Trade Liberalization, Female Labor Force Participation and Economic Growth*

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Abstract

This research argues that the interaction between international trade and female labor force participation has played a significant role in the timing of the demographic transition and, therefore, has determined differences in economic performance across countries. The theory suggests that international trade has affected the evolution of economies asymmetrically and that initial differences in capital labor ratios across countries were the source of this asymmetry. The main concern of our study is to show how differences in per household capital stocks, via international specialization, affect household choice of fertility and female labor force participation and how these decisions, in turn, feed back and affect the accumulation of capital. Surprisingly, and unlike the existing literature on international trade our model predicts a non-monotonic relation between the specialization pattern and the stock of capital of the trade partner.

Keywords: Trade, Female labor force participation, Economic Growth.

JEL Classifications: F10, F16, J13, J16.

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1 Introduction

World trade has been increasing secularly during the last two centuries. The ratio of world trade to output rose from 10% in 1870 to 21% in 1913 [Estevadeoral, Frantz, and Taylor (2003)]. Another significant feature that has accompanied the twentieth century was the increase in female labor force participation. The participation of married women in the labor market has been increasing, secularly, from around 2% in 1880 to over 70% in 2000 [Fernández (2007)].[1] The timing of the demographic transition differed significantly across regions. For today's developed countries it occurred toward the end of the 19^{th} century and in the beginning of the 20^{th} century, whereas today's developing countries experienced a decline in the rate of population growth only in the last decades of the 20^{th} century.[2]

This paper integrates these different phenomena into a Heckscher-Ohlin (H-O) trade model that highlights the interaction among international trade, female labor force participation and fertility. The main concern of the study is to show how differences in capital labor ratios across economies, via different trade patterns, affect a household's tradeoff between fertility and female labor force participation and how these decisions, in turn, feed back on growth rates of per household capital stocks. In particular, per household capital stocks are driven by two endogenous variables population growth and gross capital accumulation. Surprisingly, and unlike the existing literature on international trade, our model predicts a non-monotonic relation between the specialization pattern and the stock of capital of the trade partner.

The model builds on three basic elements from trade and demographic theory. First, capital labor ratio across countries affects specialization pattern. Second, specialization pattern affects the gender wage gap and third, the gender wage gap affects parents' optimal choice of fertility which, in turn, determines the stock of per household capital for future generations. Thus, the interaction between international trade and female labor force participation is the source for the demographic transition and, therefore, plays a significant role in determining differences in economic performance across countries. Moreover, our theory suggests that international trade has affected the evolution of economies asymmetrically. Initial differences in capital labor ratios across countries and factor intensity across sectors determine specialization patterns

[1] The numbers for mothers of preschool age children are 6% for the year 1940 and 60% for today [Fogli and Veldkamp (2007)].

[2] For a comprehensive discussion on the demographic transition see Galor (2005).

which can further intensify asymmetries of capital labor ratios.[3] A novel feature of our model is that due to differences in factors substitution across sectors capital intensity switches as economies develop. This feature produces a non-monotonic relation between the specialization pattern and the stock of capital of the trade partner.

Thus, the impact of trade on the demand for women's labor plays a key role in the model. As a result of international specialization some countries will specialize in production of goods which is particularly suitable for female workers. Somewhat surprisingly, the effect of the expansion of this "female sector" on female labor force participation is ambiguous. The reason is that, by general equilibrium forces, trade makes the importing sector contract and factors reallocate to the exporting sector. If capital is the main factor reallocated to the "female sector", female wages increase and female labor participation rises. If, however, primarily male workers enter the "female sector", this leads to a depression of wages and the exit female labor. The last mechanism will be called the "crowding out effect" hereafter. Which of both effects prevails is a determinant of the capital endowment of the country in question.

The first part of our story deals with international trade and specialization pattern. The overwhelming part of trade literature celebrates the gains from international specialization, independently whether its source is Ricardian or H-O type of comparative advantage, or whether it stem form other motives as increasing returns. Potential losses from trade are viewed as an exception that occurs under very specific conditions. Prominent examples of such conditions are those driving the infant industry argument, where trade benefits only the countries which specialize on sectors with fast, exogenous productivity growth, while hurting the others (see e.g. Young (1991) and Baldwin (1969)). Heavy economic frictions of the labor market or other institutions have also the potential to channel benefits from trade to one country only (e.g. Davis (1998) and Levchenko (2007)). By and large, however, all countries are commonly thought to gain from international trade not only due to static gains through efficient factor allocation but in addition via enhanced productivity growth through increased competition. The present paper's story builds on the H-O trade mechanism according to which countries export goods whose production is relatively intensive in their abundant factors of production and import the other goods. In our model each country in autarky produces two different goods that can be produced by physical capital, women's labor and men's labor. A key assumption is the

[3]For a related mechanism involving the interdependence of trade and capital accumulation see Ventura (1997) and Acemoglu and Ventura (2002).

