Human capital, public pensions and endogenous growth *

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October 30, 2007

Abstract

This paper investigates the implications of different social security systems on economic growth, when growth is engined by both human and physical capital accumulation. To do so, I extend the standard overlapping generations model with individuals that in their first period of life divide their time between education activities and working activities. Thus, the model allows for skill acquisition which affects economic growth. In their second period of life, individuals are old and receive pension benefits assumed to be positively related to human capital formation. It is shown that the introduction of an unfunded pension scheme in a Laissez-Faire economy decreases output growth, while a properly designed public funded pension scheme will lead to higher growth.

Keywords and Phrases: Human and physical capital accumulation, Public pensions, Overlapping Generations and Endogenous Growth

JEL Classification Numbers: D91, H55, O41

1 Introduction

This paper is concerned with interactions between human capital accumulation, social security and economic growth. The motivation for studying these interactions is especially due to two empirical observations. Firstly, education is one of the major engines behind economic growth (Coulombe et al., 2004; Cohen and Soto, 2007). This observation has stimulated and will continue to stimulate growth promoting

*I gratefully appreciate helpful comments and suggestions provided by Steinar Holden and Øystein Thøgersen. I am also indebted to Geir B. Asheim and Bjørnar Larssen for valuable comments.

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reforms in both developed and developing countries, where the reforms highlight the schooling system and education policy. Secondly, several European countries experience population ageing which in combination with an unfunded pension scheme may deteriorate public finances. This demographic development have led several countries to reform their social security system, and made pension reforms an important issue on the policy agenda in most OECD countries. However, the social security system affects both saving decisions and labor supply decisions, and consequently the growth rate in the economy.

These observations indicate that education and human capital accumulation, as well as social security programs affect economic development. However, there may also be an interaction between human capital accumulation and the social security system. If the pension benefit is linked to former working income which is positively linked to human capital, and to time spent on human capital formation, the pension system will have an impact on growth.[1] Motivated by these interactions and the empirical observations described above, the current paper studies human capital accumulation, social security systems and economic growth in a theoretical setting.

In order to do this I combine an overlapping generations model where two generations are alive at each point in time with an endogenous growth framework, where human and physical capital accumulation stimulates perpetual growth. The young generation can use a fraction of their time to studies and accumulate skills, while the remainder of their time is spent on labor market activities. The old generation is non-working and is assumed to be non-altruistic towards their children. The pension system is modeled in two different ways, one where the young generation pays contributions to the contemporaneous old agents, i.e. pay-as-you go, and one where the contributions paid by a whole young generation are invested and returned with interest to the same generation when old. However, in both systems a pension function that captures the link between human capital accumulation and pension benefits is applied. More precisely, it is assumed that human capital accumulation is a positive externality that spills over to the next generation, and that this externality is internalized by the pension system through a cross subsidy that stimulates investments in skill acquisition. For reference I also consider absence of social security, which under certain assumptions, can be interpreted as an economy with a

[1]Such a relation partially exists in Germany and has also been an issue in the Norwegian pension reform debate.

fully funded pension system.[2]

In the endogenous growth literature, growth can be engined through several channels. In the seminal paper by Romer (1986), growth is due to technological progress, endogenized by introducing knowledge as an input in production that has increasing marginal productivity.[3] On the other hand, economic growth may also be exerted through human capital accumulation. The literature on this interaction was pioneered by Lucas (1988), who added human capital accumulation to the neoclassical growth model.

Both of these two strands of endogenous growth theories can be put in an overlapping generations setting. For the Romer-type model, see Saint-Paul (1992), Wiedmer (1996) and Belan et al. (1998) among others. These contributions study growth implications of social security programs and reforms. For the Lucas-type model, see d'Autume and Michel (1994) for a general and systematic analysis, and Azariadis and Drazen (1990) for an elaboration of the standard overlapping generations model that permits multiple, locally stable balanced growth paths in equilibrium. This feature is partly due to externalities arising in the process of creating human capital. However, different social security systems and pension reforms are not discussed in these Lucas inspired contributions.

In general, it is customary to assume that the government's role in overlapping generation models with human capital either are related to subsidies in education[4] or public pension systems. In the current paper I will concentrate on transfer schemes and pension benefits, and suppress the financing of education. Earlier contributions with similar focus is for example Zhang (1995) who studies the interaction between social security and endogenous growth in a setting where agents care about their own consumption, the number of children, and the welfare of each child. The result in this analysis is that unfunded programs may cause faster growth than funded programs. The same conclusions are reached by Kemnitz and Wigger (2000). They argue that an unfunded pension scheme will provide social optimal incentives to invest in human capital. However, a funded pension program ignores that accumulation of

[2]Assuming equality between absence of social security and a fully funded pension system requires perfect capital markets and an actuarial relationship between payments and receipts (de la Croix and Michel, 2002).

[3]Romers assumption on marginal productivity is inspired by Arrow (1962), where knowledge creation is a side product of investment.

[4]Cf. Bräuninger and Vidal (2000), and Zhang (1996).

human capital over time requires that succeeding generations inherit part of their human capital stock, and thereby ignores a positive effect of actual investment in skill acquisition on the productivity of future generations. But, in that paper a funded pension scheme is assumed to be actuarial and identical to no pension system at all. This implies no role for the government and represents their Laissez-Faire economy, which is Pareto-inefficient due to these externalities.[5]

The current paper contributes to the theoretical growth and public finance literature by including different pension schemes in an endogenous growth setting, where time spent on education is essential. The question addressed is, how will different public pension schemes affect economic growth in a setting where perpetual growth is triggered by human capital accumulation as well as physical capital accumulation. I also compare the different outcomes with an economy without intergenerational transfers and governmental intervention. In contrast to Kemnitz and Wigger (2000) I model a funded and non-actuarial pension scheme with a cross subsidy that internalizes the positive intergenerational spillover of human capital accumulation. This system must be distinguished from a fully funded and actuarial system, or an economy without governmental interventions. In the current paper it is argued that a funded system modeled in this way may maintain the positive externality of investing in education. This is due to the link between investments in human capital and pension benefits. It is such a relation that prompt the results in Kemnitz and Wigger (2000), and thereby accounts for the positive spillover of education. However, this link is not dependant on a pay-as-you go pension system per se, but rather on a relation between time spent on education and pension receipts.

The paper is organized as follows. In section 2, I set up the model and present some partial results concerning the applied pension function and optimal savings. Section 3 derives equilibrium conditions, stability analysis and the endogenous growth model. Some general equilibrium results regarding the effect of studytime on individual savings are analyzed. Section 4 studies how different social security schemes will affect economic growth, and in section 5, I end by offering some concluding remarks.

[5]Note that Laissez-Faire in Kemnitz and Wigger (2000) and the current paper simply means a market economy without a government. Hence, Laissez-Faire does not exclude externalities, nor does it imply efficient allocations.

