A Politico-Economic Model of Aging, Technology Adoption and Growth*

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Abstract

This paper provides a positive theory that explains how an economy might evolve when the longevity of its citizens both influences and is influenced by the process of economic development. We propose a three periods OLG model where agents, during their lifetime, cover different economic roles characterized by different incentive structures and time horizon. Agents' decisions embrace two dimensions: the private choice about education and the public one upon innovation policy. The theory focuses on the crucial role played by heterogeneous interests in determining innovation policies, which are one of the keys to the growth process: the economy can be discontinuously innovation-oriented due to the different incentives of individuals and different schemes of political aggregation of preferences. The model produces multiple development regimes associated with different predictions about life expectancy evolution, educational investment dynamics, and technology adoption policies.

JEL Classification: D70, J10, O14, O31, O43.

Keywords: growth, life expectancy, human capital, systemic innovation, majority voting.

1 Introduction

Over the last two centuries the western world has experienced an extraordinary change in the economic environment and in all aspects of human life. During this period, OECD countries have been characterized by dramatic improvements in economic conditions, the longevity of their population and education attainments. Simultaneously, the traditional social structure has greatly

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changed: the share of both educated and retired people has increased significantly, and, as a consequence, the proportion of the working population has shrunk.

Some specific facts may provide a better description of this evolution. In the last one hundred and fifty years life expectancy has increased tremendously going from less than 60 years in the US (Lee, 2001) and 40 years in England (Galor, 2005) in 1850, to almost 80 years today (Fogel, 1994). At the same time, both the portion of lifetime devoted to education and retirement have increased. In 1850 about 10% of the population was enrolled in primary school and, on average, the time devoted to education was negligible. Considering both formal and informal schooling (domestic education), people now study for around 20 years, about a quarter of their expected lifetime. The length of time spent in retirement shows a similar trend. In 1850, less than three years were devoted to retirement. Today, especially in Europe due to the introduction of social security systems after World War II, people enjoy retirement for almost 20 years: again, one quarter of their lifetime (Latulippe, 1996). Figure 1 shows how life expectancy and its composition, in terms of agents' economic roles, have evolved between 1850 and 2000 in the United States. This trend is even more evident in the case of Europe: in particular, life expectancy has grown more rapidly (surpassing the United States), and the length of retirement has increased even more.[1]

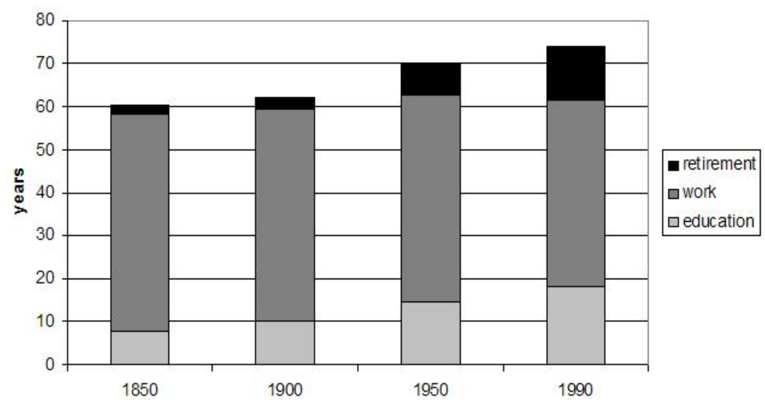

Fig.1. Life expectancy and economic roles in the US. Source: Lee (2001) and www.bls.gov.

One of the main implications of this trend is that the socio-demographic structure of developed and, to some extend, developing countries are experiencing important changes. This movement creates a system in which the preferences of both young and old people are becoming more and more important in the political debate competing with the traditional interests of adult workers. We observe the transition from a sort of "*workers' dictatorship*" - defined as a situation where the mass of the workers represents a large majority in the population - to a more diluted political representation.

[1] For European data see Galasso and Profeta (2004).

The purpose of this work is to provide an investigation of how an economy evolves when life expectancy affects both individual and aggregate preferences concerning the production side of the economy and, therefore, the growth process. We propose a three periods OLG model where agents, during their lifetime, cover different economic roles characterized by different time horizon and, consequently, incentive structures. Agents' decisions embrace two dimensions: the private choice about education and the public one related on innovation policy. The theory focuses on the crucial role played by heterogeneous interests in determining innovation policies, which are one of the keys to the growth process.

Our model economy does not create new technology, it is simply adopts those that have been created elsewhere. The adoption process is costly. We refer to a *systemic innovation* as to a type of innovation that, in order to be implemented, has to pass through the endorsement of a political mechanism, where, in general, the interests of different groups of agents do not coincide. In our framework the contrast evolves among different age groups. The public nature of systemic innovation, in contrast with the Schumpeterian view of innovations developed by firms running for the best cost-saving technology, comes from the historical point of view in which the implementation of a new technology is rarely the outcome of pure profit-maximizing by firms. Following Mokyr (1998a, 2002) and Olson (1982), in this study we focus our attention on systemic innovation as a growth-enhancing technology. Bauer (1995) points out that a decentralized market outcome seems to be a poor description of many technology breakthroughs. Economic convenience is certainly not irrelevant, but, as Mokyr (1998a) suggests: "*there usually is, at some level, a non-market institution that has to approve, license or provide some other imprimatur without which firms cannot change their production methods. The market test by itself is not always enough. In the past, it almost never was.*" (p. 219) Thus, as reported by Olson, the decision whether to adopt a new technology is likely to be resisted by those who lose by it through some kind of activism aimed at influencing the decision by the above-mentioned institutions.

Consequently, we construct a model in which technology adoption is delegated to a regulatory institution, the democratic vote. We formalize the idea that an innovation, before being introduced in large-scale production, has to be approved by some non-market institution.[2] Its adoption is ex-post disposable for all individuals in the economy, but ex-ante the choice to adopt it or not can be affected by the interests of different age groups.[3] To capture the evolving

[2] We assume that there is no uncertainty in the outcome of a new technology of this kind: once the decision to shift to the new technology is undertaken, with probability one a productivity enhancement takes place. It follows that we are not dealing with risky process of producing new ideas, but with the process of implementing existing ideas in new ways that are more efficient, although not for everybody in the same way.

[3] According to Bellettini and Ottaviano (2005), the central authority can be seen as a licensing system that has some agency to approve new technologies before they are brought to the market. Again in Mokyr (1998a)'s words: "*almost everywhere some kind of non-marketing control and licensing has been introduced*". A recent example is the creation of standard-setting agencies such as the International Organization of Standardization (ISO) or,

clash between resistive and innovative interests, we consider an economy that, at any point in time, is populated by three different overlapping groups of agents differing in terms of their life horizons and incentives structures. In fact, besides the increasing human capital accumulation, productivity improvements come from the innovation process. A systemic innovation is implemented if and only if there is a political consensus for it: because its net benefits are not equal among the different age classes, in a heterogeneous setting there is always room for suboptimal provision of the innovation itself. According to Krusell and Rios-Rull (1996) as well as Aghion and Howitt (1998), we assume that the public choice is carried out by means of a democratic majority voting where the interests of the absolute majority of the population prevail.

We find that a conflict of interests on which technology to adopt will arise between workers and students, on one side, and retired people, on the other. If the former will tend to support innovations, the latter are likely to resist technological change given that their income is not related to the current technology but rather to the previous innovation cycle. Another potential conflict opposes young people to adults. For the youngest cohort, an innovation has long lasting effects, since it affects both their future productivity in the labour market and their children's future capacity to acquire human capital. For the adults, however, a new technology will only have an effect on the ability of future generation to finance their pension. These different incentive structures would hardly coincide.

This paper contributes to two important recent research strands within the field of economic growth: life expectancy and growth (e.g. Blackburn and Cipriani (2002), Chakraborty (2004), Cervellati and Sunde (2005))[4] and vested interest and growth (e.g. Krusell and Rios-Rull (1996), Canton et al. (2002), Bellettini and Ottaviano (2005)).[5] Building on the existing literature, this paper ana-

about property rights, the European Patent Office (EPO).

[4]The important role played by life expectancy in determining the optimal education decisions of individuals has already been pointed out by models that analyzes the relationship between demographic variables and development. In a recent study, Blackburn and Cipriani (2002) endogenize life expectancy. As a result, their model generates multiple development regimes depending on initial conditions. Endogenizing life expectancy allows Blackburn and Cipriani (2002) to explain jointly the main changes that take place during the demographic transition of economies, such as greater life expectancy, higher levels of education, lower fertility and later timing of births. Cervellati and Sunde (2005) analyse a model in which human capital formation, technological progress and life expectancy are endogenously determined and reinforce each other. In a microfounded theory the authors show that the inclusion of endogenous life expectancy helps to explain the long-term development of economies and, in particular, the industrial revolution experienced by many countries as an endogenous result in the process of development. Chakraborty (2004) also endogenizes life expectancy and assumes that the survival probability depends on the public investment in health. In his model low life expectancy is detrimental for growth because on the one hand, low expectations of surviving make individuals less patient and willing to save and invest and, on the other hand, lower life expectancy also reduces the returns of investing in education. See Galor (2005) for an overview on the literature.

