Stratification, Growth, and Path Dependence in Two-Sided Markets

James W. Boudreau
Dept. of Economics, University of Connecticut
U-1063, Storrs, CT, 06269, james.boudreau@uconn.edu

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Abstract

This paper examines the impact of two-sided market interactions on social mobility and growth. To analyze the dynamic effects of two-sided matching, a heterogeneous population of agents is simulated with the match process acting as a fitness selection mechanism. Even with perfect information and substantial variety in both offspring and entrants, two-sided matching causes the population to evolve into stratified groups. Corrective measures are possible to improve mobility, but by altering the path of market evolution, a policy may have unintended negative impacts on growth, inequality, and welfare.

1 Introduction

Each human being is born with unique strengths and weaknesses, and without the capacity to insure against their shortcomings, those born with lesser degrees of talent are destined for lesser incomes. Once an uneven distribution of wealth emerges in an economy, however, parents of future generations can use their accumulated wealth to partially insure their children. Sufficient intergenerational investments allow individuals with inferior talent to attain superior employment.

From a Kantian or Rawlsian perspective, it is undesirable for a society to give substantial advantages to those who are born wealthy, and accordingly such notions of fairness often motivate government interjection into market functioning. Yet implementing policy aimed at specific welfare criteria inevitably runs the risk of adverse, albeit unintended, consequences. Despite the admirable intentions behind programs that promote equal opportunity, it is a pervasive question whether or not markets are best left to evolve on their own rather than being distorted to promote social objective functions.

Frequent targets of equal opportunity policies are two-sided markets; markets that exist to match disjoint sets of economic agents (examples include the markets for education and employment). Parental investments are highly influential in such markets since they can help make agents more appealing to potential partners. Furthermore, the match results of these markets have been shown to contribute significantly to assortative mating (Mare, 1991), which in turn may further limit intergenerational mobility (see Ermisch, Francesconi and Siedler (2006) for empirical evidence and Fernández (2002) for theoretical).

If a government makes it an objective to improve mobility by way of intervening into a two-sided market, might it hinder another objective, the promotion of economic growth, in the process? Empirical evidence of a relationship between socioeconomic mobility and economic growth is far from overwhelming (Eriksson & Goldethorpe, 1992), and theoretical treatments that examine mobility and growth (in contexts other than two-sided markets) are highly dependent on assumptions and parameter values (Bénabou (1996); Galor and Tsiddon (1997); Owen and

Weil (1998)).

The intent of this study is to examine the dynamic implications of two-sided matching, and in particular how policies that alter match outcomes affect economic growth. Work by Tesfatsion (2001) also recognizes the importance of dynamic properties in match economies, but focuses on the development of labor networks and market power when agents employ adaptive search processes. Other agent-based computational work similarly examines repeated match behavior (Haruvy, Roth, and Unver (2006);Haruvy and Unver (2004)), incorporating the use of genetic algorithms for adaptive learning and equilibrium selection. Rather than tackling such issues of search or selection, however, all match activity involved in this paper is frictionless, with emphasis instead on intergenerational investment linkages.

The model presented here is an agent-based, two-sided, overlapping generations economy in which heterogeneous agents compete in an intergenerational match game for employment. The rewards earned by the most recently matched generation are invested strategically by direct descendants to increase their productive ability and thus make those agents more attractive for the next round of matching. The concept and importance of "pre-marital" preparatory spending has recently been analyzed in work by Peters and Siow (2002) and Peters (2007), and by treating strategic investment as a behavioral rule for agents in a dynamic environment, this paper is a further contribution to that relatively young strand of literature.

A novel feature of the model is its use of the market's matching process as an evolutionary fitness selection mechanism. Agents that are unable to find a match drop out of the population and thus do not contribute to current or future productive capacity. By simulating the model economy's development under alternate policy regimes and varying assumptions, it is possible to illustrate the effects of mobility-enhancing policy on economic growth.

Simulation results suggest a robust tendency for agents in two-sided markets to swiftly move into definitive strata. Once established, deviation from the strata is unlikely, with local mobility occurring primarily among the lower classes while the upper class is more firmly entrenched. The fact that stratification is an inherent property of two-sided markets comes as no shock, but more surprising are the consequences of correcting that stratification. Altering market evolution via mobility-enhancing transfers can benefit long-run economic growth, but not under all circumstances and not in an egalitarian fashion. In fact, redistribution in the wrong conditions can be quite harmful in terms of its impact on inequality, ultimately making an economy more dependent on policy. This result emphasizes the importance of path dependence in the evolution of two-sided markets, and serves as a reminder that the effects of policy are seldom, if ever, guaranteed.

2 The Model

Two-sided markets are comprised of agents that can be grouped into two disjoint sets. Agents from one set must be matched with complementary agents in order to complete their economic objectives. [1]. The population in this model is comprised of $F = \{f^1, f^2, \ldots, f^m\}$ firms on one side of the market and $W = \{w^1, w^2, \ldots, w^n\}$ workers on the other side, $m < n$. Workers are attracted to high wages and firms require skilled labor in order to produce the economy's generic consumption good.