2 The setting

The basic framework is an overlapping generations model in the spirit of the seminal papers by Samuelson (1958) and Diamond (1965). By including human and physical capital, where human capital accumulates through studying (education), the model ensures persistent endogenous growth (Aghion and Howitt, 1999). Skill acquisition or time spent on studying is essential in the accumulation of human capital and it is assumed that it is always optimal to spend a positive amount of time to build human capital. It is also assumed that all members of a generation are identical. Hence, in equilibrium individual and average human capital coincide, i.e. $h_t = \bar{h}_t$. In the model time t is discrete and goes from 0 to ∞. It belongs to the set of integer numbers $\mathbb{N}$. The economy consists of a sequence of individuals who live for two periods. In each period t, N_t persons are born, so at each period $t \geq 1$, $N_t + N_{t-1}$ individuals are alive, where N_{t-1} are the number of old individuals. It is assumed that the number of households of each generation grows at a constant rate $n \in \langle -1, +\infty \rangle$:

$$N_t = (1+n)N_{t-1}\,. \tag{1}$$

Consequently, the total population grows also at the rate n.

2.1 Production and human capital

In each period t, production occurs according to a neoclassical production function $F(K_t, H_t)$, where F is homogenous of degree one and can thus be expressed by the mean of a function of one variable:

$$Y_t = F(K_t, H_t) = H_t F\left(\frac{K_t}{H_t}, 1\right) =: H_t f(\kappa_t)\,, \quad \text{with } \kappa_t := \frac{K_t}{H_t}\,, \tag{2}$$

where we denote by Y_t production, by K_t physical capital, by H_t labor efficiency units at time t, by κ_t the physical capital to efficient labor ratio and where:

$$y_t = f(\kappa_t) := F(\kappa_t, 1)\,, \quad \text{with } y_t := \frac{Y_t}{H_t}\,, \tag{3}$$

is the production function in its intensive form.

Assumption 1 *The production function* $f : \mathbb{R}_{++} \to \mathbb{R}_{++}$ *is positive, strictly increasing and strictly concave, i.e.* $f(\kappa) > 0$, $f''(\kappa) < 0 < f'(\kappa)$, $\forall \kappa > 0$. *On the boundary the function satisfies* $f(0) = 0$, *i.e. capital is essential, and the Inada conditions, i.e.* $\lim_{\kappa \to 0} f'(\kappa) = +\infty$ *and* $\lim_{\kappa \to +\infty} f'(\kappa) = 0$.

During the production process, the capital stock fully depreciates. Labor in efficiency units is determined by:

$$H_t = (1 - \lambda_t)\bar{h}_t N_t \,, \tag{4}$$

where $\lambda_t \in \langle 0, 1\rangle$ is endogenous and denotes the fraction of time spent on studying or education, hence $(1 - \lambda_t)$ indicates the fraction of time devoted to labor market activities and $\bar{h}_t$ is the average human capital stock at time t.[6] This implies that individuals in their first period of life divide their time between education and working, while they are non-working in their second period. At time 0, each household of the initial adult generation is endowed with human capital $h_0 > 0$. A single workers human capital, depends on studytime and the average human capital at time $t - 1$, and therefore evolves according to:

$$h_t = \psi(\lambda_t)\bar{h}_{t-1} \,. \tag{5}$$

Assumption 2 *For all $\lambda_t > 0$, $\psi(\cdot)$ is a continues, increasing and concave function defined for $\lambda_t \in \langle 0, 1\rangle$, i.e. $\psi''(\cdot) < 0 < \psi'(\cdot)$. Studytime is essential, i.e. $\psi(0) = 0$, and the function satisfies:*

$$\lim_{\lambda \to 0} \psi'(\lambda) = +\infty \,.$$

The last assumption ensures that it is always optimal to spend a strictly positive amount of time to build human capital. In each period the stock of physical capital results from total investments I_t, in the preceding period. It thus evolves according to $K_{t+1} = I_t$, in any period $t \geq 1$. Given the wage rate per efficiency unit of labor and the factor of return to capital at time t, w_t and R_t respectively, producers choose the level of capital and labor so as to maximize profits. That is:

$$\pi_t := \max_{K_t, H_t} \{H_t f(\kappa_t) - R_t K_t - w_t H_t\} \,, \tag{6}$$

where $R_t = 1 + r_t$, and r_t denotes the interest rate. Since all agents are price takers, maximization of profits implies the following first order conditions:

$$f'(\kappa_t) = R_t \quad \text{and} \quad f(\kappa_t) - \kappa_t f'(\kappa_t) = w_t \,. \tag{7}$$

From (7) it is straightforward to verify that the effect of studytime on the wage rate depends on the sign of $\psi(\lambda_t) - \psi'(\lambda_t)(1 - \lambda_t)$, i.e.:

$$\operatorname{sign} \frac{\partial w_t}{\partial \lambda_t} = \operatorname{sign} \left\{ \frac{1}{1 - \lambda_t} - \frac{\psi'(\lambda_t)}{\psi(\lambda_t)} \right\} \,. \tag{8}$$

[6]That individual human capital depends on the average human capital of the parent generation is adopted from Azariadis and Drazen (1990) and Kemnitz and Wigger (2000).

Hence, an increase in studytime does not necessarily increase the wage, as more time devoted to human capital accumulation implies lesser time devoted to working activities.

2.2 Individuals and the pension function

Each generation consists of a continuum of households, with unit mass, which are assumed to maximize their lifetime utility. Hence, individuals care about their consumption path across their lifecycle, i.e. for an individual born in period t, $c_{1,t}$ when young, and $c_{2,t+1}$ when old. It is assumed that each individual born at t has preferences described by an intertemporal utility function, $u(c_{1,t}, c_{2,t+1})$. This implies that there is no direct effect of λ_t and h_t on utility, only a indirect effect via lifecycle income. This simplification does not affect the qualitative results in the model.

Assumption 3 *The utility function $u : \mathbb{R}^2_{++} \rightarrow \mathbb{R}_{++}$ is twice continuously differentiable, strictly continuous, increasing and quasiconcave. For $(c_{1,t}, c_{2,t+1}) \gg 0$, $u(\cdot, \cdot)$ exhibits positive and diminishing marginal utility with respect to each argument. To avoid zero consumption in any period, $\lim_{c_{1,t}\rightarrow 0} u'_1 = +\infty$ for $c_{2,t+1} > 0$ and $\lim_{c_{2,t+1}\rightarrow 0} u'_2 = +\infty$ for $c_{1,t} > 0$, where u_i is the partial derivative of $u(\cdot, \cdot)$ with respect to the i-th argument.*

Individuals born in period t supplies a fraction $(1-\lambda_t)$ to the labor market, earning $h_t w_t$. Accordingly, the labor supply is in efficiency units. The social security system requires a financial input in order to convey purchasing power to the pensioners. It is assumed that this financial transfer comes from the working part of the population through a proportional tax rate $\tau_t \in \langle 0, 1\rangle$. Young agents therefore distribute their income among own consumption, taxes, and savings $S_t = s_t w_t h_t (1-\lambda_t)$, where $s_t \in \langle 0, 1\rangle$ is the fraction of income saved:

$$(1 - \lambda_t) h_t w_t = c_{1,t} + (\tau_t + s_t) w_t h_t (1 - \lambda_t)\,. \tag{9}$$