[5]To the best of our knowledge only Canton *et al.* (2002) have analyzed the relationship between vested interest and economic growth with the focus on the role played by aging population in determining the optimal technology adoption. The authors argue that when older people face a higher cost of adopting new technologies, political pressure in a democratic

lyzes an economic model in which the interactions among endogenously changes in life expectancy, education, technological change and economic growth, suggest that (a) a poverty traps can arise in the accumulation process of human capital and have long-lasting effects on aggregate output; (b) at *individual* level an higher life expectancy increases the incentive to innovate for both young and adults; (c) at *aggregate* level different configurations can arise depending on endogenous demographic structures; (d) depending on initial conditions and parameter values in the long run both "Innovation" and "No Innovation" can be feasible steady state. Much interesting, due to interplay between demographic structures and the private incentives that endogenously change, the transition path to steady state can be characterized by three switch between "No Innovation" and "Innovation" regimes.

The paper is organized as follows. In section 2 we presents the mechanics of the model, describing the economic environment and solving both the individual education problem and aggregate innovation one. Section 3 contains a simple dynamic example. Section 4 concludes.

2 The model

Time is discrete and indexed by $t \in \mathbb{N}^+$. The economy is populated by a finite number of overlapping generations of homogeneous agents. Each generation consists of a unit mass of individuals ($N_t = N = 1$) living up to three periods. Every agent born at time t survives with probability one from youth to adulthood and with probability p_{t+2} to old age. When people of generation t are young they split their unit time endowment between schooling (e_t) and working as unskilled ($1 - e_t$). Their income comes from their productivity multiplied by time spent working. It is, in case, taxed in order to finance a new productive technology to be implemented in the next period. This *"innovation tax"* is a fix share of income and takes the values $i \in (0, 1)$ or zero in case the innovation is decided or not, respectively. We define the indicator function of i_t, denoted by $\Phi(i_t)$, as follows:

$$\Phi(i_t) = \begin{cases} 1 & if \;\; i_t = i \\ 0 & if \;\; i_t = 0 \end{cases} .$$

Each adult works as skilled and has a single child. Adults' human capital is a function of average human capital of the previous period and the effort they made when young. They produce combining their human capital with a TFP parameter that increases if a new technology is endorsed the period before. This income is divided between consumption, a constant share s that goes, in a PAYGO fashion, in paying their parents' pensions[6] and, in case, the innovation tax i_{t+1}. When old, they consume the pension that their children pass to them,

system may slow down innovation adoption in an ageing society.

[6] We do not discuss the way in which the pension system is implemented and if it can be politically self-sustaining, as, among others, Bellettini and Berti Ceroni (1999) do. We assume that a commitment between generations is in place and no one can default on it.

net of the innovation tax i_{t+2}. The scheme of the timing for an agent born at time t is represented in figure 2.

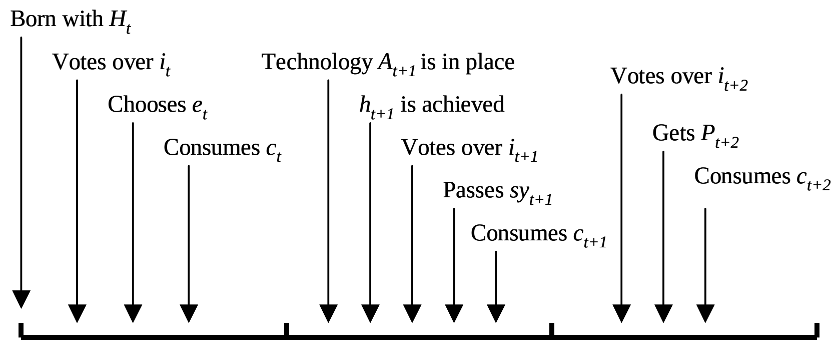

Fig.2. Timing for an agent born at time t.

In every period, adult agents individually produce a single homogenous good employing human capital as the sole input, using the publicly available technology A_t. Agents' political lever is characterized by their ability to vote, every period of their life, for a systemic innovation to be implemented in the next period. In order to take into account the increasing power of retired people, we assume that young people show a lower turnout rate at elections – defined as the percentage of people who actually vote among those having the right to – with respect to adults and old.[7] Thus, their weight in the political process is represented by an exogenous parameter $\eta \in (0,1)$. All adults and old vote at each period t, so their measure is 1 and p_t, respectively, where p_t is the share of old alive.

2.1 Production by Skilled Adults

Each skilled adult produces with a decreasing return function of human capital, combined with the available technology vintage. At time t she produces, y_t:

$$y_t = A_t h_t^{\gamma} \tag{1}$$

where $\gamma \in (0,1)$. h_t is adult endowment of human capital and A_t is the technological coefficient. Changes in A_t reflect therefore TFP changes. The level of technology employed at time t in the production of output, A_t, depends on the political choice of the previous period $(t-1)$. The TFP parameter A_t is equal to A_{t-1} in case a new technology is not implemented $(\Phi(i_{t-1}) = 0)$, while $A_t = (1+\theta)A_{t-1}$ in case a new technology is implemented $(\Phi(i_{t-1}) = 1)$. At

[7] Interestingly, as Galasso e Profeta (2004) report, not all potential electors actually vote. In some countries, elderly voters have a higher turnout rate at elections than the young, thus leading to an overrepresentation of the elderly. This voting pattern is strongest in the US, where turnout rates among those aged 60–69 years is twice as high as among the young (18–29 years). Significant differences appear also in other countries: in France, the turnout rate of the elderly (60–69 years) is almost 50% higher than that of the young (18–29 years).

time $t = 0$, $A_0 = A > 0$. A compact formulation for the dynamic evolution of technology parameter, A_{t+1}, is

$$A_{t+1} = (1 + \theta\Phi(i_t)) A_t \tag{2}$$

where θ denotes the growth rate of the technology and is a strictly positive scalar.

Remark 1 *Since $N_t = N = 1$ the aggregate production function at time t, Y_t, is $Y_t = y_t$*

2.2 Investment in Human Capital

In the first period of her life, a member of generation t invests in human capital. The acquisition of skills requires the individual's effort in schooling and a stock of existing human capital, whose average level is $\frac{H_t}{N_t} = H_t$ because $N_t = 1$. The human capital that an adult gets at time $t + 1$, h_{t+1}, is

$$h_{t+1} = \Upsilon(e_t, H_t, i_t) = \lambda\left((1 - \delta\Phi(i_t))e_t H_t^{\epsilon}\right) \tag{3}$$

The properties of the production function of human capital are as follows:

1. *The individuals' level of human capital is an increasing function of the individual's effort in schooling* (*i.e.* $\frac{\partial\Upsilon}{\partial e_t}(e_t, H_t, i_t) > 0$).

   The importance and the empirical significance of the individual's effort in schooling inputs is well documented in the literature. For a comprehensive survey of the related literature see Mincer (1974).

2. *The individuals' level of human capital is an increasing function of the parental level of human capital* (*i.e.* $\frac{\partial\Upsilon}{\partial H_t}(e_t, H_t, i_t) > 0$).

   The importance of the parental education input in the formation of the human capital of the child has been explored theoretically as well as empirically. The empirical significance of the parental effects has been documented by Becker and Tomes (1986), as well as others.

3. *There exist diminishing returns to the parental human capital effect* (*i.e.* $\frac{\partial^2\Upsilon}{\partial H_t^2}(e_t, H_t, i_t) < 0$).

4. *The level of human capital depreciates by a factor* $(1 - \delta)$ *in case an innovation is decided at time t.*

   The assumption is that when new technologies are implemented, human capital produced in schools based upon previous types of technology is less useful. The concept of vintage human capital has been explicitly used in the 90s to treat some specific issues related to technology diffusion, inequality and economic demography. In a world with a continuous pace of innovations, a representative individual faces the typical question of whether to stick to an established technology or to move to a new and better one. The trade-off is the following: switching to the new technique would allow him to employ a more advanced technology but he would lose the expertise, the specific human capital, accumulated on the old technique. For a comprehensive survey of *vintage human capital* literature see Boucekkine et al. (2006).

5. *Ranges for the parameters are* $\lambda > 0$, $0 \leq \delta < 1$ *and* $0 < \epsilon < 1$.

2.3 Utility Function and Budget Constraints

Individuals' preference are defined over the vector of consumption in all three periods of their lives, $\left(c_t^t, c_{t+1}^t, c_{t+2}^t\right) \equiv c^t$. The preferences of an individual born at time t are represented by the intertemporal, non altruistic utility function

$$u_t^t = \log c_t^t + \alpha \log c_{t+1}^t + p_{t+2} \beta \log c_{t+2}^t \tag{4}$$

where $\alpha, \beta \in (0,1)$ are the weight attached to adult and old age consumption[8], respectively. p_{t+2} is the probability to survive until old age.