To prevent any bias in results, the job market in this model is assumed to be frictionless; the most skilled workers are always matched with the highest paying firms via a one-to-one and invertible matching function $\mu : W \to F$. One-to-one matching and strict preferences are assumed for simplicity and tractability, but the remaining assumptions of the model, allow conclusions to readily extend to cases of many-to-one and many-to-many matching, with or without strict preferences (Roth & Sotomayor, 1990). Time is discrete, indexed as $t = 0, 1, 2, \ldots, T, \ldots$, and generations of workers are overlapping so that firms are able to fill employment in every period after time zero.

[1] A vast literature has been dedicated to these markets and is expertly surveyed in Roth and Sotomayor (1990) and Roth (2007)

2.1 Firms

There are m infinitely lived firms that are divided into K types or "industries," with industries differing in terms of their production possibilities. The production of firm j in industry k at time t is determined by the production function $Y^j_{k,t} = Y(X_k; \theta^j_t(\hat{a}^{\mu_j}_t))$, where X_k is an industry-specific factor that is fixed for all firms in industry k and θ^j_t is a firm-specific technology which grows as a function of the currently employed worker's ability $\hat{a}^{\mu_j}_t$. Each firm is limited to employing only one worker at a time, but labor is assumed essential for production. Firms keep all residual output after wages are paid, and since the firm-specific technological factors grow with worker ability, this implies homogeneous preferences for firms: workers are ordered according to their adult ability level.

Wages are the same for all firms within an industry, established at the beginning of each period and based on the industry average marginal product of skilled labor in the previous period. This behavior for firms is admittedly stylized in the model, but is ultimately benign because it is merely the presence of a wage hierarchy which drives results[2]. The lagged structure allows wages to be posted in advance so that they are common knowledge to all workers.

Since the highest paying industry consistently attracts the most able workers, these firms will have the highest technological growth, and thus will always offer the highest wages. Worker preferences over firms therefore remain constant over time. To keep worker preferences strict, it is assumed that same industry firms with lower indices are preferred by workers. Such a simple tie-break rule is benign so long as it is not conditioned on past match outcomes.

Identical preferences for firms (based on ability) and for workers (based on wage offers) mean that only one stable matching of workers to firms will exist in any one period. Stability in a matching is the condition in which no worker-firm pair, not currently matched with each other, can improve by leaving their current partners and forming a new relationship. Due to this single-valued core property, truthful revelation of preferences is a dominant strategy for all participants involved (Sonmez, 1999). More complex preferences do not alter results, but analysis is simplified

[2]For a more rigorous analysis of firm strategy in static two-sided markets, see Peters (2007)

when truthful revelation is assured. In fact, the structure of preferences in this particular model is perhaps not such a fantastic assumption, given the limited core and strategy-proof properties uncovered in real world two-sided markets (Roth and Peranson (1999); Teo, Sethuraman, and Tan (2001)).

2.2 Workers

Workers live for two periods, optimizing the utility they gain from consumption in the first period of life and the inheritance they are able to leave their offspring. This is done according to a common utility function $U(c_t, e_{t+1})$, which is assumed to be strictly increasing, strictly concave, and twice differentiable in both arguments with the boundary conditions $\lim_{c_t \to 0} U_1(\cdot, e_{t+1}) = \lim_{e_{t+1} \to 0} U_2(c_t, \cdot) = \infty$. Each individual worker is born with a unique asset and ability endowment $\{e_t^i, a_t^i\}$, inherited from their parents (or simply existing in the case of the initial generation). Assets come in the form of the generic consumption good, which fully depreciates at the end of each period. Since there are no savings or credit channels in the economy, workers must base their first period consumption on their endowment, and bequests on their compensation from employment.

In order to obtain the best wage available, a worker can make themselves more attractive to firms by investing in their own ability. This form of human capital investment requires that the worker sacrifice a part of their asset endowment however, implying a classic intertemporal tradeoff. Formally, let $\hat{a}_t^i = g(a_t^i, I_t^i)$, where $\hat{a}_t$ is augmented ability, I_t^i is the amount invested and $g(\cdot, \cdot)$ is increasing, concave, and differentiable in both arguments. The problem for worker i, born at time t, can then be stated as

$$\max_{I_t^i} \{U(c_t^i, e_{t+1}^i) | c_t^i = e_t^i - I_t^i, e_{t+1}^i = \mu_{t+1}(\hat{a}_t^i), I \geq 0\}.$$

Worker i's wage is indicated by $\mu_{t+1}(\hat{a}_t^i)$ since the firm they are matched to implies their wage, and their match is a function of their ability in the second period. The inheritance a parent can leave for their child therefore hinges upon the outcome of the job matching procedure. Without loss of generality, assume that among the K industries, wages rank as $f_1^j > f_2^r > \ldots > f_K^s, \ \forall t = 0, 1, 2, \ldots, T, \ldots ; j < r < s \in \{1, 2, \ldots, m\}$.

