0.995	0.661	0.619
2000				0.963	0.783	0.956	0.495	0.912
2001					0.427	0.493	0.104	0.835
2002						0.780	0.950	0.729
2003							0.982	0.440
2004								0.589

Table 7: P-values of the Kolmogorov-Smirnov test of the local time invariance of the conditional distribution of age given capital and labor are equal to their 25th quantile in the population. P-values smaller than 0.05 indicate that changes in the distribution were statistically significant.

t	1998	1999	2000	2001	2002	2003	2004	2005
1997	0.945	0.200	0.711	0.232	0.508	0.806	0.172	0.075
1998		0.445	0.575	0.149	0.269	0.345	0.121	0.043
1999			0.454	0.005	0.619	0.200	0.283	0.540
2000				0.233	0.296	0.664	0.255	0.238
2001					0.131	0.466	0.112	0.046
2002						0.925	0.724	0.404
2003							0.714	0.439
2004								0.977

Table 8: P-values of the Kolmogorov-Smirnov test of the local time invariance of the conditional distribution of age given capital and labor are equal to their 50th quantile in the population. P-values smaller than 0.05 indicate that changes in the distribution were statistically significant.

t	1998	1999	2000	2001	2002	2003	2004	2005
1997	0.999	0.923	0.684	0.385	0.282	0.091	0.109	0.782
1998		0.985	0.682	0.314	0.194	0.051	0.072	0.632
1999			0.986	0.259	0.126	0.029	0.106	0.877
2000				0.686	0.315	0.087	0.254	0.997
2001					0.801	0.651	0.313	0.754
2002						0.981	0.973	0.944
2003							0.946	0.519
2004								0.816

Table 9: P-values of the Kolmogorov-Smirnov test of the local time invariance of the conditional distribution of age given capital and labor are equal to their 75th quantile in the population. P-values smaller than 0.05 indicate that changes in the distribution were statistically significant.

Appendix B

Estimation of production functions under simultaneity and sample selection

In what follows, we present the estimation procedure for production functions in the presence of simultaneity and sample selection problems. Therefore, we consider the model (13) and decompose the error term ε_t^j into two elements, i.e., $\varepsilon_t^j = \omega_t^j + \epsilon_t^j$, where ω_t^j is the productivity shock and ϵ_t^j is the true error term. Further, we distinguish between firm j's age a_t^j and the remaining $R' = R - 1$ attributes $A_t'^j$. Hence, we can write

$$Y_t^j = \alpha_0 + \alpha_1^k k_t^j + \alpha_2^k (k_t^j)^2 + \alpha_1^l l_t^j + \alpha_2^l (l_t^j)^2 + \alpha_1^a a_t^j + \alpha_2^a (a_t^j)^2 + \sum_{r=1}^{R'} \alpha_r^{A'} A_{r,t}'^j + \omega_t^j + \epsilon_t^j, \tag{17}$$

The simultaneity problem arises if ω_t^j is correlated with at least one of the regressors. In the recent literature on the estimation of production functions, one generally assumes that the demand for labor is the only input which is potentially correlated with ω_t^j as capital stocks are assumed to be predetermined. As a remedy, Olley and Pakes (1996) propose a three-stage estimation procedure for (17) in which they advocate the use of a firm's log investment i_t^j to identify the productivity shock. In doing so, they define the investment function ι such that $i_t^j = \iota_t(\omega_t^j, k_t^j, a_t^j)$. If investments are monotonically increasing in the technology shock for a given value of age and capital, this allows to express the unobservable technology variable as a function of contemporaneous investments, capital and age. Hence, they define the inverse investment function by h_t so that $\omega_t^j = h_t(i_t^j, k_t^j, a_t^j)$. Thus, one can rewrite (17) as

$$Y_t^j = \alpha_0 + \alpha_1^k k_t^j + \alpha_2^k (k_t^j)^2 + \alpha_1^l l_t^j + \alpha_2^l (l_t^j)^2 + \alpha_1^a a_t^j + \alpha_2^a (a_t^j)^2 + \sum_{r=1}^{R'} \alpha_r^{A'} A_{r,t}'^j + h_t(i_t^j, k_t^j, a_t^j) + \epsilon_t^j. \tag{18}$$

Further, we define

$$\phi_t(i_t^j, k_t^j, a_t^j) := \alpha_0 + \alpha_1^k k_t^j + \alpha_2^k (k_t^j)^2 + \alpha_1^a a_t^j + \alpha_2^a (a_t^j)^2 + h_t(i_t^j, k_t^j, a_t^j)$$

and approximate this term by a third order polynomial series in k, i, and a.[19] Consequently, we can write

$$Y_t^j = \alpha_1^l l_t^j + \alpha_2^l (l_t^j)^2 + \sum_{r=1}^{R'} \alpha_r^{A'} A_{r,t}'^j + \phi_t(i_t^j, k_t^j, a_t^j) + \epsilon_t^j. \tag{19}$$

[19] In particular, we define

$$\phi_t(i,k,a) = \theta_0 + \sum_{p=1}^{3} (\theta_p^i i^p + \theta_p^k k^p + \theta_p^a a^p + \theta_p^{ik} (ik)^p + \theta_p^{ia} (ia)^p + \theta_p^{ka} (ka)^p + \theta_p^{ika} (ika)^p).$$

Since we control for contemporaneous movements in productivity by the inverse investment function, OLS estimation of (19) yields consistent estimates of α_1^l, α_2^l, and $\alpha_r^{A'}$, $r = 1, \ldots, R'$.