The budget constraints in the three periods are as follows. Note that in every period the incomes are taxed in case a new technology is decided to be implemented in the next period.

$$c_t^t = \omega(1 - e_t)(1 - i_t) \tag{5}$$

Consumption of a member of generation t at time t, c_t^t, is the income generated working as unskilled net of the *innovation tax*. When young each agent works as unskilled getting a constant wage ω that, for simplicity, we normalize to 1. The time devoted to work is $(1 - e_t)$. Because of the assumptions $N_t = 1$ and setting $\omega = 1$, young's gross income is $(1 - e_t)$.

$$c_{t+1}^t = y_{t+1}(1 - s - i_{t+1}) \tag{6}$$

Consumption of a member of generation t at time $t+1$, c_{t+1}^t, is the income received in the skilled sector net of the *innovation tax* and the pension contribution, required to finance the pension of her parent. s must satisfy the condition: $s < (1 - i)$.

$$c_{t+2}^t = P_{t+2}(1 - i_{t+2}) \tag{7}$$

Consumption of a member of generation t at time $t+2$, c_{t+2}^t, is the pension benefit net of the innovation tax. In the third period of her life, a member of generation t receives

$$P_{t+2}^t = \frac{s y_{t+2}^{t+1}}{p_{t+2}} = \frac{s A_{t+2} h_{t+2}^{\gamma}}{p_{t+2}} \tag{8}$$

[8] In the less general case of $\beta = \alpha^2$ some of the dynamic features described in the next sections disappear. We leave to those sections the formal discussion.

The pension is the share s of income that an adult of generation $t+1$ disbursed in the PAYGO system, multiplied by the coefficient $\frac{1}{p_{t+2}}$ that takes into account the share of people surviving to old age.

Remark 2 *Ceteris paribus, the pension benefit for an old agent decreases with the lengthening of life expectancy.*

2.4 Individual optimization with given innovation policy

In every period of her life an agent takes the innovation policy as given.[9] Agents choose the optimal schooling time when young. Maximization of (4) subject to the budget constraints (5), (6), (7), in which we previously plugged (8) and the human capital production function (3), yields the optimal schooling time, e_t^*,

$$e_t^* = \frac{\gamma[\alpha + \beta \epsilon p_{t+2}]}{1 + \gamma[\alpha + \beta \epsilon p_{t+2}]} \tag{9}$$

Remark 3 *The longer is the life expectancy, the higher is the time investment needed to finance their prolonged consumption, consistently with existing literature.*[10]

The positive effect of p_{t+2} on e_t^* arises because agents know that the only way to get higher pension benefits from their children is to invest in their own education. This, in turns, positively affects their children's human capital and, ultimately, their children's income.

Substituting (9) in (3) and writing h_t instead of H_t (since in equilibrium $h_t = H_t/N_t$ and by assumption $N_t = 1$) we get the accumulation function of human capital as a function of the previous level of human capital, the innovation policy chosen the period before and the fraction of time young spend in education. We obtain

$$h_{t+1} = \lambda \left(1 - \delta \Phi\left(i_t\right)\right) \frac{\gamma[\alpha + \beta \epsilon p_{t+2}]}{1 + \gamma[\alpha + \beta \epsilon p_{t+2}]} h_t^{\epsilon} \tag{10}$$

The human capital accumulation function shows a concave shape (given that $0 < \epsilon < 1$) and undergoes a reduction in case an innovation takes place ($\Phi\left(i_t\right) = 1$).

2.5 Endogenous life expectancy

In this subsection we allow for the level of life expectancy to increase with the aggregate human capital level.[11] For an agent born at time t the probability

[9] We will add the case of endogenous innovation policy in the paragraph 2.6.

[10] The positive effect of longevity on education is emphasized by Blackburn and Cipriani (2002), Chakraborty (2004) and Cervellati and Sunde (2005). For further evidence on the effect of health and living conditions on education attainments, see De la Croix and Licandro (1999), Lagerlof (2003) and Galor (2005).

[11] As in, among others, Blackburn and Cipriani (2002), Boucekkine et al. (2002), Cervellati and Sunde (2005).

to reach old age is, therefore, $p_{t+2} = p(H_t)$. We impose some restrictions on $p(H)$, in order to get simple results. $p(0) = \underline{p} > 0$ avoids the extreme case of a disappearing old age, while $\frac{\partial P(H)}{\partial H} > 0$ replicates the empirical evidence of a positive correlation between life expectancy and human capital.[12] Since p is a probability, we assume that $\lim_{H\to+\infty} p(H) = \overline{p} \leq 1$. For simplicity, we set $\overline{p} = 1$. Simple algebra and the identity $h_t \equiv H_t$ allow us to rewrite the expression of human capital accumulation (10):

$$h_{t+1} = \Gamma(h_t; i_t)h_t^{\epsilon}$$

The function Γ takes always positive values ($\forall(h_t; i_t) \in R^2_{++}$, $\Gamma : R^2_+ \to R_+$), increases in h ($\Gamma_1(h_t; i_t) > 0$) and, for the restrictions imposed on the function p, is limited from above by some finite number. In figure 3 and 4 we show the peculiar case in which the introduction of an innovation leads the economy from a unique steady state case to a multiple steady state one.

Proposition 4 *For each value of $h^F \in \mathbb{R}_+$ and for a given i_t it is always possible to explicitly find a continuous increasing function $h_{t+1} = \Gamma(h_t; i_t)h_t^{\epsilon}$ that shows multiple steady states.*

Proof. For simplicity, we drop the time index and substitute H with h. Let $p(h) = \frac{\underline{p}+\overline{p}\left(\frac{h}{h^F}\right)^{\sigma}\left(\frac{\sigma-1}{\sigma+1}\right)}{1+\left(\frac{h}{h^F}\right)^{\sigma}\left(\frac{\sigma-1}{\sigma+1}\right)}$, with $\sigma > 1$. Straightforward calculations lead to $p'(h) > 0$, $p''(h) \gtreqless 0$ for $h \lesseqgtr h^F$, and $p'(h^F) = \frac{(\overline{p}-\underline{p})}{4h^F}\left(\frac{\sigma^2-1}{\sigma}\right)$. h^F is therefore the value of h such that $p(h)$ shows an inflection point. Note that $p'(h^F) > 0$ and $\frac{\partial p'(h^F)}{\partial \sigma} > 0$. From (10) we build the function $\tilde{\Gamma}(p(h); h) = \Gamma(h_t; i_t)h_t^{\epsilon}$, where we (i) separate the dependency from human capital and life expectancy and (ii) drop the innovation variable i_t. We impose parameters' values such that for $h = h^F$ the limiting functions $\tilde{\Gamma}(\underline{p}; h)$ and $\tilde{\Gamma}(\overline{p}; h)$ take values below and above h^F, respectively (i.e. $\tilde{\Gamma}(\underline{p}; h^F) < h^F$ and $\tilde{\Gamma}(\overline{p}; h^F) > h^F$). Since $\lim_{\sigma\to+\infty} p'(h^F) \to +\infty$, the function $\tilde{\Gamma}(p(h); h)$ takes values $\tilde{\Gamma}(p(h^F - \triangle h); (h^F - \triangle h)) < h^F$ and $\tilde{\Gamma}(p(h^F + \triangle h); (h^F + \triangle h)) > h^F$ for any $\triangle h = o(h) > 0$ and $1 < \sigma^M(\triangle h) < \sigma < +\infty$. For continuity of $\tilde{\Gamma}(p(h); h)$ there is a steady state in h^F where function $\tilde{\Gamma}(p(h); h)$ crosses the 45 degrees line from below. This steady state is therefore unstable. Calculus inspection shows that $\frac{\partial\left(\tilde{\Gamma}(p(h);h)\right)}{\partial h} > 0$

[12] Empirically, both private and aggregate endowment of human capital are conductive to a longer life, although we focus on the aggregate view: on the one hand, demographic and historical evidence suggests that the level of human capital profoundly affect the longevity of people. For example, the evidence presented by Mirowsky and Ross (1998) supports strongly the notions that better educated people are more able to coalesce health-producing behaviour into a coherent lifestyle, are more motivated to adopt such behaviour by a greater sense of control over the outcomes in their own lives, and are more likely to inspire the same type of behaviour in their children. Schultz (1993, 1998) evidences that children's life expectancy increases with parent's human capital and education. On the other hand, there is evidence that the human capital intensive inventions of new drugs increases life expectancy (Lichtenberg, 1998, 2003) and societies endowed with an higher level of human capital are more likely to innovate, especially in research fields like medicine (Mokyr, 1998b).

in $[0;\infty)$, $\lim_{h\to 0^+} \frac{\partial\left(\tilde{\Gamma}(p(h);h)\right)}{\partial h} \to +\infty$ and $\lim_{h\to +\infty} \frac{\partial\left(\tilde{\Gamma}(p(h);h)\right)}{\partial h} \to 0^+$. With $\tilde{\Gamma}(p(0);0) = 0$ we can prove that the function $\tilde{\Gamma}(p(h);h)$ shows four steady states, alternatively unstable and stable. These are $h^{U0} = 0$, $0 < h^{S1} < h^F$, $h^{U1} = h^F$ and $h^F < h^{S2} < +\infty$. ■

In figure 3 we represent the case of *innovation (i.e.* $i_t = i$). h^{S1} and h^{S2} are stable equilibria, while h^{U1} is the unstable, positive one. By assumption in the above-mentioned case the whole graph of h_{t+1} lies below the one of no innovation: it can be, therefore, the case that if innovation takes place there is room, due to the depreciation of human capital, for two stable steady states, while in the case of no innovation only one stable steady state occurs. In figure 4 we show the case of *no innovation* (i.e. $i_t = 0$). The graph of h_{t+1} is higher and only one stable steady state, h^{S3}, arises.