Due to the non-differentiable nature of the matching function $\mu_{t+1}(\cdot)$, agent behavior is determined by threshold levels of investment where the cost of investment is equal to the benefit of a higher wage:

$$U(c_t^i, e_{t+1}^i) = U[(e_t^i - I_t^{i,k}), \mu_{t+1}(g(a_t^i, I_t^{i,k}))], \forall k \in K.$$

Worker i is not willing to invest more than $I_t^{i,k}$ to achieve employment in industry k. In addition to the individual's ability, the matching function $\mu_{t+1}(\cdot)$ also depends upon the ability levels of all workers in the economy as of $t+1$. Assuming that the ability and asset levels of all workers are common knowledge, an individual worker must account for the decisions of all others when making their own decision on how much to actually invest.

The optimal I_t^* investment scheme is therefore a Nash equilibrium, based on the post-investment potential ability of each worker. Workers will invest *just enough* to outshine competitors, as long as that amount is less than or equal to their threshold where the costs of investing are equal to the benefits. For example, if the economy consists of 12 workers and 10 firms with 4 firms in each of the higher paying industries and 2 in the lowest paying industry, investment is determined as follows:

1. Workers are ranked 1-12 according to their ability after full investment.
2. The two lowest ranked workers will fail to find employment and thus will not invest at all.
3. The workers ranked 9 and 10 will invest just enough to match the full investment ability level of the 11th ranked worker.
4. The workers ranked 5-8 will invest just enough to match the 9th ranked worker's threshold investment, given that he has invested enough to secure his 9th position.
5. The workers ranked 1-4 will have to invest just enough to match the 5th ranked worker at his threshold level for the highest wage, given that he invested enough to secure the 5th spot.

This process then generalizes according to the number of workers and firms, and according to the size of industries.

Due to the standard properties of the utility function, it is imposed in the model that those who are rich and powerful will attain better matches, and therefore higher levels of utility. Declining marginal utility means that their threshold level of investment will exceed that of poorer workers with lower ability who would otherwise attempt to usurp their position in the rank order. It is similarly imposed that individuals with lower ability, but assets high enough to compensate, are able to attain matches better than those who have high ability but low assets.

2.3 Population Dynamics

The first generation of the economy begins with an exogenously given distribution of asset and ability endowments in $t = 0$. After the workers' investment decisions, the match mechanism determines the first employment scheme for their second period of life, $t = 1$. Since the number of workers exceeds the number of jobs in the economy, some workers remain unmatched. These individuals exit the job market (or "die"), leaving only the employed to reproduce and pass on their traits. The workers' asset and ability endowments thus effectively act as evolutionary strategies, determining fitness in the match environment and implicitly dictating which workers are able to reproduce.

Since the focus of this study is on intergenerational linkages and match behavior over time, reproduction is a key aspect of the model. Traditional overlapping generations models frequently assume cloning, with each successive generation comprised of the same individuals; if population growth is positive, types of agents replicate proportionately. In this model, to prevent overly deterministic results, workers mate assortatively in pairs[3]. Each pair produces two new workers, and each individual's ability is heavily influenced by their inherited genetic composition. Entry and exit into the market allows new strategies to be incorporated into each generation without forcing "mutations" to occur in the worker gene pool. This is an effort to keep the model applicable to human behavior and markets, and is a response to criticisms of other models which perhaps overuse evolutionary techniques (Borgers, 1996).

[3]The implicit restriction of m to an even number is necessary for this mating scheme, but results would not change if an odd m were used with randomized mating, as long as workers mate only with those from the same industry as themselves.

Workers mate according to job market success, meaning that the worker matched with firm 1 will mate with the worker matched with firm 2, and so on[4]. Concurrent with the birth of the next generation, the remaining $n-m$ vacancies in the economy are filled by *new* agents, unrelated to any prevailing dynasties, and who enter the market fresh with randomly given asset and ability endowments. The market is assumed fixed in size, and thus the number of possible entries per period is restricted to the number of exits, so the total population remains constant over time.

After the initial generation, a child's asset inheritance is the same as their parents' wage. This means that by excelling in the job matching market workers better provide for their children. The K different industries imply K different socioeconomic classes of workers.

Ability is more complicated, and is conveyed across generations by the genetic inheritance of two quantitative alleles[5]. All new entrants (including the first generation) begin heterozygous, with each allele having a different quantity value drawn from a distribution $\zeta(\overline{a}, \sigma_a)$. Each agent's phenotypic realization of ability is the average of their two individual alleles, plus a "luck" factor drawn from $\zeta(0, \sigma_a)$. If agent i has alleles A^i and B^i, then, their ability is

$$a_t^i = \frac{A^i + B^i}{2} + \zeta(0, \sigma_a).$$

The quantitative representation of the alleles allows for the incomplete dominance of traits; higher ability does not strictly dominate lower. Offspring of previously employed workers obtain their alleles according to the Punnet square probability.