In a second stage, in order to correct for a possible sample selection bias, we estimate the survival probability π_{t-1} from period $t-1$ to t. This estimation is carried out in a probit regression of the survival indicator on a polynomial series in i_{t-1}^j, k_{t-1}^j, and a_{t-1}^j.

Finally, a third stage is necessary to identify α_1^k, α_2^k, α_1^a and α_2^a. Therefore, we assume that productivity follows a first order Markov chain, i.e, $\omega_t^j = E(\omega_t^j|\omega_{t-1}^j) + \xi_t^j$, where ξ_t^j denotes the innovation in the productivity and is assumed to be uncorrelated with capital in period t. Defining v_t as output net of the contributions of labor and the attributes $A_t'^j$ and substituting π_{t-1}^j and $h_{t-1}(i_{t-1}^j, k_{t-1}^j, a_{t-1}^j)$ into a function

$$g(\pi_{t-1}^j, \phi_{t-1}^j - \alpha_1^k k_{t-1}^j + \alpha_2^k (k_{t-1}^j)^2 + \alpha_1^a a_{t-1}^j + \alpha_2^a (a_{t-1}^j)^2),$$

we can write

$$v_t^j = \alpha_0 + \alpha_1^k k_t^j + \alpha_2^k (k_t^j)^2 + \alpha_1^a a_t^j + \alpha_2^a (a_t^j)^2 + g(\pi_{t-1}^j, \cdot) + \xi_t^j + \epsilon_t^j. \tag{20}$$

Note that we restrict capital and lagged capital, as well as age and lagged age to have the same coefficients, respectively. Consequently, as these coefficients enter the regression equation twice we estimate them efficiently and consistently by applying to (20) a non-linear least squares procedure.

Appendix C

Derivation of the aggregate relation in terms of $\log \bar{K}$ and $\log \bar{L}$

Let $x_t^j = (k_t^j, l_t^j)'$ denote the observable firm-specific explanatory variables with the corresponding mean vector $\bar{x}_t$. Further, $\Sigma_t = \begin{pmatrix} (\sigma_t^k)^2 & \sigma_t^{kl} \\ \sigma_t^{kl} & (\sigma_t^l)^2 \end{pmatrix}$ denotes the covariance matrix of x_t^j across J_t. According to Hildenbrand and Kneip (2005) the growth rate g_t of the aggregate response variable is given by

$$g_t = \beta_{t-1}'(\bar{x}_t - \bar{x}_{t-1}) + tr[\Delta_{t-1}(\Sigma_t^{1/2}\Sigma_{t-1}^{-1/2} - \mathbb{I})] + \text{ other effects,} \tag{21}$$

where $\mathbb{I}$ is the identity matrix, $\beta_{t-1} = (\beta_{t-1}^k, \beta_{t-1}^l)'$ is a vector and $\Delta_{t-1} = \begin{pmatrix} \delta_{t-1}^k & \delta_{t-1}^{kl} \\ \delta_{t-1}^{kl} & \delta_{t-1}^l \end{pmatrix}$ is a matrix of coefficients. Under coordinate-wise standardization (in Assumption 1) Σ_t is replaced by $\tilde{\Sigma}_t = \begin{pmatrix} (\sigma_t^k)^2 & 0 \\ 0 & (\sigma_t^l)^2 \end{pmatrix}$ and the first two rhs terms in (21) simplify to

$$\beta_{t-1}^k(\bar{k}_t - \bar{k}_{t-1}) + \beta_{t-1}^l(\bar{l}_t - \bar{l}_{t-1}) + \delta_{t-1}^k(\frac{\sigma_t^k - \sigma_{t-1}^k}{\sigma_{t-1}^k}) + \delta_{t-1}^l(\frac{\sigma_t^l - \sigma_{t-1}^l}{\sigma_{t-1}^l}), \tag{22}$$

where

$$\delta_{t-1}^k = \frac{1}{\bar{Y}_{t-1}} \int (k - \bar{k}_{t-1})\partial_k \bar{f}_{t-1}(k, l, A)\, dG_{t-1,klA}$$

and

$$\delta_{t-1}^l = \frac{1}{\bar{Y}_{t-1}} \int (l - \bar{l}_{t-1})\partial_l \bar{f}_{t-1}(k, l, A)\, dG_{t-1,klA}.$$