existence of different degree of substitutability between these factors in production of the two goods. Consequently, as economies accumulate capital, labor moves from one sector to another, causing capital intensity to switch which, in turn, affects the specialization pattern.[4]

The second part of our story deals with the specialization pattern and women's wage. Following Galor and Weil (1996) we adopt a framework in which, first, female have relative advantages in raising children and second, capital complements female labor more than mens'. Consequently, an increase in capital intensity in the "female sector" decreases the wage gap and increases female labor force participation. To asses the effects of trade take, for example, a country with a low capital stock that imports goods from the "female sector". The export sector expands and men leave the "female sector", which can thus accommodate a higher number of female workers - female wages increase. Yet if the same country specializes on production in the "female sector", male workers reallocate to the "female sector" thus driving down female wages. In this case, somewhat paradoxically, specialization in "female sectors" does nothing but attracting men to these sectors, increasing the gender wage gap and, thus, delay the integration of women in the labor market.

The third part of our story deals with link between women's relative wages and fertility.[5] In such a framework the pure effect of an increase in household income holding the price of children constant is to raise the demand for children. If all childrearing is done by women, an increase in women's wage raise both household income and the price of children, and so have offsetting income and substitution effects on the demand for children. In our model, if both men's and women's income increase proportionately, then the substitution effect driven by the increase in the cost of rasing children precisely cancels out the income effect and leaves fertility unchanged, and thus, closing the genders wage gap is the cause for fertility decline.[6] It is important to note here that an increase in the capital intensity in the "female sector" raises the relative wages of women. This element is captured by assuming

[4] In this respect our paper is close to Grossman and Maggi (2000) who argue that trade pattern reflects differences in the distribution of talent across countries. However, our model differs by the dynamic aspect of capital accumulation that changes intensity across sectors and thus gives new insights for the existence of different trade patterns.

[5] The analysis of fertility in the context of relative wages dates beck to Becker (1960), Mincer (1963) Becker (1985) and Becker (1991), and for contributions in long-run dynamics of fertility see Galor and Weil (1999), Galor and Weil (2000) and Galor and Moav (2002)

[6] For such a mechanism see Galor and Weil (1996) and Cavalcanti and Tavares (2004) and for examples of its application see Heckman and Walker (1990), Butz and Ward (1979) and Schultz (1985).

that the complementarity between women's labor and physical capital is higher than it is between men's labor and physical capital. According to Goldin (1990) the dramatic increase in the relative wages of women during the nineteenth century was industrialization. Thus, our explanation for the rise in the relative wages of women during the process of development is that women's productivity increases and, therefore, the rewards to women's labor input increase as well. Technically speaking, and for the sake of simplicity we follow Galor and Weil (1996) by assuming that while women and men have equal quantities of brains, men have more brawn and that the bigger is the stock of capital, the higher the relative returns to brains endowments.[7]

There is very little research on the links and interaction between demography and international trade. In a H-O model, Findlay (1995) shows that countries specializing in unskilled labor production will tend to see a decline in the incentive to invest in education. In the long run this negatively affects the accumulation of human capital in these countries. The developed economies on the other hand, start out with a higher skill level and therefore tend to specialize in high-skilled production. This expands the demand for high-skilled labor and therefore provides an incentive to further accumulate human capital.[8] In a Ricardian model Galor and Mountford (2006) endogenize education choice as well as fertility choice and argue that the gains from trade were channeled towards population growth in non-industrial countries while in the industrial countries they were directed towards investment in education and growth in output per capita.[9] Thus, our theory fits into this literature that aims to reveal the asymmetric impact of trade on different countries.

The rest of the paper is organized as follows. Section two formalizes our argument.

[7] O'Neill (2003) shows that there is still a 10% differential in female and male wages in the U.S. in 2000 that remains unexplained by gender differences in schooling, actual experience and job characteristics. For empirical evidence as well as theoretical explanations for gender wage gap see Albanesi and Olivetti (2005), Altonji and Blank (1999) and Goldin (1990) among others.

[8] In an econometric analysis of data on about 90 countries during 1960-90 Wood and Ridao-Cano (1999) finds that greater openness tends to cause divergence of secondary and tertiary enrolment rates between more-educated and less-educated countries, and also between land-scarce and land-abundant countries.

[9] Their theory suggests that international trade enhanced the specialization of industrial economies in the production of skilled intensive goods. The rise in the demand for skilled labor induced an investment in the quality of the population, expediting the demographic transition, stimulating technological progress and further enhancing the comparative advantage of these industrial economies in the production of skilled intensive goods. Thus the pattern of trade enhances the initial paten of comparative advantages and disadvantages. For other contributions focusing on the dynamics of comparative advantage see Findlay and Keirzkowsky (1983), Grossman and Helpman (1991) and Atkeson and Kehoe (2000) among others.

Section 3 presents some concluding remarks and proofs of the main claims appear in the appendix.