Punnet square examples are given in Figure 1. The offspring of any two workers (w_t^i, w_t^r), have an equal chance at inheriting any of four possible allele combinations. If workers (w_t^i, w_t^r) are homozygous, with alleles $\{A^i, A^i\}$ and $\{A^r, A^r\}$, they will definitely yield heterozygous offspring, unless $A^i = A^r$. On the other hand, if the two workers are heterozygous with alleles $\{A^i, B^i\}$ and $\{A^r, B^r\}$, homozygous offspring have a 25% chance of occurring if $A^i = A^r$ or if $B^i = B^r$,

[4]Assortative mating is well documented, for example see Mare (1991). All results are qualitatively the same with a more relaxed mating scheme, as long as employment status plays a significant role.

[5]See Bartels et al. (2002) for evidence of genetic influence on intelligence.

Figure 1: Punnet Square Examples: Homozygous Agents on left, Heterozygous on right

	A^r	A^r
A^i	A^i, A^r	A^i, A^r
A^i	A^i, A^r	A^i, A^r

	A^r	B^r
A^i	A^i, A^r	A^i, B^r
B^i	B^i, A^r	B^i, B^r

and a 50% chance of occurring if both equalities hold. Of course, depending on the workers involved, it may be the case that $B^i = A^r$ and/or $B^r = A^i$ as well, with the probabilities of inheritance adjusting accordingly.

2.4 Equilibrium

A time $t \geq 0$ *equilibrium* in this economy is a family of decisions, $\{I_t^{i*}, c_t^{i*}\}, \forall i = 1, 2, \ldots, n$, and a corresponding matching function, $\mu_{t+1}(\cdot)$ such that all workers act optimally given the asset and ability levels of all other workers and the wages posted by firms. Given the market's frictionless two-sided structure, by standard arguments (Gale & Shapley, 1962) a stable assignment will exist in each time period, the exact composition of which is predicated upon the current population's preference rankings.

Model analysis is focused on how market evolution is altered when time t equilibria are altered. Whether or not the economy reaches a stationary state in terms of growth or mobility depends on the specification of firm technology. If the growth of technology diverges, but at different rates in different industries, then there will be very little mobility in the limit as the difference in wages will also diverge. Conversely, if technology growth converges to some productive limit, then the rate of mobility will also converge to a stationary rate, one that is determined by the limiting wage differential. Both cases of technology growth are considered below.

3 Simulation Settings

Simulation of the economy allows for the observation of its dynamic behavior in a controlled environment[6]. For functional forms, let first period ability be augmented additively by asset investments so that $\hat{a}^i_{t+1} = a^i_t + I^i_t$. The utility function for workers takes the form

$$U(c^i_t, e^i_{t+1}) = c^i_t e^i_{t+1} = (e^i_t - I^i_t)\mu_{t+1}(a^i_t + I^i_t).$$

With the Cobb-Douglas specification, workers are always *willing* to invest their entire endowment if necessary to secure employment. How much each worker *actually* invests is based on the ability and asset levels of all other workers, with Nash equilibrium serving as their behavioral rule.

For firms, two cases of technology are considered. In both cases, the production function for firm j in industry k takes the form $Y^j_{k,t} = L\theta^j_t X_k$, where L is an indicator function noting the presence of an employee. The difference in specifications is in the growth of technology, θ^j_t. In the first specification, technology grows sigmoidally according to $\theta^j_t = \exp(-\exp(\frac{\sum_t \hat{a}^{\mu_j}_t}{Q_1}))$. With such a form, technology grows at an exponential rate initially and then tapers off so that the productivity of each firm converges to a maximum of X_k. The second specification has technology growing exponentially over time according to $\theta^j_t = \exp(\frac{\sum_t \hat{a}^{\mu_j}_t}{Q_2})$, meaning that productivity increases indefinitely. Q_1 and Q_2 are large, positive constants set to 20,000 and 100,000 respectively.

Population parameters for the number of workers, firms and industries are set to $n = 12$, $m = 10$, and $K = 3$, respectively[7]. The three industries are separated with four firms in the first industry, four firms in the second industry, and two firms in the third, with $X_k \in \{70, 60, 50\}$. Each generation loses two of its number since they will fail to find employment and exit the market, soon to be replaced by new agents. New entrants have equal odds of receiving each asset endowment, so they may find themselves rich, poor, or middle class. Genetic alleles for the

[6] All simulations are programmed in Matlab, copyright *The Mathworks, Inc.*, 1984-2004.

[7] Larger populations are of course possible, but do not substantively alter dynamic results. For generalizations of pre-marital investment strategies in very large populations and in the absence of perfect information, see Peters and Siow (2002), and Peters (2007)

first generation, and for each new market entrant, are randomly drawn from a normal distribution with mean $\overline{a} = 60$ and a variance of σ_a =10 or 25. The random factor that is added to each worker's ability is accordingly drawn from a normal distribution with mean zero and a variance of σ_a, so they may be helped or hurt by luck.