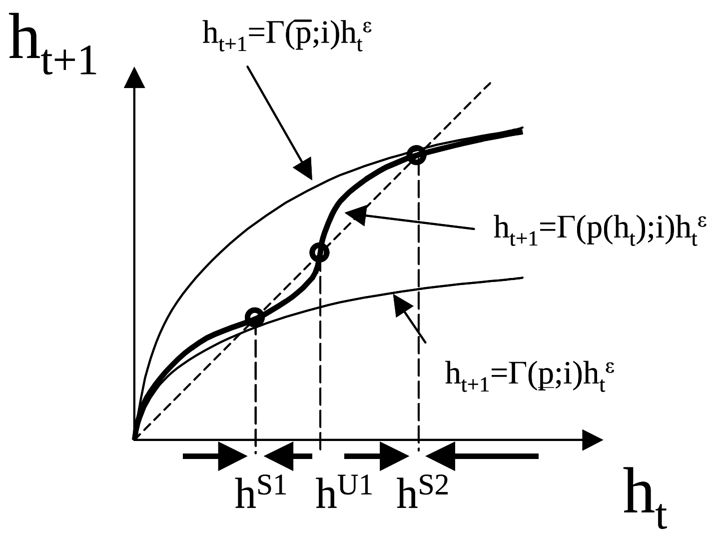

Fig.3. Equilibria of human capital level in the case of innovation and endogenous life expectancy.

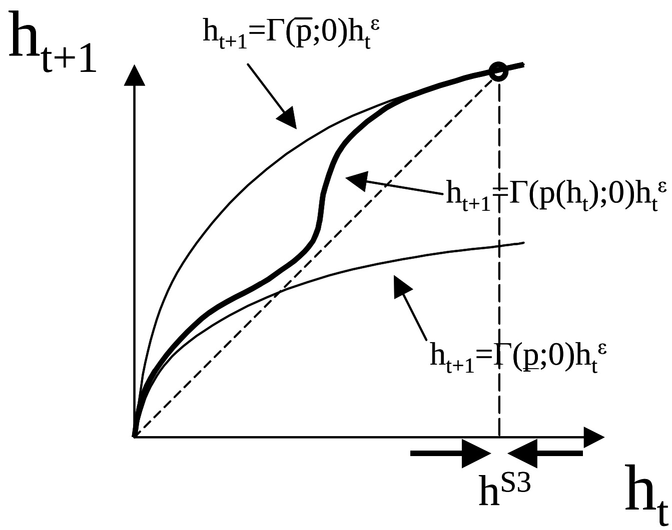

Fig.4. Equilibrium of human capital level in the case of no innovation and endogenous life expectancy.

Apart from the innovation policy, increases in the weight of both adult (α) and old age (β) consumption, the constant of proportionality (λ), the productivity of human capital in final good production (γ) and the elasticity of past human capital in the production of new human capital (ϵ) shift h_{t+1} upward, leading both to higher level of human capital for any steady state and, in case, to the disappearance of the low steady state, h^{S1} in figure 3.

Remark 5 *The fact that (i) the growth of human capital is bounded and (ii) human capital is the only factor of production and its accumulation function does not depend upon the level of the TFP parameter allows us to study, in an "additive" way, how human capital and production evolve.*

For example, once human capital reaches a steady state, using (1) we can keep track of the final production looking solely at the innovation policy undertaken. Therefore the steady state production is a constant level in the case of no innovation ($y^* = A_0(h^{S*})^\gamma$), while it will increase at the constant rate θ in the case of innovation ($y_t = A_0(1+\theta)^t(h^{S*})^\gamma$). The value h^{S*} represents one of the stable steady states reached by the human capital function.

2.6 Endogenous innovation policy: aggregation rule and individual choices

In this section we endogenize the process of technology adoption by means of a majority voting mechanism. At every point in time the agents belonging to the three age classes vote for a new technology to be implemented in the next period. The decision to adopt a new technology is endorsed if the majority of agents votes in favor of it.[13] At time t young of generation t, adults of generation $(t-1)$ and (survived) old born at time $(t-2)$ are alive. Their political weights, whose sum is normalized to one, are

$$\frac{\eta}{\eta+1+p_t}, \frac{1}{\eta+1+p_t} \text{ and } \frac{p_t}{\eta+1+p_t},$$

respectively.

Remark 6 *The longer the life expectancy is, the larger is the political weight of old and the smaller is that of both young and adults.*

Lemma 7 *For values of old's life expectancy p_t smaller than the threshold p^O:*

$$p_t < p^O = 1 - \eta$$

a "workers' dictatorship" arises at time t: no matter what young and old prefer, adults alone will set the agenda in terms of innovation. There are no values of p_t such that another age class alone can decide upon innovation.

Proof. Adults get the absolute majority if and only if their share is bigger than $\frac{1}{2}$: imposing $\frac{1}{\eta+1+p_t} > \frac{1}{2}$ we obtain, solving for p_t, $p_t < 1-\eta$. For similar considerations it is possible to show that both $\frac{\eta}{\eta+1+p_t}$ and $\frac{p_t}{\eta+1+p_t}$ can not exceed $\frac{1}{2}$. ■

In early stages of development (i.e. $p_t < p^O$) the political power is, therefore, in the hands of adult alone. Meanwhile the accumulation of human capital leads to longer life expectancy and ultimately to a smaller shares of both young and adults. Once p_t reaches and passes p^O decisions about innovation cannot be supported by adults alone. In order to implement a new technology, the economy needs the consensus of at least two age classes. We call this subsequent stage of development *"diluted power"*. Note that the specific cost-benefit setup of the innovation implies that old people are always *against* innovation: they are supposed to pay today a fraction of their income for a new technology that will be available once they are dead. In the case of *"workers' dictatorship"* this feature is not influential, since adults have the absolute majority. On the contrary in the case of $p_t > p^O$ an innovation is implemented if and only if both young and adults vote in favor of a progressive policy.[14] Therefore, if either young, adults or both these age classes vote against innovation, a conservative policy will be put in place.

Definition 8 *v_t^j is the individual preference over the innovation policy voted by an agent of age j at time t (with $j \in \{Y; A; O\}$ standing for young, adults and old, respectively). v_t^j can take the states $\{\iota; \nu\}$, indicating a vote in favor of innovation and a vote against innovation, respectively.*

[13] It is possible to restate the mechanism of deciding upon technology adoption in terms of median voter, but we find this approach much clear and intuitive.

[14] With *progressive policy* we indicate the adoption of a new technology. Conversely, *conservative policy* means no adoption.

Note that old's choice is always to vote against innovation, as will be shown below: $v_t^O = \nu, \forall t \in \mathbb{N}^+$. The function M_t aggregates the votes of the three generations alive at time t and its outcome is the majority choice:

$$M_t(v_t^Y; v_t^A; v_t^O) = \begin{cases} I & \text{if} \begin{cases} v_t^Y = v_t^A = \iota \text{ and } p_t \geq p^O \\ v_t^A = \iota \text{ and } p_t < p^O \end{cases} \\ \\ N & \text{otherwise} \end{cases} \tag{11}$$

Whenever $M_t = I$ the innovation tax applies to every agent alive at time t and the new technology A_{t+1} is disposable at time $(t+1)$. Conversely, if $M_t = N$ agents do not pay any tax and they produce, at time $(t+1)$, with technology A_t. The majority choice $M_t = \{I; N\}$ maps, through the biunivocal function $i_t = i(M_t)$, into the set $\{0; i\}$.

In order to have an intertemporal voting equilibrium it is required that, in every period, agents optimally choose the innovation policy, taking future outcomes as given. Since people live up to three periods, young face three-period sequences of policies, adults two-periods ones and old have just one policy choice to do.[15]

Now we turn to the analysis of how each age class votes taking into account the optimal future political and economic choices. An agent belonging to age class j at time t bases her choice on the difference between the utility she gets in the case she votes in favor or against innovation. The stream of future majority choices and outcomes over which the agent forms correct expectations is $V_{t+1} = \{M_{t+1}; e^*_{t+1}; M_{t+2}; e^*_{t+2}; ...\}$. For every age class $j \in \{Y; A; O\}$ we define the differential utility as

$$\Delta u_t^j(V_{t+1}) = u_t^j(v_t^j = \iota; V_{t+1}) - u_t^j(v_t^j = \nu; V_{t+1})$$

that collapses to

$$\Delta u_t^j(V_{t+1}) = u_t^j(v_t^j = \iota) - u_t^j(v_t^j = \nu)$$

because of the specification of the utility function described above. In fact, the outcome of future innovation policies and educational choices do not influence agent's differential utility: income and substitution effects of the innovation cost cancel out. Since at the beginning of their life agents cannot commit themselves to a specific stream of votes, at each moment in time each of them votes to maximize her expected future lifetime utility. For a young agent born at time t the expected future lifetime utility is

$$u_t^Y = \log c_t^t + \alpha \log c_{t+1}^t + p_{t+2}\beta \log c_{t+2}^t \tag{12}$$

[15] Being the two values of policy variable $M = \{I; N\}$ ("innovation" and "no innovation", respectively), young born at time t face eight possible streams of policies: $\{I_t; I_{t+1}; I_{t+2}\}$; $\{I_t; I_{t+1}; N_{t+2}\}$; $\{I_t; N_{t+1}; I_{t+2}\}$; $\{I_t; N_{t+1}; N_{t+2}\}$; $\{N_t; I_{t+1}; I_{t+2}\}$; $\{N_t; I_{t+1}; N_{t+2}\}$; $\{N_t; N_{t+1}; I_{t+2}\}$; $\{N_t; N_{t+1}; N_{t+2}\}$. Adults at time t face four possible streams: $\{I_t; I_{t+1}\}$; $\{I_t; N_{t+1}\}$; $\{N_t; I_{t+1}\}$; $\{N_t; N_{t+1}\}$. Old people just face the decision $\{I_t\}$ or $\{N_t\}$.

that coincides with (4). Expected future lifetime utility for an adult born at time $(t-1)$ is defined as

$$u_t^A = \alpha \log c_t^{t-1} + p_{t+1}\beta \log c_{t+1}^{t-1} \tag{13}$$

while the one of an old agent born at time $(t-2)$ is

$$u_t^O = \beta \log c_t^{t-2} \tag{14}$$

In the last expression the probability p_t does not appear because only survived old choose. The single age classes choose how to vote as follows.