As the economy is assumed to be in autarky, all savings are allocated to investments. During old-age individuals are retired and receive the proceeds of their savings along with their pension benefits, P_t. The financing of these receipts depends on the pension system. In this paper I will consider both one unfunded and one funded pension scheme. Both systems contain subsidizing of education. The funded system are thus not equivalent to absence of social security programs. As aforementioned I also consider an economy without any governmental interference. Old-age consumption is consequently:

$$c_{2,t+1} = R_{t+1}S_t + P_{t+1}\,. \tag{10}$$

The pension function I apply is closely related to the one used in Kemnitz and Wigger (2000). Pension payments are positively linked to former wage and the level of human capital for each individual. For analytical purposes I use the following simple functional form:[7]

$$P_t = \Theta w_{t-1} h_{t-1}(1-\lambda_{t-1})\,, \tag{11}$$

where $0 < \Theta < 1$ denotes the constant pension ratio. Note that the pension function is non-actuarial in the sense that an increase in human capital affects pension payments in two ways. First, investment in human capital affects pensions via the effect on wage income, as human capital increases the wage per hour but also reduces the time spent on paid work. Second, investment in human capital is assumed to have a direct positive effect on pensions apart from the effect via wage income. This channel accounts for the intergenerational cross subsidy. This element of the model ensures that the positive spillover of investments in human capital on the productivity of future generations is captured.

Proposition 1 *The effect of studytime on pension benefit is concave. If* $(1-\lambda_t)^{-1} > \psi'(\lambda_t)/\psi(\lambda_t)$*, the pension benefit is decreasing in studytime unambiguously. If* $(1-\lambda_t)^{-1} < \psi'(\lambda_t)/\psi(\lambda_t)$*, the effect depends on the relative size of the terms in* $\partial P_{t+1}/\partial\lambda_t$*.*

Proof. Let $w_t = w_t(h_t)$. By inserting equation (5) into the pension function and taking the partial derivative with respect to λ_t yields:

$$\frac{\partial P_{t+1}}{\partial \lambda_t} = \Theta\Big\{ w_t'(\cdot)\psi'(\lambda_t)\bar{h}_{t-1}\psi(\lambda_t)\bar{h}_{t-1}(1-\lambda_t) + w_t(\cdot)\bar{h}_{t-1}\left[\psi'(\lambda_t)(1-\lambda_t) - \psi(\lambda_t)\right]\Big\}\,,$$

which implies that:

$$\frac{1}{1-\lambda_t} > \frac{\psi'(\lambda_t)}{\psi(\lambda_t)} \quad\Rightarrow\quad \frac{\partial P_{t+1}}{\partial \lambda_t} < 0\,.$$

[7] The choice of functional form does not alter the qualitatively conclusions.

But, if $\psi'(\lambda_t)/\psi(\lambda_t) > (1-\lambda_t)^{-1}$, then $\partial P_{t+1}/\partial\lambda_t > (<)\, 0$, if the second term is greater (less) than the first term in the derivative. ■

Proposition 1 can intuitively be explained as follows. An increase in studytime has two effects on pension benefits working in opposite directions. One effect is simply that an increase in studytime increases human capital,[8] and thereby increases pension payments. However, the wage rate is not necessarily increasing in studytime,[9] and as efficient wage is derived by multiplying the wage rate with human capital, the outcome of this factor is ambiguous. The other effect is related to total wage and working hours. An increase in studytime reduces time spent on labor market activities and thereby reduces labor income. The result can therefore be interpreted to imply that studytime increase pension benefits up to a certain point (or certain age), and thereafter the relation is decreasing. If one chooses to devote period 1 for studying only, the pension receipt is nil. But, as $\lambda_t \in \langle 0,1\rangle$, we can only infer that if $\lambda_t \to 1 \Rightarrow P_t \to 0$ on the boundary.

As time spent on human capital accumulation is endogenously determined, it is necessary to study the relationship between studytime and pension benefits, as well as studytime and lifecycle income. That studytime is uniquely determined can be shown by the first order condition of the utility function with respect to λ_t. However, as the first order condition for lifecycle income with respect to λ_t, necessarily must yield the same result, since there is no direct effect on utility of λ_t, it is sufficient to study the latter. By substituting (10) into (9) we obtain the intertemporal budget constraint:

$$c_{1,t} + \frac{c_{2,t+1}}{R_{t+1}} = (1-\tau_t)h_t w_t (1-\lambda_t) + \frac{P_{t+1}}{R_{t+1}}\,.$$

By inserting the accumulation of human capital (5) and the pension function (11), into the consolidated budget constraint we get the following expression for lifecycle income:

$$W_t := \psi(\lambda_t) w_t \bar{h}_{t-1}(1-\lambda_t)\left(1-\tau_t+\frac{\Theta}{R_{t+1}}\right), \tag{12}$$

where $w_t = w_t(h_t)$ and h_t is given by (5). Differentiating (12) with respect to time spent on human capital accumulation and set equal to zero yields the following:

$$\frac{\partial W_t}{\partial \lambda_t} = 0 \;\Rightarrow\; \psi'(\lambda_t) = \frac{\psi(\lambda_t) w_t}{(1-\lambda_t)\left[w_t + \psi(\lambda_t) w_t'(h_t)\bar{h}_{t-1}\right]} > 0\,, \tag{13}$$

[8] Cf. Equation (5) and Assumption 2.

[9] Cf. equation (8).

where the last inequality follows from Assumption 2. The inequality implies that $w_t > |\psi(\lambda_t)w_t'(h_t)\bar{h}_{t-1}|$. Equation (13) represents the tradeoff between studying and working. It implicitly defines the optimal length of studytime that maximizes lifecycle income. This relationship implies that the time spent on human capital accumulation depends positively on pension benefits.

As we are interested in growth effects and capital accumulation we need an expression for optimal individual savings. In order to illustrate the results I simplify the model by using a logarithmic additive separable lifetime utility function:

$$u(c_{1,t}, c_{2,t+1}) = \log c_{1,t} + \rho \log c_{2,t+1}\,, \tag{14}$$

where $\rho = 1/(1+\xi) \in \langle 0, 1\rangle$ is the psychological discount factor, where $\xi > 0$ stands for the constant pure rate of time preference of an individual, which varies inversely with ρ. It follows that the utility function $u(\cdot,\cdot)$ satisfies Assumption 3 and has an intertemporal elasticity of substitution equal to 1. The consumption choice and thereby optimal savings behavior is obtained by maximizing the lifetime utility function (14) subject to the intertemporal budget constraint in (12). By combining the two first order conditions for consumption we obtain the following intertemporal Euler equation for consumption:

$$\frac{u_1'(c_{1,t}, c_{2,t+1})}{u_2'(c_{1,t}, c_{2,t+1})} = R_{t+1} \;\Rightarrow\; c_{2,t+1} = \rho R_{t+1} c_{1,t}\,. \tag{15}$$

Inserting (9), (10) and the pension function (11) into the Euler equation (15) determines the optimal fraction saved as $s_t = (1+\rho)^{-1}\left[\rho(1-\tau_t) - \Theta(R_{t+1})^{-1}\right]$. Optimal savings per individual young agent is therefore:

$$S_t = \frac{1}{1+\rho}\left[\rho(1-\tau_t) - \frac{\Theta}{R_{t+1}}\right] w_t h_t (1-\lambda_t)\,. \tag{16}$$

This expression demonstrates individual optimal savings, and implies the following partial equilibrium effects.