The two-sided market structure of the model economy makes it highly susceptible to stratification, irrespective of population size or number of industries. The only necessary restriction on parameter values is that industry wages are sufficiently different from one another, relative to the mean ability level. If wages are too close together, asset endowments lose their heterogeneity and the economy becomes de facto one large industry with intra-mobility driven solely by ability. Distributional specifications do impact the economy over time if the genetic variance is made to be extraordinarily large relative to the mean genetic ability level, however, even an exceptional variance can not completely eliminate stratification.

To eliminate any possible dependence on the initial conditions of the random number generator (used for stochastic processes), an economy with a particular specification is simulated 100 times, with identical random processes for each run of simulations. Each experimental economy lasts for 1200 generations, a suitable horizon for observable dynamic behavior given the specified parameters. That length of time allows production possibilities to converge for the first case of technology while preventing them from diverging completely in the case of exponential growth.

As a measure of stratification, economies are represented by intergenerational Markov transition matrices. This is done by keeping track of what industry each worker matches with (remaining unmatched counts as industry $K + 1$), and categorizing them according to the industry of their most recent ancestor. The figures are added up and averaged over the possible number of workers in each industry. Averaging those figures over 500 generations and 100 trials yields a $(K + 1) \times (K + 1)$ transition matrix. Each entry (l, k) represents the fraction of workers that matched to industry k, given that their direct ancestor matched to industry l. The transition matrix thus provides a summary of socioeconomic mobility in a simulated economy. Higher values in the diagonal elements correspond with a greater tendency for descendants to

Table 1: Transition Matrices from Simulated Economies

σ_a	10	25
	$\begin{pmatrix} 0.70606 & 0.24915 & 0.03456 & 0.01023 \\ 0.21727 & 0.48165 & 0.19515 & 0.10593 \\ 0.03877 & 0.22531 & 0.28922 & 0.44670 \\ 0.11457 & 0.31309 & 0.25137 & 0.32097 \end{pmatrix}$	$\begin{pmatrix} 0.58933 & 0.30235 & 0.07348 & 0.03484 \\ 0.28607 & 0.40138 & 0.18098 & 0.13157 \\ 0.12149 & 0.30698 & 0.25578 & 0.31574 \\ 0.12770 & 0.28555 & 0.23531 & 0.35144 \end{pmatrix}$

have matched within the same industry as their like-numbered parent. Alternatively, smaller values of diagonal and off-diagonal elements correspond with greater mobility.

Table 1 provides the transition matrices for the first technology specification with $\sigma_a = 10$ and $\sigma_a = 25$. Most notable in both tables is that workers with successful parents (matched to the highest paying industry) generally stay successful. The remaining strata also demonstrate persistence, but persistence that declines along with income. New entrants, with randomly assigned endowments, are equally likely to gain employment in all except for the (first) highest paying industry. Such asymmetry in mobility is consistent with empirical estimates of transition probability, indicating that upward mobility from the bottom is more likely than downward mobility from the top. In fact, parameters (in particular the value of $\sigma_a = 25$) are chosen specifically so that mobility results with the first technology specification are similar both qualitatively and quantitatively to the transition matrices presented in Table 5 of Dearden, Machin and Reed (1997), as well as those presented in Table 2 of Gottschalk and Spolaore (2001), for the UK and US economies respectively. Such mobility estimates can also be obtained with various other parameter combinations (more workers, more industries, etc.), but to maintain simplicity the small population of $n = 12$, $m = 10$ and $K = 3$ is used in the next section when making comparisons to the case of a transfer scheme.

4 Correcting Stratification

Whether or not stratification is ultimately detrimental to an economy depends upon the welfare criterion used. If it is the case that a particular economic activity necessitates an egalitarian structure, limited mobility is quite undesirable. An obvious example is the market for public education, where children with varying levels of natural ability and wealth must be assigned to schools (see Balinski and Sonmez (1999) and Chen and Sonmez (2006)). Those with higher initial wealth endowments are able to engage in various preparatory activities which can improve specific attributes that factor in to schools' preferences.

In addition to notions of distributive justice, the elimination of stratification may be in an economy's best interest if production technology is positively affected by the ability of past generations. If naturally "talented" workers are able to take advantage of their potential and rise in the ranks to the most productive industry, it seems reasonable to assume that such reallocation lead to an overall increase in production possibilities. Increased mobility could then have the potential to be growth enhancing.

4.1 Introducing a Transfer Scheme

To compare the development of the economy with improved mobility with that of the standard model, it is necessary to construct a redistribution policy with minimal incentive distortions. The following is only one example of such a policy, but one that serves adequately as an illustration of unintended consequences.

At any time t an individual may be born with an ability endowment greater than that of all others in the time t cohort, but with an asset endowment less than that which enables them to attain employment in the first industry. Call this individual $\widehat{w}_t$. If $\widehat{w}_t$ exists, a lump sum tax is levied evenly on all workers, with the total amount of the tax just enough to guarantee $\widehat{w}_t$ employment at the highest wage. The tax is then transferred directly to $\widehat{w}_t$ in period t so that the funds are available for investment. Workers are taxed rather than firms in this scenario so that the total amount of assets available for investment remains unaltered. The tax is lump sum so that other than the change for $\widehat{w}_t$, the wealth hierarchy is unchanged.