2 The Model

The economy is populated with a mass of L_t households, each containing one husband and one wife. Individuals live for three periods: Childhood, adulthood and old age. At childhood, each individual consumes a fixed quantity of time from her parents. at adulthood, individuals raise children and supply labor to the market, earning a wage. For convenience, we assume that they do not consume in this period. At old age, individuals do not work, and they consume their savings. The capital stock in each period is equal to the aggregate savings in the previous period.

A key assumption in our model is the difference between male and female. This difference is not reflected at childhood or at old age. At adulthood, however, men and women differ in their labor endowments. Workers can supply both raw physical strength and mental input. We assume that while men and women have equal endowments of mental input, men have more physical strength than women. Note that despite that differences between men and women are a major issue, we do not model differences in preferences between men and women but rather we assume that each household which is composed of a husband and a wife has a utility function by which this household chooses a joint consumption. [10]

2.1 Production

A final good Y is the composite of two intermediate goods, X_1 and X_2

$$Y_t = X_{1,t}^{\theta} X_{2,t}^{1-\theta} \tag{1}$$

The intermediate goods are produced with three factors: capital K, physical labor L^p, and mental labor L^m. For each of the two sectors we adopt the specification from Galor and Weil (1996) for the constant returns to scale technologies by assuming

$$X_i = aK_{i,t}^{\alpha_i}(L_{i,t}^m)^{1-\alpha_i} + bL_{i,t}^p$$

[10] For an alternative approach, see Lundberg, Pollak, and Wales (1997) and Basu (2006)

In the present paper we focus on the effect of trade on sectors with different demand for male and female labor; hence we introduce differences in gender labor intensity across sectors in a very simple way by setting $\alpha_1 = \alpha \in (0, 1)$ and $\alpha_2 = 1$. Thus, the technologies are

$$\begin{aligned} X_1 &= aK_{1,t}^{\alpha}(L_{1,t}^{m})^{1-\alpha} + bL_{1,t}^{p} \\ X_2 &= aK_{2,t} + bL_{2,t}^{p} \end{aligned} \tag{2}$$

2.1.1 Labor Supply

Men and women are equally efficient in raising children. On the labor market, however, each women supplies a unit of mental labor L^m while men supply a unit of mental labor L^m plus a unit of physical labor L^p. Thus, as long as physical labor has a positive price, men receive a higher labor wage than women and therefore the opportunity cost of raising children is higher for a man than for a women. Consequently, men only raise children when women are doing so full-time. Parameters are assumed to be such that men never raise children.

Finally, we assume that male workers cannot divide mental and physical labor and must allocate both units to one and the same sector. This means, in particular, that men employed in the X_2-sector waste their mental labor.

2.2 Preferences

Each period t households derive utility from the number of their children n and old-age consumption c of a final good Y.[11]

$$u_t = \gamma \ln(n_t) + (1 - \gamma) \ln(c_{t+1}) \tag{3}$$

It is assumed here that the only input required to raise children is time and thus the opportunity cost of raising children is proportional to the market wage. Let w_t^F and w_t^M be the female wage and the male wage respectively. The full income of a household is $w_t^M + w_t^F$, which is spent on consumption and raising children. Let z be the fraction of the time endowment of one parent that must be spent in order to raise one child. If the wife spends time raising children, then the marginal cost of

[11]Note that since the basic unit is a household which consists a husband and a wife, n_t is in fact the number of couples of children that a couple has.

a child is zw_t^F. If the husband spends time raising children, then the marginal cost of a child is zw_t^M. The household's budget constraint is therefore

$$\begin{aligned} w_t^F zn_t + s_t \leq w_t^M + w_t^F \qquad & if \qquad zn_t \leq 1 \\ w_t^F + w_t^M zn_t + s_t \leq w_t^M + w_t^F \qquad & if \qquad zn_t \geq 1 \end{aligned} \tag{4}$$

where s_t is the household's savings. In the second period, the household consumes their savings

$$c_{t+1} = s_t(1 + r_{t+1}) \tag{5}$$

where r_{t+1} is the interest rate.

2.3 Optimality

Cost minimizing final good producers choose factor requirements according to factor prices. Under perfect competition, production cost equals the final good's price P_t, which we normalize to one. Writing $p_{i,t}$ as the price of intermediate good X_i in period t, this means

$$P_t = \left(\frac{p_{1,t}}{\theta}\right)^{\theta} \left(\frac{p_{2,t}}{1-\theta}\right)^{1-\theta} = 1 \tag{6}$$

Household maximizing problem yields

$$zn_t = \begin{cases} \gamma(1 + w_t^M/w_t^F) \quad if \quad \gamma(1 + w_t^M/w_t^F) \leq 1 \\ 2\gamma \quad if \quad 2\gamma > 1 \\ 1 \quad otherwise \end{cases} \tag{7}$$