Proposition 2 *Private savings increases in the interest rate, given wages and contributions. When the public pension ratio satisfies $\Theta < \rho(1-\tau_t)R_{t+1}$, private savings increases in wages, given the interest rate and contributions. The effect of studytime on individual savings is ambiguous and depends on the relation between $\psi(\lambda_t)$ and $\psi'(\lambda_t)(1-\lambda_t)$, given the interest rate, wages and that $\Theta < \rho(1-\tau_t)R_{t+1}$.*

Proof. The positive effect of the interest factor on private savings is verified by the following partial derivative:

$$\frac{\partial S_t}{\partial R_{t+1}} = \frac{\Theta h_t w_t (1-\lambda_t)}{(1+\rho)(R_{t+1})^2} > 0\,.$$

The effect of wages on private savings is unambiguous if and only if we impose a restriction on the maximum value of the pension benefit rate. Thus,

$$\frac{\partial S_t}{\partial w_t} = \frac{1}{1+\rho}\left[\rho(1-\lambda_t) - \frac{\Theta}{R_{t+1}}\right] h_t(1-\lambda_t) > 0 \;\;\Leftrightarrow\;\; \Theta < \rho(1-\tau_t)R_{t+1}\,,$$

which implies that savings is an increasing function of wage income if and only if the pension benefit rate is lower than $\Theta(1-\tau_t)R_{t+1}$. The relation between private savings and time spent on human capital accumulation is given by:

$$\frac{\partial S_t}{\partial \lambda_t} = \frac{1}{1+\rho}\left[\rho(1-\tau_t) - \frac{\Theta}{R_{t+1}}\right] w_t \bar{h}_{t-1}\left[\psi'(\lambda_t)(1-\lambda_t) - \psi(\lambda_t)\right]\,,$$

which implies that:

$$\text{If}\quad \psi'(\lambda_t)(1-\lambda_t) > (<)\;\psi(\lambda_t) \;\;\Rightarrow\;\; \partial S_t/\partial\lambda_t > (<)\;0\,,$$

and thereby completes the proof. ■

Except for the maximum condition on the pension benefit rate, the result that an increase in the wage rate increases savings is fairly established (Blanchard and Fischer, 1989). However, the relation between the interest rate and savings is ambiguous in a standard overlapping generations model without a public sector. This follows from the fact that a change in the interest rate has two effects for the consumer, a substitution effect and an income effect. As all individuals initially must be considered as savers, these two effects work in opposite directions, and the net effect is ambiguous. Nevertheless, as aforementioned the utility function employed in this paper is logarithmic and thus has an intertemporal elasticity of substitution equal to 1. In that case, and if there are no governmental interaction, the income effect exactly compensates the substitution effect and young individuals' savings are independent of the interest rate (de la Croix and Michel, 2002). This exactly balancing effect is not the case here however, as the social security system yields an incentive to substitute consumption between the two periods and take advantage of changes in the interest rate. The relation between studytime and individual savings are ambiguous within this partial equilibrium analysis. However, if one determines the sign on $(\psi'(\lambda_t)(1-\lambda_t)-\psi(\lambda_t))$, the relation follows unambiguously. These implications will not hold in general equilibrium, as will be explored in section 3.2.[10]

As the growth factor in this model is determined by both physical and human capital accumulation, equation (16) will be fundamental in showing the dynamics of capital in the economy. Before looking further at the growth factor let's present the set up of the public sector.

2.3 The public sector

2.3.1 National wealth

In this closed economy national wealth (Ω_t^n) consists of a country's human and physical capital. This implies the definition, $\Omega_t^n := K_t + H_t$. In this model it is assumed that the capital stock consists of both capital owned by households and the governments wealth (Ω_t^G). The government can only accumulate wealth in form of a pension fund that is built up from tax receipts from the young. This requires that there exists a time-lag between the contributions of the young and the transfers to the old agents. Human capital accumulation is assumed to only take place within the private sector. This simplification is rationalized by the purely distributional role of the government, and that human capital is owned by each individual. The government's wealth in the beginning of period $t+1$ can in general terms be expressed as:

$$\Omega_{t+1}^G = R_t\Omega_t^G + \tau_t w_t h_t(1-\lambda_t)N_t - \Theta w_{t-1}h_{t-1}(1-\lambda_{t-1})N_{t-1}\,,$$

where the second and the third term on the RHS is the government's income from taxes and payments to pensioners respectively. The growth factor presented in this paper will be on per capita form. Accordingly, it is desirable to derive the government's wealth in per capita form:

$$(1+n)\omega_{t+1}^G = R_t\omega_t^G + \tau_t w_t h_t(1-\lambda_t) - (1+n)^{-1}\Theta w_{t-1}h_{t-1}(1-\lambda_{t-1})\,, \qquad (17)$$

where $\omega_t^G := \Omega_t^G/N_t$ is the government's wealth per individual.

[10] It can also be shown that social security may have a negative impact on individual savings, as both an increase in taxes and the social security pension rate will decrease savings. These relations are due to a consumption smoothing effect.

2.3.2 Social security systems

Two different public pension systems will be distinguished: (i) a pay-as-you go system that is unfunded in the sense that the contributions paid by the young individuals at time t are used to pay pensions to the contemporaneous old agents. (ii) a funded system where the contributions paid by the young individuals at time t are invested and returned with interest at time $t+1$. In order to compare the results with Zhang (1995) and Kemnitz and Wigger (2000) among others, I also consider an economy without any social security system. This implies that $\Theta = P_t = \tau_t = \omega_t^G = 0$, for all t. The budget restrictions presented in this section closely follows Thøgersen (2001).

Pay-as-you go pension scheme

Within this pension scheme the government cannot contribute to accumulation of wealth as the income from the proportional taxes are paid out as pension benefits in the same period. Therefore $\omega_t^G = \omega_{t+1}^G = ... = 0$. The budget restriction is:

$$N_t \tau_t w_t h_t (1 - \lambda_t) = \Theta w_{t-1} h_{t-1} (1 - \lambda_{t-1}) N_{t-1} \,. \tag{18}$$

The LHS of (18) reflects the governments income in period t, while the RHS reflects pension expenditures $P_t N_{t-1}$, within the same period.

Funded pension scheme

The funded pension scheme considered must be distinct from both a pay-as-you go scheme and a fully funded and actuarial scheme. The system is similar to a pay-as-you go system regarding the direct effect of skill acquisition on pensions, but it differs with respect to the funding element. And it is exactly the funding feature that entails physical capital accumulation, which is absent in an unfunded system. Accumulation of physical capital is however present in the funded and actuarial pension system. Thus, physical capital accumulation is triggered both within the funded actuarial system, and the specific pension system considered here. But, these systems differs regarding the distribution of pension benefits, as the former is actuarial and the latter is non-actuarial. Note also that while an actuarial system, provided that capital markets are perfect, generates the same results regarding capital accumulation and economic growth as absence of governmental interventions, i.e. Laissez-Faire, the employed public pension scheme does not generate Laissez-Faire results. This follows as human capital accumulation is subsidized through the pension function.