Because of the model's highly nonlinear evolutionary process, the impacts of redistribution are not immediately clear. Although the policy in this case ensures that the worker with the

Table 2: Transition Matrices from Simulated Economies with Redistribution, the Case of Convergent Technology Growth

σ_a	10	25
	$\begin{pmatrix} 0.69897 & 0.25612 & 0.03462 & 0.01029 \\ 0.21624 & 0.48359 & 0.19457 & 0.10559 \\ 0.05369 & 0.21123 & 0.28868 & 0.44640 \\ 0.11588 & 0.30935 & 0.25293 & 0.32184 \end{pmatrix}$	$\begin{pmatrix} 0.58471 & 0.30416 & 0.07549 & 0.03564 \\ 0.28712 & 0.40021 & 0.18046 & 0.13221 \\ 0.12409 & 0.30466 & 0.25412 & 0.31713 \\ 0.13227 & 0.28660 & 0.23396 & 0.34717 \end{pmatrix}$

highest pre-investment ability in each period is always allocated to the most productive sector, it also alters the investment incentives and mating behavior of the remaining workers in the market.

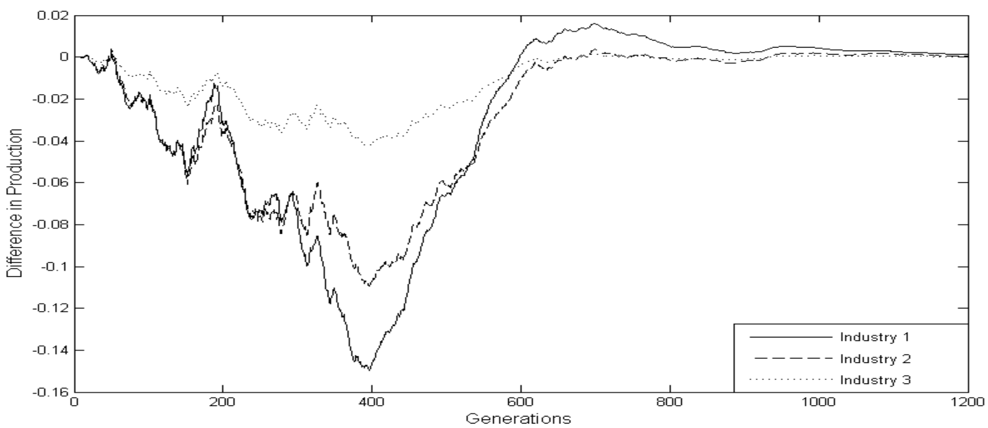
First consider the effects on strategic investment. When subsidization raises $\widehat{w}_t$ to the first industry, it forces all agents that otherwise would have been employed in that industry to invest more since one must be relegated to the second industry. The increased competition then trickles down to whichever industry $\widehat{w}_t$ would have achieved without any transfer. At the same time, however, there are less funds total for all workers except $\widehat{w}_t$ to invest. In a single period, then, redistribution can lead to either higher or lower levels technology growth in each industry.

Next consider the effects of altered matching on future generations. Just because $\widehat{w}_t$ has the highest ability in the market for a given period does not mean that their genetics are the best. It could be the case that one of $\widehat{w}_t$'s alleles is large and the other small, or that both alleles are inferior and $\widehat{w}_t$ is simply a lucky individual. Moreover, the rise of $\widehat{w}_t$ changes the mating prospects of many other agents, thus the genetic composition of the next generation. It may also mean that a different set of genes exit the market completely. Policy implementation thus has the potential for substantial echo or domino effects that may either help or hinder growth.

4.2 Comparisons of Growth and Inequality

To compare the economy's evolution with and without the policy, first consider the case of technology growth that converges to finite limits. Table 2 displays the transition matrices for simulated economies with redistribution. Compared with the numbers in Table 1, there is a

Figure 2: Transfer Scheme vs. Laissez Faire Economy: Convergent Technology Growth, $\sigma_a = 10$



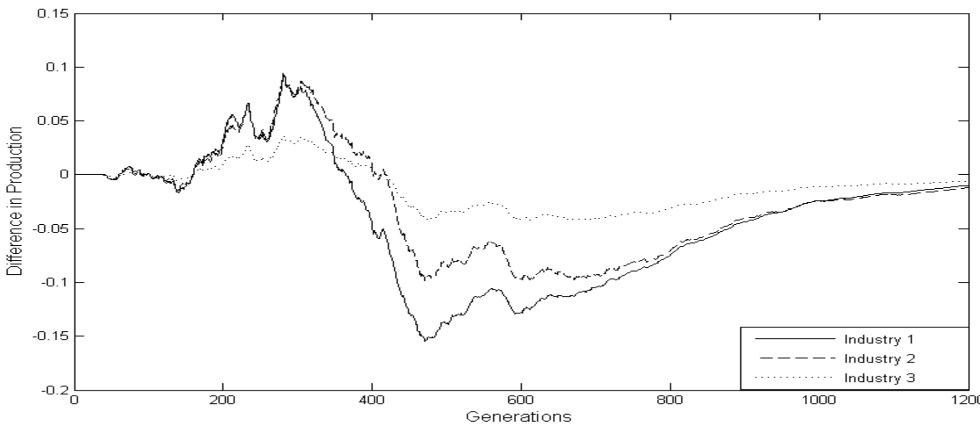
small but significant increase in intergenerational mobility. The effects of those changes are illustrated in Figures 2 and 3, which depict the difference in production between the economy with the transfer scheme and the economy without as they progress through time[8]. A positive trend indicates that the economy tends to grow faster in the presence of the transfer scheme and vice versa.