For a relatively high gender wage gap (7) implies that women raise children full-time. As the gender gap decreases women may join the labor force and decrease fertility. In the limit when w_t^F approaches w_t^M women spend a fraction $\min(1, 2\gamma)$ of their time in raising children. If $\gamma > 1/2$, then women devote themselves in rasing children. In order to capture the dynamics of female labor force participation observed in the data we assume that $\gamma < 1/2$. Consequently, for high gender wage gap, women do not participate in the labor market. However, as this gap falls households find it optimal to decrease fertility and increase the participation of women in the labor market. Thus, (4) collapses to

$$s_t = (1 - zn_t)w_t^F + w_t^M \tag{8}$$

and (7) collapses to

$$zn_t = \min\left\{\gamma\left(1 + w_t^M / w_t^F\right), 1\right\} \tag{9}$$

2.3.1 Factor Prices

Notice that both consumption goods X_i are essential in final good production. This means that they are produced in positive quantities so that $L_{1,t}^m > 0$ holds. Consequently, the first unit of capital in sector X_1 is infinitely productive and $K_{1,t} > 0$ in equilibrium. Thus, we can write with (2) the returns to capital in the two sectors as

$$r_t = p_{1,t}\alpha a (K_{1,t}/L_{1,t}^m)^{\alpha-1} \tag{10}$$

$$r_t = p_{2,t} a \qquad if \qquad K_{2,t} > 0 \tag{11}$$

Similarly, male wages are derived form (2) and reflect the marginal productivity of their respective labor contribution

$$w_t^M = p_{1,t} b [(1-\alpha) a/b (K_{1,t}/L_{1,t}^m)^{\alpha} + 1] \qquad if \qquad L_{1,t}^p > 0 \tag{12}$$

$$w_t^M = p_{2,t} b \qquad if \qquad L_{2,t}^p > 0 \tag{13}$$

$$w_t^F = p_{1,t}(1-\alpha) a (K_{1,t}/L_{1,t}^m)^{\alpha} \qquad if \qquad zn_t < 1 \tag{14}$$

It will prove useful to the analysis in terms of per household variables. Therefore we define in these variables

$$\begin{aligned} k_t &= K_t/L_t \\ k_{i,t} &= K_{i,t}/L_t \\ m_t &= L_{1,t}^m/L_t \\ l_{i,t} &= L_{i,t}^p/L_t \end{aligned} \tag{15}$$

as total capital, sectorial capital, mental labor and sectorial physical labor per household, respectively. Finally, we define

$$\kappa_t = k_{1,t}/m_t \tag{16}$$

as the ratio of capital to mental labor employed in the first sector. This ratio will play a central role in the following analysis.

2.4 The Integrated Economy

Production (1) of the final good Y implies that expenditure on intermediate goods is constant in prices $p_{i,t}$. In this closed economy, where total output equals total production in each sector, the relative price therefore satisfies

$$\frac{p_{2,t}}{p_{1,t}} = \frac{1-\theta}{\theta}\frac{X_1}{X_2} = \frac{1-\theta}{\theta}\frac{a\kappa_t^{\alpha}m_t + bl_{1,t}}{ak_{2,t} + bl_{2,t}} \tag{17}$$

The full equilibrium allocation is determined by looking at two regime separately - the first in which women do not, and the second in which women do participate in the formal labor market.

To simplify the analysis, we assume that the second sector is too small to accommodate all male labor in equilibrium. More specifically, we assume

$$2 - 1/\theta > \alpha \tag{18}$$

to be satisfied in throughout the following analysis. A sufficient condition for male labor in the first sector to be always positive, the relative price (17) must fall short of the ratio of marginal rates of transformation at $l_{1,t} = 0$, i.e.

$$(1-\alpha)\frac{a}{b}\kappa_t^{\alpha} + 1 > \frac{1-\theta}{\theta}\frac{a}{b}\kappa_t^{\alpha}(1 - zn_t)$$

This condition is whenever $(1-\alpha) > (1-\theta)/\theta$ or (18) holds. Under this assumption, the ratio of male to female (shadow) wage can be computed by the marginal productivities in the first sector. The relevant ratio is

$$\frac{w^M}{w^F} = 1 + \frac{b}{(1-\alpha)a\kappa_t^{\alpha}}$$

which determines female labor force participation $1 - zn_t$ through (9):

$$zn_t = \min\left\{\gamma\left(2 + \frac{b}{(1-\alpha)a\kappa_t^{\alpha}}\right), 1\right\} \tag{19}$$

Equilibrium - the first regime $zn_t = 1$. Women do not participate in the formal labor market so that $m_t = l_{1,t}$. This implies that that conditions $m_t > 0$ and $k_{1,t} > 0$ hold.