Within the transfer scheme considered there is a time-lag between the government's income and expenditures. This entails that the government can contribute to the national wealth by investing the tax income in productive use, before making the transfers to the old individuals in the next period. Utilizing this aspect of the modeling implies that $\Theta w_{t-1}h_{t-1}(1-\lambda_{t-1})N_{t-1} = R_t\Omega_t^G$, so that the government's wealth in per capita is:

$$(1+n)\omega_{t+1}^G = \tau_t w_t h_t(1-\lambda_t)\,. \tag{19}$$

The budget restriction is accordingly:

$$\tau_t w_t h_t(1-\lambda_t)N_t R_t = \Theta w_t h_t(1-\lambda_t)N_t\,. \tag{20}$$

The LHS of (20) reflects the governments income in period t, and the RHS reflects their pension liabilities.

3 Equilibrium conditions and the growth model

In this section I will derive an analytical expression for the growth factor in the model economy. The growth factor will be expressed in per capita terms. Before turning to the analysis of different pension systems, it is necessary to study the equilibrium conditions in both the labor market and the capital market. I will also do steady state analysis to characterize stability.

3.1 Market equilibrium

Now I consider equilibrium conditions for the model economy. There are three markets in this economy: The final good market, the labor market and the capital market. The labor market equilibrium is derived from equation (5). Combining this equation with the assumption that all members of a generation are identical, and the training technology in (5) becomes:

$$H_t = (1-\lambda_t)\psi(\lambda_t)h_{t-1}N_t\,, \tag{21}$$

which determines the equilibrium condition in the labor market.

In this model the government is able to supply capital within a funded pension scheme. Accordingly, the general set up described here requires that the supply of capital stems from both the private and the public sector. The equilibrium condition in the capital market is thus:

$$K_{t+1} = N_t S_t + \Omega_{t+1}^G\,.$$

Since $K_{t+1} = \kappa_{t+1} H_{t+1}$, and we are interested in capital in efficiency units, using the equilibrium condition in (21) yields:

$$K_{t+1} = \kappa_{t+1}(1 - \lambda_{t+1}) h_{t+1} N_{t+1} = N_t S_t + \Omega^G_{t+1} \, . \tag{22}$$

According to Walras' law in period t, the equilibrium of the labor market and the capital market implies that of the final good market:

$$Y_t = H_t f(\kappa_t) = N_t (c_{1,t} + S_t) + N_{t-1} c_{2,t} \, ,$$

where the RHS displays the sum of aggregate consumption and aggregate savings. The demand of the good is the sum of the old individuals born in $t - 1$ and of the young individuals that consume and save in the final good.

The equilibrium conditions derived in this subsection will be explored to analyze optimal savings and economic growth under different social security strategies in section 4. But first we will have a closer look at savings, capital accumulation and the growth rate in the general setting.

3.2 Savings, stability and the growth factor

To obtain an expression for optimal individual savings we take as point of departure the expression in equation (16) and insert the first order conditions from (7), the training technology in (5) and the assumption that $h_t = \bar{h}_t$:

$$S_t = \frac{1}{1+\rho} \left[\rho(1 - \tau_t) - \frac{\Theta}{f'(\kappa_{t+1})} \right] \left[f(\kappa_t) - \kappa_t f'(\kappa_t) \right] (1 - \lambda_t) \psi(\lambda_t) h_{t-1} \, . \tag{23}$$

Based on equation (23) the following proposition examines the general equilibrium relation between individual optimal savings and time spent on human capital accumulation.

Proposition 3 *In general, the effect of studytime on individual savings is ambiguous and depends on the relative size of the terms in $\partial S_t / \partial \lambda_t$. The ambiguity is due to the effect from $\partial \kappa_t / \partial \lambda_t$ which affects savings in opposite directions.*

Proof. Let $k_t := K_t / N_t > 0$. It follows from the definition of κ_t and from the equilibrium condition in the labor market (21) that $k_t = (1 - \lambda_t) \psi(\lambda_t) h_{t-1} \kappa_t$. Accordingly,

$$\frac{\partial \kappa_t}{\partial \lambda_t} = \kappa_t \left[\frac{1}{1 - \lambda_t} - \frac{\psi'(\lambda_t)}{\psi(\lambda_t)} \right] .$$

As will become clear after the next step of the proof, the sign on this derivative is essential in the analysis of how saving responds to changes in studytime. The derivative of optimal savings with respect to λ_t is:

$$\frac{\partial S_t}{\partial \lambda_t} = \underbrace{\frac{1}{1+\rho}\left[\rho(1-\tau_t) - \frac{\Theta}{f'(\kappa_{t+1})}\right] h_{t-1}}_{(1)}$$
$$\times \left\{ \underbrace{\left\{ -\kappa_t^2 f''(\kappa_t) \left[\frac{1}{1-\lambda_t} - \frac{\psi'(\lambda_t)}{\psi(\lambda_t)}\right]\right\}}_{(2)} \underbrace{\psi(\lambda_t)(1-\lambda_t)}_{(3)} \right.$$
$$\left. + \underbrace{\left[f(\kappa_t) - \kappa_t f'(\kappa_t)\right]}_{(4)} \underbrace{\left[\psi'(\lambda_t)(1-\lambda_t) - \psi(\lambda_t)\right]}_{(5)} \right\},$$

with the following properties. (1), (3) and (4) are all strictly positive due to earlier assumptions. Notice that $\kappa_t > 0$. $(1) > 0$ follows as $\Theta < \rho(1-\tau_t)f'(\kappa_{t+1})$, which is assumed in Proposition 2. $(3) > 0$ follows from $\lim_{\lambda \to 0} \psi'(\lambda) = +\infty$ in Assumption 2. $(4) > 0$ follows as $w_t > 0$. The sign on $\partial S_t / \partial \lambda_t$ then depends on (2) and (5). In factor (2), notice from Assumption 1 that $f''(\kappa_t) < 0$. The determination of the sign therefore partly depends of the sign on $\partial \kappa_t / \partial \lambda_t$. That the sign on $\psi'(\lambda_t)(1-\lambda_t) - \psi(\lambda_t)$ does not fully determine the sign on $\partial S_t / \partial \lambda_t$, is verified by the following:

$$\text{If} \quad \frac{\partial \kappa_t}{\partial \lambda_t} < 0 \;\Rightarrow\; \psi(\lambda_t) < \psi'(\lambda_t)(1-\lambda_t) \;\Rightarrow\; \frac{\partial S_t}{\partial \lambda_t} \gtreqless 0\,,$$

where the same inequality holds for $\partial \kappa_t / \partial \lambda_t > 0$. Accordingly, the sign on $\partial S_t / \partial \lambda_t$ is determined by the relation between the two products $(2) \times (3)$ and $(4) \times (5)$. If the absolute value of the former is greater (lesser) than the absolute value of the latter, S_t is increasing (decreasing) in λ_t. Thus, the proof is complete. ■