With a relatively small variance, the economy with improved mobility grows just a bit slower than the standard version until technology growth reaches its inflection point and productive capacity catches up. With higher variance in ability, transfers occur less frequently since exceptional agents are more able to succeed with ability alone. It is interesting then, that when redistribution is more selective it causes the economy to grow faster while technology grows rapidly. This suggests that smaller, more deserving alterations in match structure have less potential for adverse effects. For either variance level it is important to note that the differences in productivity follow qualitatively similar patterns across industries. This does not hold for the second case of technology growth.

When technology growth is exponential, the impact of the transfer scheme is much more

[8]All figures are averaged over 100 experimental simulations

Figure 3: Transfer Scheme vs. Laissez Faire Economy: Convergent Technology Growth, $\sigma_a = 25$



drastic. Obviously the ever-increasing difference between industries limits mobility as time goes on, making transfers both larger and more frequent, and thus making the policy more influential. Figures 4 and 5 show that influence, illustrating the difference in productivity in economies with and without redistribution for the case of exponential technology growth.

After negligible effects for the first five hundred generations, mobility-enhancing redistribution significantly alters the economy's evolution. The reallocation of skilled workers consistently benefits the top industry, but only at the expense of the others. Redistribution increases the post-investment ability levels of the most qualified workers by making them compete with $\widehat{w}_t$, but the benefits of increased competitiveness are insufficient to improve industries other than the first. Thus, although the dramatic increase in the top sector's productivity due to the transfer scheme (about 10%) is sufficient to make total growth higher in economies with improved mobility, inequality is also dramatically elevated. Ironically, the increased wage dispersion makes the economy under mobility-enhancing policy much more susceptible to stratification should the policy ever be removed.

Figure 4: Transfer Scheme vs. Laissez Faire Economy: Exponential Technology Growth, $\sigma_a = 10$

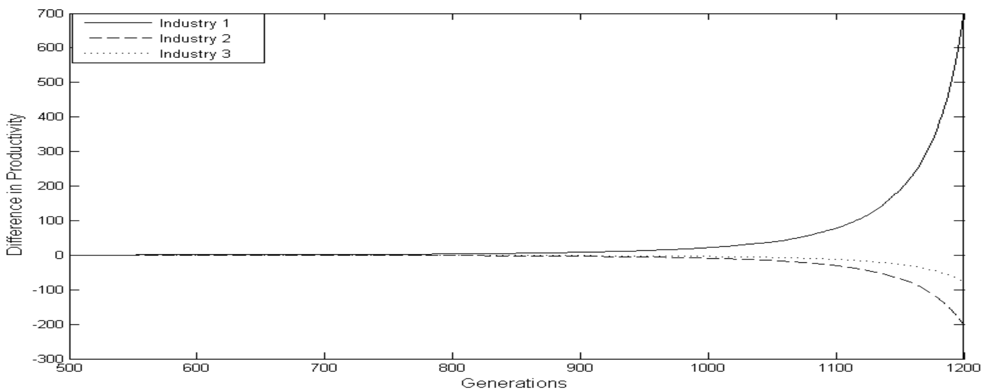


Figure 5: Transfer Scheme vs. Laissez Faire Economy: Exponential Technology Growth, $\sigma_a = 25$