Old Old people, in the case of a progressive policy, only incur in costs: once the new technology is in place, they will be dead. Their differential utility is therefore

$$\Delta u_t^O = u_t^O(v_t^O = \iota) - u_t^O(v_t^O = \nu) = \beta \log(1-i) < 0, \forall i \in (0,1]$$

where we plugged (7) and (8) in (14).

Remark 9 *Old's optimal choice is to always vote against innovation.*

Adults When adult, agents vote over the innovation that will be implemented the next period. As described above, their differential utility depends only on present innovation choices.

$$\Delta u_t^A = u_t^A(v_t^A = \iota) - u_t^A(v_t^A = \nu) \tag{15}$$

By substituting (6), (7), (10) and (13) into (15), we get:

$$\Delta u_t^A(p_{t+1}) = \alpha \log(1-i-s) + p_{t+1}\beta \log(1+\theta) + p_{t+1}\beta\gamma \log(1-\delta) - \alpha \log(1-s) \tag{16}$$

The first and fourth terms jointly show the differential negative impact of the innovation tax on the net income when adult: in the case of innovation the share of income going to finance adult age consumption shrinks. The second term represents the gain in productivity attached to the pension income when old, weighted by the probability to survive. The third term is the negative impact of an innovation on the stock of human capital acquired by adult's child: this translates in smaller pensions benefits for the adult herself when old.

Let us assume from now on that

$$(1+\theta)(1-\delta)^{\gamma} > 1 \iff \theta > (1-\delta)^{-\gamma} - 1 \tag{17}$$

This condition on the relative magnitude of TFP improving parameter and human capital depreciation parameter states that the productivity improvements in the production of final good (θ) exceeds an increasing function of both the rate of depreciation of the human capital in the case of innovation (δ) and its productivity in the production of the final good (γ). We rewrite (16) in a compact way, since it will be useful in the next subsection.

Remark 10 *Adults' differential utility can be represented by a linear positive relation linking Δu^A to p, dropping the time index for simplicity:*

$$\Delta u^A(p) = m^A(\theta, \delta, \gamma, \beta)p + q^A(s, i, \alpha) \tag{18}$$

where $m^A = \beta \log\left((1+\theta)(1-\delta)^\gamma\right)$ and $q^A = \alpha \log\left(\frac{1-s-i}{1-s}\right)$.

Lemma 11 *Adults vote for the adoption of a new technology if and only if they achieve a life expectancy p_{t+1} larger than the threshold p^A, defined as*

$$p^A = \frac{\alpha \log\left(\frac{1-s}{1-s-i}\right)}{\beta \log\left((1+\theta)(1-\delta)^\gamma\right)} \tag{19}$$

Conversely, if $p_{t+1} < p^A$, they are against.

Proof. The expression of p^A is obtained from (16) solving $\Delta u_t^A(p_{t+1}) = 0$ for p_{t+1}. Given (17) and $i > 0$, the graph of $\Delta u_t^A(p_{t+1})$ has a negative intercept and crosses the $\Delta u_t^A = 0$ axis from below, proving the Lemma. ■

Adults vote for an innovation if and only if they will get higher resources (net of innovation costs) when old, in the form of pensions paid by their adult children. The threshold p^A is a positive function of i: the more expensive is the adoption of a new innovation, the less the adult will be innovation-prone. The same consideration holds for δ: due to the adoption of a new technology, the more the human capital depreciates, the less the adult will be in favour of implementing the new technology itself. Conversely, higher growth rates of TFP make adults to prefer innovations. Note that the elasticity of past human capital in the production of the new human capital (ϵ) is not involved in adult's decisions: we will see below that only young take into account how the past level of human capital affects the next period's human capital accumulation. The higher the share of adult's income going to finance old's pensions is (s), the less the adult will be innovation-prone. The higher is the preference for adult age consumption (α), the more they will be against innovation. Conversely, preference for old age consumption (β) leads to preference for innovation. This is because of the structure of innovative process: it is a cost today and it gives benefits tomorrow. Lastly, an increase in the elasticity of human capital in the production of final good (γ) works against innovation: innovation makes part of the human capital achieved during youth to depreciate, and the higher its effectiveness in production is, the higher the loss is in terms of pensions paid by adults' adult children.

Young Young vote over innovation taking into account their expected future lifetime utility but, for the same arguments stated above, what will happen at time $(t+1)$ and $(t+2)$ does not influence young's vote today. Young's differential utility is therefore:

$$\Delta u_t^Y = u_t^Y(v_t^Y = \iota) - u_t^Y(v_t^Y = \nu) \tag{20}$$

By substituting (12), (5), (6), (7) and (10) into (20), we get:

$$\Delta u_t^Y(p_{t+2}) = \log(1-i)+\alpha\log(1+\theta)+\alpha\gamma\log(1-\delta)+p_{t+2}\beta\log(1+\theta)+p_{t+2}\beta\epsilon\gamma\log(1-\delta) \tag{21}$$

Young, in case of innovation, again directly benefit from the technologic parameter θ, but now it impacts both on their labour income when adults and on their pension benefits when retired. In this last case the benefit from innovation is proportional to p_{t+2}, so a longer life gives them more time to enjoy higher consumption. The cost structure is similar: a constant cost is due to the depreciation of human capital when young become adults, through a smaller marginal productivity in the production of final good. Another cost, proportional to p_{t+2}, takes into account the depreciation of human capital of young's children: two periods later, in fact, today's young will get a pension that will be, in terms of human capital, depreciated because of today's choice to innovate. Therefore the depreciation term is mitigated by two terms, ϵ and γ: the first takes into account the elasticity between the production of new human capital and the past stock of human capital, the latter the elasticity of human capital in the production of final good.

Consistently with the case of adults, we rewrite (21) in the same fashion.

Remark 12 *Young's differential utility can be represented by a linear positive relation linking Δu^Y to p, their life expectancy:*

$$\Delta u^Y(p) = m^Y(\theta,\delta,\gamma,\epsilon,\beta)p + q^Y(\theta,\delta,\gamma,i,\alpha) \tag{22}$$

where $m^Y = \beta\log\left((1+\theta)(1-\delta)^{\gamma\epsilon}\right)$ and $q^Y = \log\left((1-i)\left((1+\theta)(1-\delta)^{\gamma}\right)^{\alpha}\right)$.

Lemma 13 *Young vote for the adoption of a new technology if and only if they achieve a life expectancy p_{t+2} larger than the threshold p^Y, defined as*

$$p^Y = \frac{-\left[\log\left((1-i)\left((1+\theta)(1-\delta)^{\gamma}\right)^{\alpha}\right)\right]}{\beta\log\left((1+\theta)(1-\delta)^{\gamma\epsilon}\right)} \tag{23}$$

Conversely, if $p_{t+2} < p^Y$, young are against innovation.

Proof. The expression of p^Y is obtained from (21) solving $\Delta u_t^Y(p_{t+2}) = 0$ for p_{t+2}. ■

Young's choices over innovation shows similar determinants as adult's. Again the threshold level is negatively correlated with the TFP growth rate (θ) induced by innovation. The depreciation of human capital in the case of innovation (δ) is a factor that discourages young, as long as adult, to vote for innovation. For young, increases in both adult and old age consumption preferences makes them to be more prone to innovation.

The effect of the elasticity of past human capital in the production of human capital (ϵ) on p^Y is positive: $\frac{\partial p^Y}{\partial \epsilon} > 0$. A high inertia in the transmission of human capital from one generation to the other leads to less interest in innovation because, as in Boucekkine et al. (2002). *Ceteris paribus*, the more the accumulation of human capital relies on past human capital, the more it depreciates in case of innovation. Differently from the case of adults, for young preference for both adult (α) and old (β) age consumption are conducive to innovation.

	p^O	p^A	p^Y	p^*
political weight of young people (η)	-	0	0	0
preference for adult age consumption (α)	0	+	-	+
preference for old age consumption (β)	0	-	-	+
productivity gains from innovation (θ)	0	-	-	0
frictional costs of innovation (i)	0	+	+	-
depreciation of human capital due to innovation (δ)	0	+	+	-
productivity of human capital in final good production (γ)	0	+	+	+
elasticity of past h-capital in the production of new h-capital (ϵ)	0	0	+	+
share of adults' income used to pay parents' pensions (s)	0	+	0	0
constant of proportionality in h-capital production (λ)	0	0	0	+

Tab.1. Partial effects of parameters on thresholds.