In order to do stability analysis it is necessary to derive an expression for the dynamic behavior of efficient capital in the economy. Accordingly, we define a dynamic equilibrium as a sequence $\{\kappa_t\}_{t=0}^{\infty}$ given by:

$$(1+n)\kappa_{t+1} = \frac{1}{(1-\lambda_{t+1})h_{t+1}} \left\{ \frac{1}{1+\rho}\left[\rho(1-\tau_t) - \frac{\Theta}{f'(\kappa_{t+1})}\right] \times \left[f(\kappa_t) - \kappa_t f'(\kappa_t)\right](1-\lambda_t)h_t + (1+n)\omega_{t+1}^G \right\}, \tag{24}$$

where the initial condition κ_0 is exogenously given. This is an autonomous, nonhomogeneous and nonlinear difference equation of the first order. To study the behavior of this equation we define a steady state equilibrium as a stationary ratio between capital and efficient labor where all variables are constant. Consider first the fixed point tax rate obtained from the fixed point value of (17):

$$\tau = \frac{\omega^G(1+n-R)}{wh(1-\lambda)} + \frac{\Theta}{1+n}\,, \tag{25}$$

where $w = f(\kappa) - \kappa f'(\kappa)$ and $R = f'(\kappa)$ according to (7). In equilibrium the government stabilizes public wealth through their tax policy, i.e. (25) is satisfied. Hence, an implicit steady state equilibrium of (24) is:

$$\begin{aligned}(1+n)\bar{\kappa} = \frac{1}{(1-\lambda)h}\Bigg\{ & \frac{1}{1+\rho}\left[\rho(1-\tau) - \frac{\Theta}{f'(\bar{\kappa})}\right] \\ & \times \left[f(\bar{\kappa}) - \bar{\kappa}f'(\bar{\kappa})\right](1-\lambda)h + (1+n)\omega^G\Bigg\}\,,\end{aligned} \tag{26}$$

where $\bar{\kappa}$ is a fixed point, τ and ω^G are given by (25), and h is given by the fixed point value of equation (5). The nonlinear equation can be approximated by a linear equation by taking a first order Taylor expansion of κ_{t+1} about $\bar{\kappa}$:

$$\kappa_{t+1} \simeq \bar{\kappa} + \left(\left.\frac{d\kappa_{t+1}}{d\kappa_t}\right|_{\kappa_t=\bar{\kappa}}\right)(\kappa_t - \bar{\kappa}) = \kappa_t \left.\frac{d\kappa_{t+1}}{d\kappa_t}\right|_{\kappa_t=\bar{\kappa}} + \bar{\kappa}\left(1 - \left.\frac{d\kappa_{t+1}}{d\kappa_t}\right|_{\kappa_t=\bar{\kappa}}\right)\,, \tag{27}$$

where $\bar{\kappa}$ and κ_{t+1} are given as implicit functions in (26) and (24) respectively. The linear approximation in (27) implies that the dynamical system in (24) is locally asymptotically stable around $\bar{\kappa}$, if and only if:[11]

$$\left|\left.\frac{d\kappa_{t+1}}{d\kappa_t}\right|_{\kappa_t=\bar{\kappa}}\right| < 1\,. \tag{28}$$

From (25) and (24) one can derive that:[12]

$$\frac{d\kappa_{t+1}}{d\kappa_t} = \frac{VZ + UV'}{(1-\lambda_{t+1})h_{t+1}(1+\rho)(1+n) - V\Theta\frac{f''(\kappa_{t+1})}{(f(\kappa_{t+1}))^2}}\,, \tag{29}$$

where

[11] Cf. Galor (2007).

[12] See the appendix for a detailed derivation of (29).

$$
\begin{aligned}
V &:= \left[f(\kappa_t) - \kappa_t f'(\kappa_t)\right](1-\lambda_t)h_t \\
V' &:= -\kappa_t f''(\kappa_t)(1-\lambda_t)h_t \\
Z &:= \frac{\rho\omega_t^G f''(\kappa_t)}{h_t(1-\lambda_t)\left[f(\kappa_t)-\kappa_t f'(\kappa_t)\right]}\left[1-\frac{\kappa_t(1+n-f'(\kappa_t))}{f(\kappa_t)-\kappa_t f'(\kappa_t)}\right] \\
U &:= \rho\left(1-\frac{\omega_t^G\left(1+n-f'(\kappa_t)\right)}{h_t(1-\lambda_t)\left[f(\kappa_t)-\kappa_t f'(\kappa_t)\right]}-\frac{\Theta}{1+n}\right)-\frac{\Theta}{f'(\kappa_{t+1})}
\end{aligned}
$$

The dynamical system is therefore locally asymptotically stable converging to the steady state equilibrium $\bar{\kappa}$ regardless of the initial condition on κ_0, if and only if the absolute value of the numerator is less than the absolute value of the denominator in (29), i.e.:

$$
\begin{aligned}
&\Bigg| f''(\kappa_t)\Bigg\{\rho\omega_t^G\left[1-\frac{\bar{\kappa}\left(1+n-f'(\bar{\kappa})\right)}{f(\bar{\kappa})-\bar{\kappa}f'(\bar{\kappa})}\right] \\
&\quad -\bar{\kappa}(1-\lambda_t)h_t\left[\rho\left(1-\frac{\omega_t^G\left(1+n-f'(\bar{\kappa})\right)}{h_t(1-\lambda_t)\left[f(\bar{\kappa})-\bar{\kappa}f'(\bar{\kappa})\right]}-\frac{\Theta}{1+n}\right)-\frac{\Theta}{f'(\bar{\kappa})}\right]\Bigg\}\Bigg| \\
<\ &\left|(1-\lambda_{t+1})h_{t+1}(1+\rho)(1+n)-\frac{\left[f(\bar{\kappa})-\bar{\kappa}f'(\bar{\kappa})\right](1-\lambda_t)h_t\Theta f''(\bar{\kappa})}{(f(\bar{\kappa}))^2}\right|.
\end{aligned}
$$

Moreover, if $d\kappa_{t+1}/d\kappa_t$ evaluated at $\kappa_t = \bar{\kappa}$ is between 0 and 1, the analysis implies that κ converges monotonically (smoothly) to $\bar{\kappa}$. But, if $d\kappa_{t+1}/d\kappa_t$ evaluated at $\kappa_t = \bar{\kappa}$ is between -1 and 0, convergence is oscillatory. In the current paper it is sufficient to assume stability around steady state, hence equation (24) must content the condition in (28).