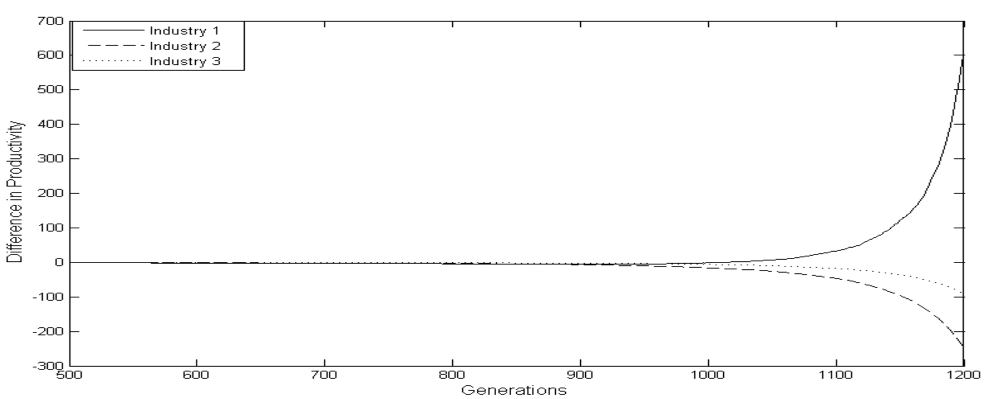
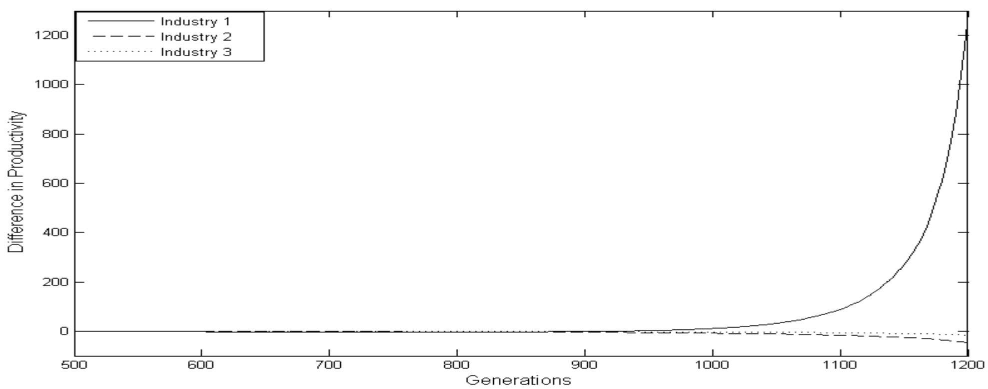


Figure 6: Manna from Heaven vs. Laissez Faire Economy: Exponential Technology Growth, $\sigma_a = 25$



4.3 Manna from Heaven

A final consideration is the possibility of a mobility-enhancing policy that does not require taking away from the wealth of workers. In the preceding section, the total amount of wealth available for investment remains unchanged and is simply redistributed. The reason for the decline in productivity in sectors other than the first may therefore be due to the lower level of wealth available for investment in those industries. If, instead, the tax is levied upon firms, or if a benevolent government simply gives additional funds to deserving workers, the detrimental effects on lower industries are reduced. They are not, however, eliminated.

Figure 6 illustrates the deviation from the standard model caused by a policy that gives wealth to a deserving worker as defined in section 4.1, but that takes nothing away from the rest of the working population. As expected, the most productive industry is enormously benefitted due to the increased investment of competing workers. Surprising, however, is the fact that the remaining two industries still suffer in the presence of the policy. Their loss is much less than in the redistributive case since all workers not receiving a transfer maintain the same amount of wealth, but altering the competitive market remains detrimental to the lower income bracket.

This example highlights the danger of implementing policy tools in markets that function as dynamic systems. Any transfer scheme implemented in this environment will involve similar degrees of uncertainty regarding its effects, since altering the path of even one individual necessarily alters the paths of others, in both present and future generations. Path dependence will be a characteristic of any two-sided market with intergenerational influences, as matching in one period impacts the incentives and capabilities of the next.

5 Discussion

The agent-based model presented here illustrates the importance of dynamic properties in two-sided markets. Though the model is obviously a stylized representation, its robust results of stratification in the absence of market frictions provide an additional explanation for empirical observations of limited mobility. While children do not always inherit their parents' successful traits genetically, those who are wealthy certainly have an advantage when it comes to investing in preparatory activity, and such advantages then make those individuals more attractive in matching markets. Intergenerational linkages in two-sided markets therefore help perpetuate socioeconomic stratification. Though it is unfortunate, limited mobility seems to be a natural characteristic of matching markets as they evolve.

Mobility enhancing policy has the potential to improve total production in an economy, but in a lopsided manner and at the expense of those in lower income brackets. Competition in two-sided markets is a crucial motivation for human capital investments, and interference with that mechanism in any period can have a far-reaching impact on the investments of future generations. Consistently correcting stratification thus does not always lead to beneficial outcomes, and can actually make the economy dependent on redistribution for any mobility at all. While it is certainly true that governments have a wide array of objectives in addition to growth, the path dependence of market evolution suggests that policy makers tread with caution. Even small changes in a market can result in unintended outcomes down the line.

In closing there are a few caveats in order. Subsidization within this model allows the most naturally able workers to fulfill their potential. In real life, similar aid programs do exist for just those purposes, and credit markets allow individuals to borrow against their initial endowments to invest in themselves. The question remains, however, whether or not it is feasible to make up the difference between those that are born elite and those that are born naturally able but without wealth. An assumption of this model is that investments equally augment all workers' ability, but compelling evidence from Berg and Krueger (2002) indicates that this is not the case; that in fact some people may benefit more from educational spending than others. Also contrary to the assumptions of this model, there are many ways to improve matching prospects (improving networking skills and making contacts, for example), and not all of those investments contribute to the growth of production. These possibilities, as well as the possibility of other avenues for two-sided markets to impact growth, are left for future research.

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