Case 1. Consider now the situation where, in addition, $k_{2,t} > 0$ and $l_{2,t} > 0$ is satisfied. Under these conditions male wages and rental rate in both sectors equalize and with (10) - (13) we have

$$\frac{p_{2,t}}{p_{1,t}} = \alpha\kappa_t^{\alpha-1} \tag{20}$$

$$\frac{p_{2,t}}{p_{1,t}} = (1-\alpha)\frac{a}{b}\kappa_t^{\alpha} + 1 \tag{21}$$

This system determines implicitly the ratio κ_t and $p_{2,t}/p_{1,t}$ as a function of the parameters α and a/b. The equilibrium allocation is then determined by equating relative prices to those that clear the goods market (17). Using the resource constraints $l_{1,t} + l_{2,t} = 1$ and $k_{1,t} + k_{2,t} = k_t$, the resulting equilibrium condition can be written as

$$\frac{a}{b}(k_t/l_{1,t} - \kappa_t) + 1/l_{1,t} - 1 = \frac{1-\theta}{\theta}\frac{\frac{a}{b}\kappa_t^{\alpha} + 1}{\alpha\kappa_t^{\alpha-1}} \tag{22}$$

Since κ_t is independent of k_t the expression on the left and, in particular, the term

$$\frac{ak_t}{l_{1,t}} + \frac{b}{l_{1,t}} = const$$

must be constant as well. Thus, we conclude that $l_{1,t}$ is an increasing function of k_t and consequently $l_{2,t} = 1 - l_{1,t}$ is decreasing in k_t. With $l_{1,t} = m_t$ this implies further that $k_{2,t} = k_t - k_{1,t} = l_{1,t}(k_t/l_{1,t} - \kappa_t)$ is increasing in k_t.

With this information we can compute the upper and lower limits on k_t for which $k_{i,t} > 0$ and $l_{1,t} > 0$ hold. The upper limit k_H is determined by taking the limit $l_{1,t} \to 1$ in (22)

$$\frac{a}{b}(k_H - \kappa_t) = \frac{1-\theta}{\theta}\frac{\frac{a}{b}\kappa_t^{\alpha} + 1}{\alpha\kappa_t^{\alpha-1}}$$

The lower limit k_L is determined by taking the limit $k_{2,t} \to 0$ in (22)

$$\kappa_t/k_L - 1 = \frac{1-\theta}{\theta}\frac{\frac{a}{b}\kappa_t^{\alpha} + 1}{\alpha\kappa_t^{\alpha-1}}$$

where κ_t is still defined by (20) and (21).[12]

Case 2. For $k_t \leq k_L$ there is no capital employed in the X_2-sector ($k_{2,t} = 0$) and the equilibrium is defined by (17) and (21) (use $k_{1,t} = k_t$ and $m_t = l_{1,t}$)

$$(1-\alpha)\frac{a}{b}\kappa_t^{\alpha} + 1 = \frac{1-\theta}{\theta}\frac{\frac{a}{b}\kappa_t^{\alpha} + 1}{1/l_{1,t} - 1} = \frac{1-\theta}{\theta}\frac{\frac{a}{b}\kappa_t^{\alpha-1} + 1/\kappa_t}{1/k_t - 1/\kappa_t} \tag{23}$$

[12] It is easy to check that $k_L < k_H$.

This condition defines κ_t as an increasing function of k_t.

Case 3. For $k_t \geq k_H$ there is no labor employed in the X_2-sector ($l_{2,t} = 0$) and the equilibrium is defined by (17) and (20)

$$\alpha \kappa_t^{\alpha-1} = \frac{1-\theta}{\theta} \frac{\kappa_t^{\alpha} + b/a}{k_t - \kappa_t} \tag{24}$$

This condition defines the ratio κ_t as an increasing function of k_t. The resource constraint determines $k_{2,t} = k_t - k_{1,t}$ and equation (20) fixes the equilibrium prices.

Thus, κ_t is increasing in the per household capital stock k_t no matter the level of k_t. Figure 1 summarizes these findings. At low k_t all capital in employed in the first sector and X_2 is produced using labor only. Relative prices $p_{2,t}/p_{1,t}$ are too low to attract capital to the second sector and (20) does not hold while prices are determined by (21) and (23). With increasing capital, k_t, output in the first sector increases and the price of the second good appreciates up to the point where (20) and (21) hold with equality (this happens at $k_t = k_L$). From this level on capital is used in both sectors and the capital labor share in the first sector is fixed by marginal rates of transformation in the X_i-sectors. If capital exceeds the level k_H it becomes cheap to the extend that it is the only profitable factor to employ in the second sector and all labor in employed in the first one. These calculations have

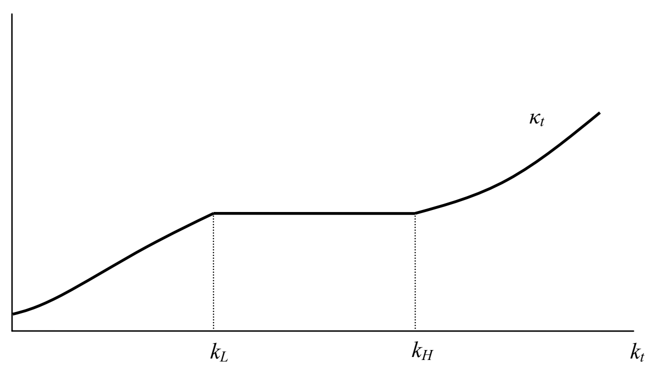

Figure 1: Capital labor ratio in the first sector under $zn_t = 1$.

been performed assuming that women do not participate in the formal labor market ($zn_t = 1$). From relation (19) we know, however, that there is a critical level of $k_{1,t}$ above which female are attracted to the formal labor market. Thus, if the capital stock k_t exceeds a certain threshold women enter the formal labor force. At this point, the second regime starts.