In order to make comparisons of different social security systems in an endogenous growth setting with human capital, the model outlined above is used to derive an expression for the growth factor in the economy. Notice that economic growth is here, as distinct from Kemnitz and Wigger (2000), engined by both human and physical capital. To derive an analytical expression for the growth factor I assume that output is given by a Cobb-Douglas production function $Y_t = AK_t^{\alpha}H_t^{\beta}$, where A is a scale parameter, and α and β denotes the capital share and labor share respectively. It is assumed that:

$$
A > 0\,,\ \alpha > 0\,,\ \beta > 0\,,\ \alpha + \beta = 1\,,
$$

where the last assumption implies constant returns to scale, and in accordance with equation (2) the production function can thus be expressed as $Y_t = AH_t\kappa_t^{\alpha}$, or in intensive terms as $y_t = A\kappa_t^\alpha$. The applied production function verifies Assumption 1, and the first order conditions in (7) are so given by:

$$R_t = \alpha A \kappa_t^{-\beta} \quad \text{and} \quad w_t = \beta A \kappa_t^\alpha \, . \tag{30}$$

Plugging (30) into (24) gives:

$$\begin{aligned} \kappa_{t+1} = \frac{1}{(1-\lambda_{t+1})h_{t+1}} \Bigg\{ & \frac{1}{(1+\rho)(1+n)} \left[\rho(1-\tau_t) - \frac{\Theta}{\alpha A \kappa_{t+1}^{-\beta}} \right] \\ & \times \beta A \kappa_t^\alpha (1-\lambda_t) h_t + \omega_{t+1}^G \Bigg\} . \end{aligned} \tag{31}$$

By letting g represent the growth factor in the economy, the following relation yields an implicit expression for economic growth in general:

$$\begin{aligned} g := \frac{\kappa_{t+1}}{\kappa_t} = \frac{1}{(1-\lambda_{t+1})h_{t+1}} \Bigg\{ & \frac{1}{(1+\rho)(1+n)} \left[\frac{\rho(1-\tau_t)}{\kappa_t^\beta} - \frac{\Theta g^\beta}{\alpha A} \right] \\ & \times \beta A (1-\lambda_t) h_t + \frac{\omega_{t+1}^G}{\kappa_t} \Bigg\} , \end{aligned} \tag{32}$$

where both human and physical capital are included.

4 Endogenous growth and social security systems

This section highlights the impact on economic growth of different types of public pension schemes. It also considers an economy without any governmental interferences, representing a Laissez-Faire economy. Comparing different social security systems is done by exploiting the different budget constraints of the government described in section 2.3. Bringing these relations into the growth factor in (32) reveals how implementation of public pension systems affect economic growth in the model economy.

Considering an economy without governmental interventions implies absence of any social security system. This is the case in a Laissez-Faire economy where old individuals only consume out of their own earlier savings.[13] In our model this case

[13] As inheritance is excluded from the model, consumption in the second period of life depends on individual savings and return on capital. See Holler (2007) for a model that includes ascending altruism and intrafamilial transfers, and Lambrecht et al. (2005) for a model with descending altruism and bequests.

is characterized by $P_t = \Theta = \tau_t = \omega_t^G \equiv 0$, and the growth factor is accordingly:[14]

$$g^{LF} = \frac{(1-\lambda_t)h_t}{\kappa_t^\beta (1-\lambda_{t+1})h_{t+1}} \frac{\rho A \beta}{(1+\rho)(1+n)} \,. \tag{33}$$

Recall that a fully funded and actuarial system is said to be neutral, provided that capital markets are perfect. Thus, in general, the intertemporal equilibrium yields the conclusion that a fully funded and actuarial social security system has no effect on total savings and capital accumulation, and therefore no effect on economic growth. Moreover, within this model g^{LF} is therefore equivalent to such a system.

4.1 Public pension systems

In a pay-as-you go pension scheme the government only plays the role of an intergenerational distributer and public wealth is in any period equal to zero, i.e. $\omega_t^G \equiv 0$. Solving the government's budget restriction in (18) for the pension ratio yields:

$$\Theta = (1+n)\tau_t \psi(\lambda_t) \Delta\kappa_t^\alpha \Delta(1-\lambda_t) \,, \tag{34}$$

where Δ defines the fraction of the subsequent variable between period t and $t-1$, i.e. $\Delta\kappa_t^\alpha := \kappa_t^\alpha / \kappa_{t-1}^\alpha$ and $\Delta(1-\lambda_t) := (1-\lambda_t)/(1-\lambda_{t-1})$. Inserting the restriction in (34) into the growth factor in (32) one obtains:

$$\begin{aligned} g^{PG} = \frac{1}{(1-\lambda_{t+1})h_{t+1}} \Bigg\{ & \frac{1}{(1+\rho)(1+n)} \left[\frac{\rho(1-\tau_t)}{\kappa_t^\beta} \right. \\ & \left. - \frac{(1+n)\tau_t\psi(\lambda_t)\Delta\kappa_t^\alpha \Delta(1-\lambda_t) g^\beta}{\alpha A} \right] \beta A (1-\lambda_t) h_t \Bigg\} \,. \end{aligned} \tag{35}$$

The analysis of how different pension systems affect economic growth within the model, is done by comparing the relevant growth expressions. The following proposition compares the pay-as-you go system with a Laissez-Faire economy.

Proposition 4 *The introduction of a pay-as-you go pension system to a Laissez-Faire economy decreases economic growth, i.e.* $g^{LF} > g^{PG}$*.*

[14] Variables with indices *LF*, *PG* and *F* respectively indicates the Laissez-Faire, pay-as-you go and funded cases.

Proof. Assume that $g^{LF} \leq g^{PG}$. Using the expressions in (33) and (35) yields the following:

$$\begin{aligned}
g^{LF} &= \frac{(1-\lambda_t)h_t}{\kappa_t^{\beta}(1-\lambda_{t+1})h_{t+1}} \frac{\rho A\beta}{(1+\rho)(1+n)} \\
&\leq g^{PG} = \frac{1}{(1-\lambda_{t+1})h_{t+1}} \left\{ \frac{1}{(1+\rho)(1+n)} \left[\frac{\rho(1-\tau_t)}{\kappa_t^{\beta}} \right.\right. \\
&\qquad \left.\left. - \frac{(1+n)\tau_t\psi(\lambda_t)\Delta\kappa_t^{\alpha}\Delta(1-\lambda_t)g^{\beta}}{\alpha A} \right] \beta A(1-\lambda_t)h_t \right\} \\
&\iff 1 \leq -\frac{(1+n)\psi(\lambda_t)\kappa_t\Delta(1-\lambda_t)g^{\beta}}{\rho\alpha A\kappa_{t-1}^{\alpha}}.
\end{aligned}$$

As the fraction on the RHS is positive the inequality fails and the proposition is proved by contradiction. ■

This result is due to two opposing effects where one of them dominates the other. In this model growth is due to both physical and human capital accumulation. The effect of a pension system on physical capital works through the effect on total savings. Unfunded public pension benefits will have a negative impact on private savings, since individuals partially rely on public pensions to finance their retirement. In a Laissez-Faire economy this negative impact is absent. The effect on human capital works in the opposite direction. As the pension benefit is positively related to studytime, this forms an incentive to spend time on building human capital. This relationship, which is absent in a Laissez-Faire economy, stimulates economic growth. However, the first effect dominates the second, and thereby reveals that the total effect of an unfunded pension scheme is lower growth compared to an economy without governmental interventions in our model.