In table 1 we sum up the partial effects of the parameters on the thresholds p^O, p^A and p^Y. Moreover, we add the effects of the same parameters on all the steady state values (p^*) of the function $h_{t+1} = \Gamma_1(h_t; i_t)h_t^{\epsilon}$, derived in the previous section. This will turn to be useful in the section where we jointly study the economic and political mechanisms.

2.7 Political Outcome

We now formalize the political outcome at each point in time, given the preferences of every age class, their political weights and life expectancies. We derive some propositions that help to understand the dynamic features of the economy, that are analyzed in the next subsection.

In figure 5 we plot the graphs of $\Delta u^Y(p)$ and $\Delta u^A(p)$ ((22) and (18), respectively) and report, on the p axis, the value of p^Y, p^A and p^O.

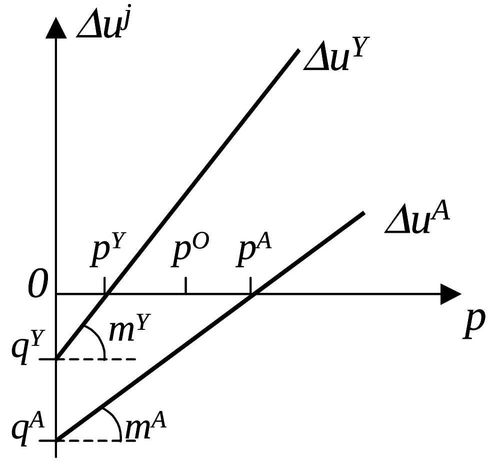

Fig. 5. Differential utilities and thresholds.

At time t the three generations alive are represented by their own life expectancies: p_t for old people born at time $(t-2)$, p_{t+1} for adults born at time $(t-1)$ and p_{t+2} for young people born at time t.[16] Life expectancy of each of the three age class is therefore compared with the corresponding threshold: p_t with p^O, p_{t+1} with p^A and p_{t+2} with p^Y.

Resuming, from (11) we know that for $p_t < p^O$ adults alone decide upon innovation. If $p_t > p^O$, innovation takes place only once that both $p_{t+1} > p^A$ and $p_{t+2} > p^Y$.

Lemma 14 *Young are in favor of innovation for any given level of life expectancy if innovation costs are small enough, i.e.* $i < 1-((1+\theta)(1-\delta)^{\gamma})^{-\alpha} \equiv i^*$.

Proof. We need that $p^Y < 0$ for some small values of i. Under assumption (17), it is enough to show that $q^Y > 0$, by graphical considerations. This is true if and only if $(1-i)\left((1+\theta)(1-\delta)^{\gamma}\right)^{\alpha} > 1$. By simple algebra the Lemma is proved. ■

Intuitively, since young get benefits in adult age and adulthood is reached with probability one, for large enough productivity improvement from innovation (θ) they are favorable to innovation if it is cheap (i^*), no matter what is their life expectancy. An implication of the previous Lemma is that the decision to adopt a new technology is therefore in the hand of adults alone when frictional costs are small. A general result is:

Lemma 15 *Whenever* $p^Y < p^A$*, the political outcomes in the case of "workers' dictatorship" and "diluted power" are the same.*

[16] More precisely, the best interpretation of old people's p_t is not in term of life expectancy, but as their mass in the political choice at time t.

Proof. Directly from inspection of (11). ■

This Lemma states that whenever young are relatively more innovation-prone than adults ($p^Y < p^A$) we can ignore the value of p^O in determining the political outcome. A useful property of adults' and young's thresholds is the following.

Lemma 16 *Adults' threshold p^A is larger than p^Y if $s > 1-\alpha\left((1+\theta)(1-\delta)^\gamma\right)^{-1} \equiv s^Y$ holds.*

Proof. We base our proof on the graphical representation in figure 5. Note that $\Delta u^Y(p)$ is steeper than $\Delta u^A(p)$ because (17) holds and $m^Y > m^A$. This inequality is ensured because $\frac{m^A}{m^Y} = \frac{\log((1+\theta)(1-\delta)^\gamma)}{\log((1+\theta)(1-\delta)^{\epsilon\gamma})} < 1$ since $0 < \epsilon < 1$. We need to prove that the intercept of $\Delta u^Y(p)$ lies above that of $\Delta u^A(p)$. Given $m^Y > m^A$, $p^Y < p^A \Leftrightarrow q^Y > q^A \Leftrightarrow ((1-i)\left((1+\theta)(1-\delta)^\gamma\right)^\alpha) > \left(\frac{1-s-i}{1-s}\right)^\alpha$, where, because of monotonicity, we have dropped the logs. Let us build $\Psi(i) = (1-i)^{\frac{1}{\alpha}}(1+\theta)(1-\delta)^\gamma - \left(\frac{1-s-i}{1-s}\right)$ by rearranging LHS and RHS terms of the previous inequality. Studying $\Psi(i)$ we find $\Psi(0) = (1+\theta)(1-\delta)^\gamma - 1 > 0$; $\Psi(1-s) = s^{\frac{1}{\alpha}}(1+\theta)(1-\delta)^\gamma > 0$; $\Psi'(i) = -\frac{1}{\alpha}(1-i)^{\frac{1-\alpha}{\alpha}}(1+\theta)(1-\delta)^\gamma + \frac{1}{1-s}$ and $\Psi''(i) > 0, \forall i \in [0; 1-s]$. Sign of $\Psi'(i)$ is ambiguous, while $\Psi'(0) = \frac{1}{1-s} - \frac{(1+\theta)(1-\delta)^\gamma}{\alpha}$ and $\Psi'(1-s) = \frac{1}{1-s} - s^{\frac{1-\alpha}{\alpha}}\frac{(1+\theta)(1-\delta)^\gamma}{\alpha}$. A sufficient condition for $\Psi(i)$ to be always positive (i.e. $q^Y > q^A, \forall i \in [0; 1-s]$) is to impose that $\Psi'(0) > 0$, that also leads to $\Psi'(1-s) > 0$ because of $s^{\frac{1-\alpha}{\alpha}} < 1$. $\Psi'(0) > 0 \iff \frac{1}{1-s} > \frac{(1+\theta)(1-\delta)^\gamma}{\alpha}$ and, together with the property $\Psi''(i) > 0$, this proves the Lemma. ■

The previous Lemma states that adults are less innovation-prone than young if the fraction of their income going in pension contributions is large enough. Standard calculus investigation shows that s^Y is negatively related to both preference for adult age consumption (α) and depreciation of human capital (δ). For example, an economy in which pension contributions are relatively large (and the political power of young is relatively small, i.e. $p^O > p^A$) shows that, for values of life expectancy between p^A and p^O, young are in favor of innovation while adults block the implementation of an innovation policy.

In the next section the dynamic behavior of the economy is discussed. An important property of the model is the following.

Proposition 17 *With standard intertemporal discounting behavior, i.e. $\beta = \alpha^2$, it is never the case of $p^A < p^Y < 1$.*

Proof. The strategy we follow is to break the two inequalities in the Lemma and to show that both can not simultaneously hold for any parametrization of the model. Let us define

$$\Psi(\alpha)_1 = \alpha^2(1-p^Y) = \alpha^2 + \alpha\frac{\log\left((1+\theta)(1-\delta)^\gamma\right)}{\log\left((1+\theta)(1-\delta)^{\epsilon\gamma}\right)} - \frac{\log\left(\frac{1}{1-i}\right)}{\log\left((1+\theta)(1-\delta)^{\gamma\epsilon}\right)}$$

and

$$\Psi(\alpha)_2 = \alpha^2(p^A - p^Y) = \alpha\left(\frac{\log\left((1+\theta)(1-\delta)^{\gamma}\right)}{\log\left((1+\theta)(1-\delta)^{\epsilon\gamma}\right)} + \frac{\log\left(\frac{1-s}{1-s-i}\right)}{\log\left((1+\theta)(1-\delta)^{\gamma}\right)}\right) - \frac{\log\left(\frac{1}{1-i}\right)}{\log\left((1+\theta)(1-\delta)^{\gamma\epsilon}\right)}$$

using (19), (23) and substituting $\beta = \alpha^2$. We only write the dependency of both Ψ_1 and Ψ_2 on α for brevity. It turns out that the two inequalities $p^A < p^Y$ and $p^Y < 1$ are both satisfied if both $\Psi_2 < 0$ and $\Psi_1 > 0$ hold, respectively. In figure 6 we show the shapes of these two functions in terms of α.