Equilibrium - the second regime $zn_t < 1$. Via equation (19) κ_t determines female labor force participation and there are two possible cases. First women enter the formal labor market at a capital stock k_t below the level k_L (low γ), or, second, women enter the formal labor market at a capital stock k_t above the level k_H (high γ).

Case 1 where $k_{2,t} = 0$. This implies $l_{2,t} > 0$. Since men keep working in the first sector (condition (18)) we conclude that equation (21) holds. Further notice that the interior solution of (19) implies

$$(1-\alpha)\frac{a}{b}\kappa_t^{\alpha} = \frac{\gamma}{zn_t - 2\gamma} \tag{25}$$

Combining equation (17) with (21) and (25) and using the resource constraints $l_{1,t} + l_{2,t} = 1$ and $m_t = l_{1,t} + (1 - zn_t)$ leads to the equilibrium condition

$$\frac{zn_t - \gamma}{zn_t - 2\gamma} = \frac{1-\theta}{\theta} \frac{\left(\frac{\gamma}{1-\alpha}\frac{1}{zn_t - 2\gamma} + 1\right)\left[\frac{1-\alpha}{\gamma}\frac{a}{b}(zn_t - 2\gamma)\right]^{1/\alpha} k_t - (1 - zn_t)}{1 - \left[\frac{1-\alpha}{\gamma}\frac{a}{b}(zn_t - 2\gamma)\right]^{1/\alpha} k_t + (1 - zn_t)} \tag{26}$$

The expression on the left is decreasing in zn_t while the term on the right is increasing in zn_t and in k_t so that the solution zn_t is unique and decreasing in k_t. By equation (25) this implies that the ratio κ_t is increasing in k_t.

Relative prices are determined by (21) and continue to increase in the capital stock k_t as long as $l_{1,t}, l_{2,t} > 0$ and $k_{2,t} = 0$ hold. As k_t grows large, however, relative prices hit the level $p_{2,t}/p_{1,t} = \alpha\kappa_t^{\alpha-1}$, i.e. (20) is satisfied. Above this level $k_{2,t} = 0$ ceases to hold and capital and male labor is used in both, X_1 and X_2 production.

Case 2 where $l_{1,t}, l_{2,t} > 0$ and $k_{1,t}, k_{2,t} > 0$. At these intermediate levels of the capital stock, the ratio κ_t is determined by the system (20) and (21) (and actually takes the same value of intermediate ranges of capital under $zn_t = 1$). As long as $l_{1,t}, l_{2,t} > 0$ and $k_{1,t}, k_{2,t} > 0$ hold, the ratio κ_t is thus constant and so are relative prices $p_{2,t}/p_{1,t}$ and female labor force participation. Hence, when the capital stock increases further, this induces a reallocation of male labor towards the first sector as under $zn_t = 1$. When this reallocation is complete all male work in the first sector.

Case 3 where $l_{2,t} = 0$. In this case the equilibrium is determined by (17), (19), and (20) and the resource constraints $m_t = 2 - zn_t$ and $k_{2,t} = k_t - k_{1,t}$. The resulting equilibrium

$$\frac{1-\theta}{\theta}\frac{2(1-\gamma)\kappa_t^{\alpha}-\frac{b\gamma}{(1-\alpha)a}+b}{a\left(k_t-\kappa\right)}=\alpha\kappa_t^{\alpha-1} \tag{27}$$

The expression to the left is increasing in κ_t and decreasing in k_t, while the expression to the right of (27) is decreasing in κ_t. Thus, this condition defines κ_t as an increasing function of k_t. With (25) this implies that zn_t is a decreasing function of k_t. Consequently, the relative prices (20) are decreasing in k_t thus preserving the allocation of male labor.

Notice finally that κ_t is unbounded if the capital stock grows infinitely large, i.e. $\lim_{k_t\to\infty}\kappa_t=\infty$. We turn to the second case where $l_{2,t}=0$ holds at the threshold