This result does not necessarily entail that introducing a public pension scheme in a Laissez-Faire economy decreases growth. Analyzing the non-actuarial funded pension scheme reveals that a properly designed social security system may trigger economic growth. Recall that the time-lag between the government's income and pension liabilities, makes it possible for the government to do profitable investments. Public wealth is given by plugging the first order condition for wages (30) into the dynamic equation for public wealth in (19): $\omega_{t+1}^{G} = (1+n)^{-1}\tau_t\beta A\kappa_t^{\alpha}h_t(1-\lambda_t) > 0$. The government's budget restriction in (20) is now given by $\Theta = \tau_t\alpha A\kappa_t^{-\beta}$. By inserting these relations into the general growth factor we get the following expression:

$$g^F = \frac{(1-\lambda_t)h_t\beta A}{\kappa_t^\beta(1-\lambda_{t+1})h_{t+1}}\frac{\tau_t(1-g^\beta)+\rho}{(1+\rho)(1+n)}\,. \tag{36}$$

As showed in the next proposition, a funded pension scheme may increase growth if the pension benefit is positively linked to the time spent on human capital accumulation.

Proposition 5 *Introducing a public funded pension system, that stimulates individuals to build human capital in their first period of life, increases economic growth, i.e.* $g^F > g^{LF}$ *for* $g^\beta < 1$.

Proof. Assume that $g^F \leq g^{LF}$. Using the expression in (33) and (36) yields the following:

$$\begin{aligned} g^F &= \frac{(1-\lambda_t)h_t\beta A}{\kappa_t^\beta(1-\lambda_{t+1})h_{t+1}}\frac{\tau_t(1-g^\beta)+\rho}{(1+\rho)(1+n)} \\ &\leq g^{LF} = \frac{(1-\lambda_t)h_t}{\kappa_t^\beta(1-\lambda_{t+1})h_{t+1}}\frac{\rho A\beta}{(1+\rho)(1+n)} \\ &\Longleftrightarrow \tau_t(1-g^\beta) \leq 0\,, \end{aligned}$$

which proves the proposition by contradiction as $\tau_t(1-g^\beta) > 0$ for $g^\beta < 1$. ■

This result is due to the link between skill acquisition in the first period of life, and pension benefits in the second period of life, and that a funded scheme designed in this way stimulates physical capital accumulation. In a Laissez-Faire economy, physical capital accumulation is engined by private savings. These savings are reduced when a public social security system is implemented, but as a funded scheme initiates accumulation of public wealth the effect on total savings in the economy is absent. In the public funded system applied, these relations are maintained in addition to the relation between time spent on human capital accumulation and old-age receipts. As the latter relation does not exist in a Laissez-Faire economy, growth is stimulated by the introduction of such a properly designed public funded scheme.

Comparing the two social security systems reveals that growth is higher under the funded program than with a pay-as-you go system, i.e. $g^F > g^{LF} > g^{PG}$. The main mechanism behind this result lies in the time-lag that follows with a funded program. The government puts the tax receipts from the young to productive use and gives rise to a positive rate of return. Consequently, even though both pension schemes relates studytime and pension benefits, only the funded program stimulates public investments.

5 Concluding remarks

In this paper I present an overlapping generations model with endogenous growth, where both human and physical capital accumulation are the engines of output growth. Moreover, human capital accumulation is assumed to spill over to the nest generation and thus represent a positive externality on economic growth. The applied pension function explicitly takes account of this externality by relating skill acquisition and pension benefits, as well as wages, human capital and pension benefits. A similar pension function is used in Kemnitz and Wigger (2000). However, they suppress all other relations but the one between pension benefits and time spent on human capital formation. In their analysis, where human capital accumulation is the engine of growth, it is shown that an unfunded social security system leads to higher growth, compared to a Laissez-Faire economy. The conclusion is driven by the pension function that stimulates human capital investment, a mechanism that is absent in a Laissez-Faire economy. Zhang (1995) reaches the same conclusions in a model where investment in human capital of children is the engine of endogenous growth. The conclusion follows as unfunded social security reduce fertility and increases human capital investment per child to per family income when private intergenerational transfers are operative.

In contrast to these papers I find that a pay-as-you go pension system generates lower growth than in a Laissez-Faire economy. This result is due to two opposing effects where the positive effect from higher saving in a Laissez-Faire economy, exceeds the negative effect from lower human capital formation. The negative effect arises as the link between pension benefits and time spent on skill acquisition is absent in an economy without a social security system. However, this does not exclude public pension as a growth promoting fiscal policy. The analysis in this paper finds that a properly designed funded pension scheme that stimulates both physical and human capital accumulation leads to higher economic growth than absence of social security and governmental interference. Physical wealth is here accumulated by the government as well as by private individuals. Formation of human capital is stimulated by the pension system, due to the relation between studytime and pension receipts.

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Appendix

By inserting the first order conditions in (7) and the governmental condition in (25) into (24) gives:

$$
\begin{aligned}
(1-\lambda_{t+1})h_{t+1}(1+\rho)(1+n)\kappa_{t+1} = &\left[\rho\left(1-\frac{\omega_t^G\left(1+n-f'(\kappa_t)\right)}{h_t(1-\lambda_t)\left[f(\kappa_t)-\kappa_t f'(\kappa_t)\right]}-\frac{\Theta}{1+n}\right)\right.\\
&\left.-\frac{\Theta}{f'(\kappa_{t+1})}\right]\left[f(\kappa_t)-\kappa_t f'(\kappa_t)\right](1-\lambda_t)h_t\\
&-(1+n)\omega_{t+1}^G(1+\rho)\,.
\end{aligned}
$$

Differentiating with respect to κ_t, and taking into account that κ_{t+1} is a function of κ_t, gives:

$$
\begin{aligned}
(1-\lambda_{t+1})h_{t+1}(1+\rho)(1+n)\frac{d\kappa_{t+1}}{d\kappa_t} = &\left\{\frac{\rho\omega_t^G f''(\kappa_t)}{h_t(1-\lambda_t)\left[f(\kappa_t)-\kappa_t f'(\kappa_t)\right]}\right.\\
&\left.\times\left[1-\frac{\left(1+n-f'(\kappa_t)\right)\kappa_t}{f(\kappa_t)-\kappa_t f'(\kappa_t)}\right]+\frac{\Theta f''(\kappa_{t+1})}{\left(f(\kappa_{t+1})\right)^2}\frac{d\kappa_{t+1}}{d\kappa_t}\right\}\\
&\times\left[f(\kappa_t)-\kappa_t f'(\kappa_t)\right](1-\lambda_t)h_t\\
&+\left[\rho\left(1-\frac{\omega_t^G\left(1+n-f'(\kappa_t)\right)}{h_t(1-\lambda_t)\left[f(\kappa_t)-\kappa_t f'(\kappa_t)\right]}-\frac{\Theta}{1+n}\right)\right.\\
&\left.-\frac{\Theta}{f'(\kappa_{t+1})}\right]\left[-\kappa_t f''(\kappa_t)(1-\lambda_t)h_t\right]\,.
\end{aligned}
$$

Using the definitions for V, V', Z and U given in (29), the derivative above can be written as:

$$
(1-\lambda_{t+1})h_{t+1}(1+\rho)(1+n)\frac{d\kappa_{t+1}}{d\kappa_t} = VZ + V\frac{\Theta f''(\kappa_{t+1})}{\left(f(\kappa_{t+1})\right)^2}\frac{d\kappa_{t+1}}{d\kappa_t} + UV'\,,
$$

which solved for $d\kappa_{t+1}/d\kappa_t$ yields the expression given in (29).