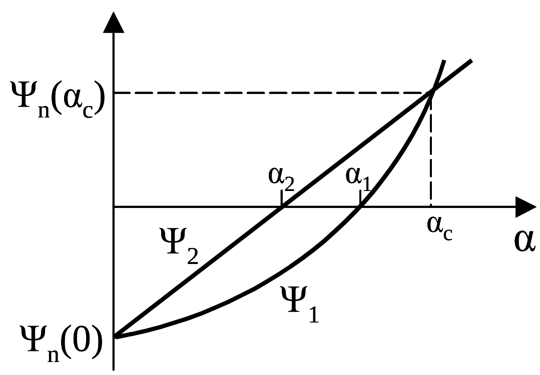

fig.6. Shapes of Ψ_1 and Ψ_2, their intersections with the axis and their crossing points.

The derivatives of Ψ_1 and Ψ_2 with respect to α are $\Psi_1' = \frac{\log((1+\theta)(1-\delta)^{\gamma})}{\log((1+\theta)(1-\delta)^{\epsilon\gamma})} + 2\alpha$ and $\Psi_2' = \frac{\log((1+\theta)(1-\delta)^{\gamma})}{\log((1+\theta)(1-\delta)^{\epsilon\gamma})} + \frac{\log\left(\frac{1-s}{1-s-i}\right)}{\log((1+\theta)(1-\delta)^{\gamma})}$. Second derivatives are $\Psi_1'' = 2$ and $\Psi_2'' = 0$, implying that Ψ_1 is a quadratic function of α and for $\alpha > 0$ it is increasing at an increasing rate. Ψ_1 is a positively sloped straight line. For $\alpha = 0$, both Ψ_1 and Ψ_2 take the same value $\Psi(0)_1 = \Psi(0)_2 = \Psi(0)_n = -\frac{\log\left(\frac{1}{1-i}\right)}{\log((1+\theta)(1-\delta)^{\gamma\epsilon})} < 0$, as shown in the graph. Moreover, Ψ_2 is steeper than Ψ_1, for any values of the parameters and for some small values of α (from an inspection of Ψ_1' and Ψ_2', until $\frac{\log\left(\frac{1-s}{1-s-i}\right)}{\log((1+\theta)(1-\delta)^{\gamma})} > 2\alpha$). Therefore for $\alpha : 0 \leq \alpha < \alpha_2$, $\Psi_2 < 0$ holds, while it is not the case for $\Psi_1 > 0$. Because of their shapes and their crossing in $\alpha = 0$, they also have to cross again for some positive value of α. If the crossing point $(\Psi(\alpha_c)_n)$ of Ψ_1 and Ψ_2 lies below the α-axis and $\alpha_c < 1$, this means that there are values of α: $\alpha_1 < \alpha < \alpha_2$ such that both $\Psi_2 < 0$ and $\Psi_1 > 0$ hold at the same time. This can not be the case because equating Ψ_1 to Ψ_2 gives $\alpha_c = \frac{\log\left(\frac{1-s}{1-s-i}\right)}{\log((1+\theta)(1-\delta)^{\gamma})}$, that plugged into Ψ_1 or Ψ_2 gives $\Psi(\alpha_c)_1 = \Psi(\alpha_c)_2 = \Psi(\alpha_c)_n = \frac{\log\left(\frac{1-s}{1-s-i}\right)^2}{(\log((1+\theta)(1-\delta)^{\gamma}))^2} + \frac{\log\left(\frac{1-s-i+is}{1-s-i}\right)}{\log((1+\theta)(1-\delta)^{\gamma\epsilon})} > 0$, for any values of $\theta, s, i, \delta, \gamma$ and ϵ in their supports. The Lemma is therefore proved. ■

Corollary 18 *With β not linked to α it is always possible to find a value β^W such that, for $\beta > \beta^W$, $p^A < p^Y < 1$ holds.*

The Lemma states that when the discount factor is independent from the time index but it only depends on the *distance* between two points in time, there are not feasible values of life expectancy such that adults are in favor of innovation while young are not. That is because young get a double benefit from innovation, both during their adulthood and old age. Discounting them in the same way, it is intuitive that once they became adult they can not "do better" than when they were young, in the sense that they discount old age consumption in the same way as before, but now the gains will only be from one side (i.e. higher pension contributions by their children) and will be only a *fraction*, depending on s, of their children gains in productivity.

Conversely, if people attach a large weight on old age consumption, for some values of life expectancy it can be the case that, when young, they are not in favor of innovation, while adults are. This is because the variable part of net gains young get with innovation (the last two terms in (23)) are only in part linked to life expectancy, and therefore they are less reactive to large values of β. Adults' variable part and constant part of net gains are instead directly linked by the parameters α and β (see (19)). By allowing $\frac{\beta}{\alpha}$ to increase, the differential expected future lifetime utility of adults increases at an higher rate than that of young, giving rise to the case of $p^A < p^Y < 1$.

2.8 Dynamics and Discussion

The transitional dynamic of the economy during the adjustment toward the steady state is the core analysis of this section. The artificial economy we describe is one in which initial life expectancy is small but increasing and people are not yet in favor of innovation, in order to give an example of some dynamic behaviors of the economy. We leave to the reader the analysis of other kinds of dynamics, easily derivable from the economy's properties described up to this point.[17] We therefore assume two restrictions to hold for initial life expectancy p_0:[18]

$$p_0 < \min\{p^O; p^A; p^Y\} \tag{24}$$

$$p(h^{U_0}) < p_0 < p(h^{S_0}(i_t)) \tag{25}$$

where (25) means that initial life expectancy p_0 lies between the values that $p(h)$ takes at the two successive unstable and stable steady states of $h_{t+1} = \Gamma_1(h_t; i_t)h_t^{\epsilon}$, respectively. These assumptions ensure that at time $t = 0$ life expectancy is monotonically increasing until its steady state value $p^S = p(h^{S_0}(i_t))$. The preferred policy is N because, due to (19) and (23), agents vote against

[17] As an example, other kinds of dynamics include cases in which initial life expectancy is *decreasing* toward a lower steady state or cases in which the initial undertaken policy is I.

[18] Without loss of generality, we assume that p_0 is the life expectancy of young born at time $t = 0$. Note that $p_0 \neq \underline{p}$: the former is the value of life expectancy that the economy shows at time $t = 0$, the latter is the value of life expectancy that function $p(h)$ takes for $h = 0$.

innovation. We further assume that two more restrictions hold: $p^A < 1$ and $p^Y < 1$, so that both adults and young can, in principle, be in favor of innovation for large enough values of life expectancy.[19] We keep track of the evolution of p_t knowing that it converges monotonically toward its steady value p^S. The evolution of p_t allows us to describe the (possible) variations in the innovation policy adopted. The political outcome, defined by (11), depends on (i) the relative ordering of $\{p^O; p^A; p^Y\}$ and (ii) the one-to-one comparison between the triplets $\{p_t; p_{t+1}; p_{t+2}\}$ and $\{p^O; p^A; p^Y\}$. Moreover, where p^S is located with respect to p^O, p^A and p^Y affects the long run policy implemented.

Given assumptions (17), (24) and (25), up to four dynamic scenarios are possible.

Proposition 19 *The evolution over time of an economy characterized by an increasing life expectancy, whose initial value is $p_0 < \min\{p^O; p^A; p^Y; p^S\}$, shows up to four different configurations in term of innovation policy. The four configurations depend on the four thresholds' ordering and are the followings:*

1. *The economy never engages in innovation, ending up in a steady state in which output is constant over time.*

2. *The economy at some point switch to a regime of steady innovation adoption, ending up in a steady state in which output grows over time at rate θ.*

3. *The economy experiences innovation for a limited time span. Before and after this limited period of enhanced output growth the economy evolves without innovating, ending up in a steady state in which output is constant over time.*

4. *The economy experiences two waves of innovation, the second of which lasts forever. The economy behaves as in the previous point (3), but before reaching the human capital steady state h^{S_0} (and, therefore, $p = p^S$) it again incurs in preference for innovation. Its output's steady state growth rate is θ.*

Proof. As the proposition, this proof will be divided in four points.

1. This is the case whenever (a) $p^S < \min\{p^O; p^A; p^Y\}$, (b) $p^O < \min\{p^A; p^Y\} \cap p^S < \max\{p^A; p^Y\}$ or (c) $p^Y < \min\{p^O; p^A; p^S\} \cap p^S < p^A$. In (a) the steady state level of human capital and life expectancy is very small. This is due to small values of λ, that eventually can lead to only one, low steady state for the function $h_{t+1} = \Gamma(h_t; 0)h_t^{\epsilon}$. We are in sub-case (b) if young have large political power but their incentive to innovate is small (large η, i or δ, small θ, α or β) and, as in (a), the steady state level of human capital and life expectancy is small. In (c) innovation does not take

[19]The inequalities $p^A < 1$ and $p^Y < 1$ resolve in $\frac{1-s}{1-s-i} < ((1+\theta)(1-\delta)^{\gamma})^{\frac{\beta}{\alpha}}$ and $\frac{1}{1-i} < (1+\theta)^{\alpha+\beta}(1-\delta)^{\gamma(\alpha+\beta\epsilon)}$, respectively.

place at any point in time because adults are very conservative (and young are very progressive) and the steady state level of human capital and life expectancy is reached before adults switch from innovation aversion to preference for it. This is for large values of s and small values of i.