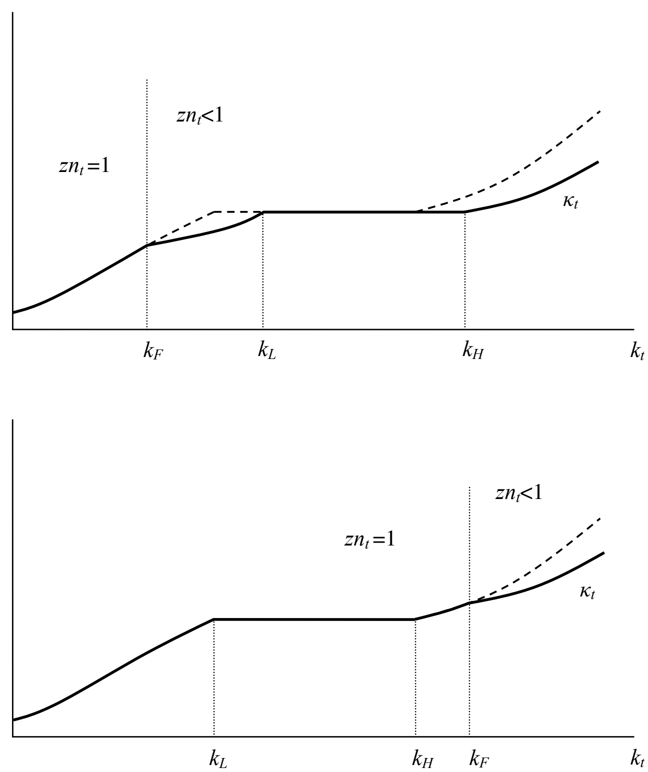

Figure 2: Capital labor ratio in the first sector under $zn_t<1$. (The dashed line indicates the case under which $znt=0$)

capital stock where women enter the formal labor market. The equilibrium is readily established. Since $l_{2,t}=0$ implies $k_{2,t}>0$ (20) holds. Together with (17) this implies again (27) and hence zn_t and the ratio κ_t are decreasing and increasing function of k_t, respectively.

These findings are summarized in Figure 2. The left panel illustrates the case of small γ where female enter the labor force at the threshold k_2 with $k_F < k_H$, i.e. at low levels of per household capital stock already. As the capital stock increases, its price drops up to the point where it is cheap enough to use it as production factor in the second sector. As capital increases further, male exit the second sector and are replaced by capital. In this reallocation process the ratio κ_t is fixed by the marginal rates of transformation and zn_t ceases to react to changes in per household capital stock. For high levels of the capital stock only capital is employed in the second sector. The ratio κ_t continues to rise in k_t, which further attracts women to the labor market. The right panel illustrates the case of large γ where female enter the labor force at a higher threshold k_F under $l_{2,t} = 0$. At these levels, any increase of k_t raises κ_t and attracts more women to the labor market. In both panels the dotted line represent the values of κ_t when fixing $zn_t = 1$ exogenously. Whenever female labor participation is positive, this dotted line lies below the bold line, representing the equilibrium κ_t.

Notice finally that relative price $p_{2,t}/p_{1,t}$ depends on k_t in a non-monotonic way: for $k_t < k_L$ the relative price is determined by (21) and increasing in κ_t, for $k_t > k_H$ it is determined by (20) and decreasing in κ_t, while finally for $k_t \in [k_L, k_H]$ it is jointly determined by (20) and (21). By the weakly increasing function $\kappa_t(k_t)$ represented in Figure 2, this implies that for $k_t < k_L$ the relative price $p_{2,t}/p_{1,t}$ is increasing in k_t, for $k_t > k_H$ the relative price is decreasing in k_t, and for $k_t \in [k_L, k_H]$ the relative price is constant in k_t. Figure 3 illustrates this result. The fundamental reason for this non-monotonicity is that capital intensity of the goods changes over the full range of k_t. Due to the high substitutability of factors in X_2-production, the good X_2 is relatively labor intensive at high rental rates while it is capital intensive at low rental rates. Since further a falling rental rate decreases the price of the capital intensive good by more than the price of the labor intensive good, this generates a U-shaped behavior of prices as a function of the rental rate. Finally, the rental rate is decreasing in the capital stock which leads to the hum-shaped relation of relative prices as a function of per household capital stock represented in Figure 3.

With the static equilibrium of the closed economy well understood, we turn to the dynamic system next.

Dynamics. The dynamic system is determined by the period-by-period equilibria

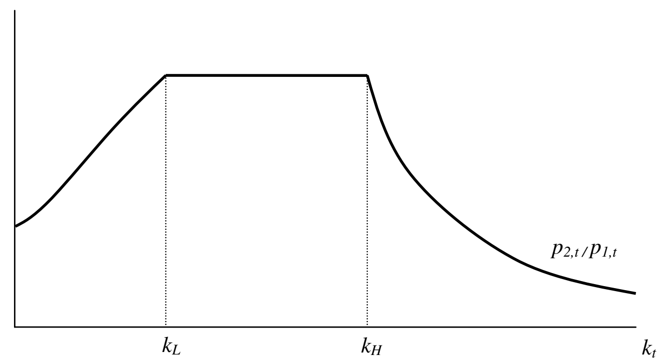

Figure 3: Relative prices as a function of k_t in the closed economy.