For the sake of comparability with conventional growth models, we are interested in a relationship like (21) but in terms of changes in aggregate levels $\bar{K}$ and $\bar{L}$ rather than in terms of aggregate *log* levels $\bar{k}$ and $\bar{l}$. More specifically, we want to arrive at a relationship for the growth rate containing

$$\beta_{t-1}^k(\log \bar{K}_t - \log \bar{K}_{t-1}) + \beta_{t-1}^l(\log \bar{L}_t - \log \bar{L}_{t-1}).$$

We start[20] with the definition of $\log \bar{K}_t$.

$$\log \bar{K}_t = \log \Big[\int K dG_{t,K} \Big] = \log \Big[\int \exp(k) dG_{t,k} \Big]. \tag{23}$$

For two periods t and $t-1$ Assumption 1 (Structural stability of G_{kl}) implies

$$G_{t-1,k}\Big(\frac{\sigma_t^k}{\sigma_{t-1}^k}(k - \bar{k}_{t-1}) + \bar{k}_t\Big) = G_{t,k}(k).$$

Hence, we can rewrite (23) by

$$\begin{aligned} \log \bar{K}_t &= \log \Big[\int \exp\Big(\frac{\sigma_t^k}{\sigma_{t-1}^k}(k - \bar{k}_{t-1}) + \bar{k}_t\Big) dG_{t-1,k} \Big] \\ &= \bar{k}_t + \log \Big[\int \exp\Big(\frac{\sigma_t^k}{\sigma_{t-1}^k}(k - \bar{k}_{t-1})\Big) dG_{t-1,l} \Big] \end{aligned}$$

Now, we define a function q from $\mathbb{R}_+$ to $\mathbb{R}$ such that

$$q(\sigma^k) := \log \Big[\int \exp\Big(\frac{\sigma^k}{\sigma_{t-1}^k}(k - \bar{k}_{t-1})\Big) dG_{t-1,k} \Big].$$

[20]The derivation for $\log \bar{L}_t$ can be carried out analogously.

By the definition of q we have $q(\sigma_t^k) = \log \bar{K}_t - \bar{k}_t$ and simple algebra yields $q(\sigma_{t-1}^k) = \log \bar{K}_{t-1} - \bar{k}_{t-1}$. From these properties of q it follows that

$$\bar{k}_t - \bar{k}_{t-1} = \log \bar{K}_t - \log \bar{K}_{t-1} - [q(\sigma_t^k) - q(\sigma_{t-1}^k)].$$

Further, by the first order Taylor approximation of $q(\sigma^k)$ at σ_{t-1}^k we obtain

$$\begin{aligned} q(\sigma_t^k) &\approx q(\sigma_{t-1}^k) + \partial_{\sigma^k} q(\sigma^k)\big|_{\sigma^k=\sigma_{t-1}^k} \cdot (\sigma_t^k - \sigma_{t-1}^k) \\ &= q(\sigma_{t-1}^k) + \frac{1}{\sigma_{t-1}^k \bar{K}_{t-1}} \int (k - \bar{k}_{t-1}) \exp(k) dG_{t-1,k} \cdot (\sigma_t^k - \sigma_{t-1}^k). \end{aligned}$$

Consequently,

$$\beta_{t-1}^k(\bar{k}_t - \bar{k}_{t-1}) = \beta_{t-1}^k(\log \bar{K}_t - \log \bar{K}_{t-1}) - \frac{\beta_{t-1}^k}{\bar{K}_{t-1}} \int (k - \bar{k}_{t-1}) \exp(k) dG_{t-1,k} \cdot \Big(\frac{\sigma_t^k - \sigma_{t-1}^k}{\sigma_{t-1}^k}\Big).$$

Doing analogous derivations for $\log \bar{L}_t$, we obtain

$$\begin{aligned} g_t &= \beta_{t-1}^k(\log \bar{K}_t - \log \bar{K}_{t-1}) + \beta_{t-1}^l(\log \bar{L}_t - \log \bar{L}_{t-1}) \\ &+ \gamma_{t-1}^k\Big(\frac{\sigma_t^k - \sigma_{t-1}^k}{\sigma_{t-1}^k}\Big) + \gamma_{t-1}^l\Big(\frac{\sigma_t^l - \sigma_{t-1}^l}{\sigma_{t-1}^l}\Big) + \text{ other effects,} \end{aligned}$$

where

$$\gamma_{t-1}^k = \delta_{t-1}^k - \frac{\beta_{t-1}^k}{\bar{K}_{t-1}} \int (k - \bar{k}_{t-1}) \exp(k) dG_{t-1,k}$$

and

$$\gamma_{t-1}^l = \delta_{t-1}^l - \frac{\beta_{t-1}^l}{\bar{L}_{t-1}} \int (l - \bar{l}_{t-1}) \exp(l) dG_{t-1,l}.$$

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