2. This is the case whenever (a) $p^A < p^S < \min\{p^O; p^Y\}$, (b) $p^S > \max\{p^A; p^Y\} \cap (p^O > \max\{p^A; p^Y\} \cup p^O < \min\{p^A; p^Y\})$, or (c) $p^Y < \min\{p^O; p^A; p^S\} \cap p^S > p^A$. In (a) adults alone decide to implement innovation due to large preference for old age consumption (β), then the economy reach the steady state level of human capital. In (b) either adults alone set an innovation policy and the steady state level of human capital is large, or young coalesce with adults from the beginning, due to large political power, in setting a progressive policy. In (c) young alone set up an innovation policy and the steady state level of human capital is reached before the conservative adults veto the policy.

3. This case takes place if and only if $p^A < p^O < p^S < p^Y$ holds. Here adults alone set up the innovation policy, then conservative young step in once they get enough political power, restoring the no-innovation regime. This is the case of large preference for old age consumption (β), small political power and willingness to innovate of young and relatively small steady state level of human capital.

4. This case takes place if and only if $p^A < p^O < p^Y < p^S$ holds. As before conservative young stop the first wave of innovation, but at some point they turn to be in favor of innovation, since the steady state level of human capital, and therefore life expectancy, is large.

■

The four scenarios depicted in the previous proposition describe the four different regimes that an economy characterized by endogenous increase in life expectancy and centralized decisions upon innovation policy can show. Regimes 3 and 4 and, in general, configurations of $\{p^O; p^A; p^Y; p^S\}$ in which $p^A < p^Y$ are not feasible in the case the intertemporal discounting behavior of agents is characterized by $\beta = \alpha^2$.

3 Conclusions

Over the past century, all OECD countries have been characterized by a dramatic increase in economic conditions, life expectancy and education attainments. This paper examines the unexplored interactions among aging, human capital formation, technology adoption and economic growth. Assuming that longevity is positively correlated with the level of human capital, it demonstrates that an increase of life expectancy is, in principle, growth-enhancing factor. However, its effectiveness can be harmed by two phenomena, one related to human capital accumulation and the other to aggregation issues about technology adoption.

We reach Blackburn and Cipriani's (2002) same conclusions about the pure economic effects of an increase in longevity. Due to the positive causal effect of human capital on expected life expectancy, it can be the case that small levels of human capital lead to a short life, and this in turns disincentives people to invest in education, giving rise to a poverty trap. At this stage of development, life expectancy is short and human capital stock is small.

About the political features of our economy, we find that a variation in life expectancy affects both the individual incentives to innovate and the aggregate choices of the economy, since political representativeness of different age classes changes. At individual level an higher life expectancy increases the incentive to innovate for both young and adults. However, at the aggregate level different configurations can arise due to the endogenous changes in the demographic structure.

Relatively to the predictions about the transition toward the steady state, we find that during first stages of development, when (i) human capital is negligible, (ii) life expectancy is short and (iii) retired people are few, the political power is in the hand of adult workers alone. The decision to innovate or not coincides, therefore, with adults' choice. In the case their incentives to innovate are small (i.e. a large share of labour income going to finance the pension system, a large elasticity of the human capital used in production or a high concern in adult age consumption) they impose to the whole economy a no innovation regime. In developed economies, where (i) life expectancy is long, (ii) human capital endowment is large and (iii) retired people are several, a political majority that enforces an innovation policy can be achieved only by means of a coalition. Since elderly people are innovation averse, the only way for an innovation to be implemented is that both young and adult are in favour of innovation. Therefore, if on the one hand a longer life expectancy pushes people's incentives toward innovation, on the other hand it makes the political weight of old to increase, making the achievement of a consensus for innovation potentially more difficult. This is true, in particular, when young's incentives for innovation are lower than the ones of adult, in the case of a high inertia in the transmission of human capital from one generation to the next one and when the preference for old age consumption is large. However, if intertemporal discounting is standard, the case of adults in favor of innovation and, at the same time, young against is not feasible.

With this paper we provide the basis for joining together two strands of the literature on economic growth that are gaining importance in the research and political debate: technologic innovation and aging population. We stress how different links run between these two phenomena, defining the possible conflict of interests among different generations and showing how the lengthening of life expectancy changes the way this conflict of interests is solved. Moreover, we stress how private and public choices combine (or not) in order to give birth to a human capital abundant, growing economy.

References

[1] Aghion, P., Howitt, P. (1998). "Endogenous Growth Theory", MIT Press.

[2] Bauer, M. (1995). "Resistance to New Technology", Cambridge University Press.

[3] Becker, G. S., Tomes, N. (1986). "Human Capital and the Rise and Fall of Families", Journal of Labor Economics, 4, 1-39.

[4] Bellettini, G., Berti Ceroni, C. (1999). "Is Social Security Really Bad for Growth?", Review of Economic Dynamics, 2, 796-819.

[5] Bellettini, G., Ottaviano, G. I. P. (2005). "Special Interest and Technological Change", Review of Economic Studies, 72, 43-56.

[6] Blackburn, K., Cipriani, G. P. (2002). "A Model of Longevity, Fertility and Growth", Journal of Economic Dynamic and Control, 26, 187-204.

[7] Boucekkine, R., de la Croix, D., Licandro, O. (2002). "Vintage human capital, demographic trends, and endogenous growth", Journal of Economic Theory, 104, 340-375.

[8] Boucekkine, R., Licandro, O., Puch, L. A., del Rio, F. (2005). "Vintage Capital and the Dynamics of the AK Model", Journal of Economic Theory, 120, 39-72.

[9] Canton, E. J. F, de Groot, H. L. F., Nahuis R. (2002). "Vested Interests, Population Ageing and Technology Adoption", European Journal of Political Economy, 18, 631-652.

[10] Cervellati, M. Sunde, U. (2005). "Human Capital Formation, Life Expectancy and Process of Economic Development", American Economic Review 95, 1653–1672.

[11] Chakraborty, S. (2004). "Endogenous Lifetime and Economic Growth", Journal of Economic Theory, 116, 119–137.

[12] de la Croix, D., Licandro, O. (1999). "Life Expectancy and Endogenous Growth". Economics Letters, 65, 255–263.

[13] Fogel, R.W. (1994). "Economic Growth, Population Theory and Physiology: The Bearing of Long-Term Process on the Making of Economic Policy", American Economic Review 84, 369-395.

[14] Galasso, V., Profeta, P. (2004). "Politics, Ageing and Pensions", Economic Policy, April, 63-115.

[15] Galor, O. (2005). "Handbook of Economic Growth", Vol. 1A, edited by P. Aghion and N. Durlauf.

[16] Galor, O., Weil, D. N. (2000). "Population, Technology and Growth: From Malthusian Stagnation to the Demographic transition and Beyond", American Economic Review 90, 807-828.

[17] Jovanovic, B., (1998). "Vintage Capital and Inequality", Review of Economic Dynamics, 1, 497-530.

[18] Krusell, P., Rios-Rull, J. V. (1996). "Vested Interest in a Positive Theory of Stagnation and Growth", Review of Economic Studies 63,301-329.

[19] Lagerlof, N.-P. (2003). "From Malthus to Modern Growth: The Three Regimes Revisited", International Economic Review, 44(2), 755–777.

[20] Latulippe, D. (1996). "Effective Retirement Age and Duration of Retirement in the Industrial Countries Between 1950 and 1990", ILO Issues in Social Protection Discussion Paper No. 2.

[21] Lee, C. (2001). "The Expected Length of Male Retirement in the United States, 1850-1990", Journal of Population Economics, 14, 641-650.

[22] Lichtenberg, N. P., (1998). "Pharmaceutical Innovation, Mortality Reduction and Economic Growth", NBER Working Paper, 6569.

[23] Lichtenberg, N. P, (2003). "The Impact of New Drugs Launches on Longevity: Evidence from Longitudinal, Disease-Level Data from 52 Countries, 1998-2001", NBER Working Paper, 9754.

[24] Mincer, J. (1974). Schooling, Experience and Earnings. Columbia University Press.

[25] Mirowsky, J., Ross, C.E., (1998). "Education, Personal Control, Lifestyle and Health - A Human Capital Hypothesis", Research on Ageing 20, 415-449.

[26] Mokyr, J. (1998a). "The Political Economy of Technological Change: Resistance and Innovation in Economic History", in M. Bergand and K. Bruland (eds.) Technological Revolutions in Europe (Cheltenham: Edward Elgar) 39–64.

[27] Mokyr, J., (1998b). "Induced technical innovation and medical history: an evolutionary approach", Journal of Evolutionary Economics, Vol. 8, 119-137.

[28] Mokyr, J. (2002). "The Gifts of Athena: Historical Origins of the Knowledge Economy", Princeton University Press, 218-284.

[29] Olson, M. (1982). The Rise and Decline of Nations - Economic Growth, Stagflation and Social Rigidities, Yale University Press.

[30] Schultz, P. T., (1993). "Mortality Decline in the Low-Income World: Causes and Consequences", American Economic Review, 83 (2), 337-342.

[31] Schultz, P. T., (1998). "Health and Schooling Investments in Africa", Journal of Economic Perspectives, 13 (3), 67-88.