and the law of motion for capital and labor

$$k_{t+1} = \frac{s_t}{n_t} \tag{28}$$

Male and female wages are calculated from marginal productivities in the first sector

$$w_t^M = (1-\alpha)a\kappa_t^\alpha + b \qquad \text{and} \qquad w_t^F = (1-\alpha)a\kappa_t^\alpha \tag{29}$$

so that per household saving (8) is

$$s_t = p_{1,t} b \left[(2 - zn_t)(1-\alpha)\frac{a}{b}\kappa_t^\alpha + 1 \right] \tag{30}$$

To determine price $p_{1,t}$ use the normalization of the ideal price index (6) and write

$$p_{1,t} = \theta^\theta (1-\theta)^{1-\theta} \left(\frac{p_{2,t}}{p_{1,t}} \right)^{\theta-1} \tag{31}$$

Relative prices depend on whether $k_{2,t} > 0$ or $l_{2,t} > 0$ and are determined by (20) or (21) accordingly. Again, two regimes are to be distinguished.

The First Regime $zn_t = 1$. In the case of $zn_t = 1$ we combine (31) with (20) or (21) (according to $k_{2,t} > 0$ or $l_{2,t} > 0$) to write per household saving from (30) as

$$s_t = \begin{cases} \vartheta \left((1-\alpha)\frac{a}{b}\kappa_t^\alpha + 1 \right)^\theta & if \;\; l_{2,t} > 0 \\ \vartheta \left((1-\alpha)\frac{a}{b}\kappa_t^\alpha + 1 \right) \left(\alpha\kappa_t^{\alpha-1} \right)^{\theta-1} & if \;\; k_{2,t} > 0 \end{cases}$$

where we abbreviate $\vartheta = b\theta^\theta(1-\theta)^{1-\theta}$. The dynamics of k_t are then determined by (28) together with either (20) and (21), or with (24) or with (23), depending on whether $k_t \in (k_L, k_H)$, $k_t \leq k_L$, or $k_t \geq k_H$ holds. Notice that, since the ratio κ_t is non-decreasing in k_t, the function $k_{t+1}(k_t)$ is so as well. However, in the range $k_t \in (k_L, k_H)$ the ratio κ_t is constant in k_t. Consequently, in this range, small increases in today's capital stock do not increase the total savings and leave tomorrow's capital stock unaffected. The function $k_{t+1}(k_t)$ has a flat part, a plateau. Due to constant returns to both factors in the second sector the increase in capital stock does not affect labor productivity and leaves wages - and thus savings - unchanged.

The Second Regime $zn_t < 1$. With the interior solution of (19) and the relative prices (20) or (21) (according to $k_{2,t} > 0$ or $l_{2,t} > 0$) per household savings (30) are

$$s_t = \begin{cases} \vartheta(1-\gamma)\frac{2(1-\alpha)\frac{a}{b}\kappa_t^\alpha+1}{\left((1-\alpha)\frac{a}{b}\kappa_t^\alpha+1\right)^{1-\theta}} & if \quad l_{2,t} > 0 \\ \\ \vartheta(1-\gamma)\frac{2(1-\alpha)\frac{a}{b}\kappa_t^\alpha+1}{\left(\alpha\kappa_t^{\alpha-1}\right)^{1-\theta}} & if \quad k_{2,t} > 0 \end{cases}$$

The dynamics of k_t are then determined by (28) together with either (20) and (21) or with (25) and (26) or with (27) depending on whether $k_t \in (k_L, k_H)$, $k_t \leq k_L$, or $k_t \geq k_H$.

As in the first regime, per household savings are weakly increasing in the capital stock k_t since κ_t is non-decreasing in k_t under $zn_t > 0$ as well and s_t is an increasing function of κ_t. This implies that, just as in standard Ramsey-type or OLG models, there is no leapfrogging of closed economies and a country that starts out with an initially lower stock of capital does not fully catche up in finite time with another country that is initially endowed with a higher stock of capital.

2.5 A Two-Country World Economy

Assume the world economy consists of two countries, Home and Foreign, which engage in free and costless trade in intermediate goods, Neither of the two factors capital and labor is allowed to cross national borders. Both countries have the identical technologies and motives to trade arise only through differences in per household capital stock. The demand structure of intermediate goods is generated by final good production (1) and implies that relative world prices reflect aggregate output. Denoting Foreign's variables with a star, the world prices are then, parallel to (17)

$$\frac{p_{2,t}}{p_{1,t}} = \frac{1-\theta}{\theta} \frac{a\left[\kappa_t^\alpha m_t + \lambda_t (\kappa_t^*)^\alpha m_t^*\right] + b\left[l_{1,t} + \lambda_t l_{1,t}^*\right]}{a\left[k_{2,t} + \lambda_t k_{2,t}^*\right] + b\left[l_2 + \lambda_t l_{2,t}^*\right]} \tag{32}$$

where $\lambda_t = L_t^*/L_t$ is the relative population size of Foreign to Home. To understand the world equilibrium of this two-country economy we start by looking at the two cases where relative factor prices equalize and where they do not. The relevant results are formulated in two Claims. The first claim treats the case where relative factor prices equalize.

Claim 1 *If relative factor prices equalize under free trade, i.e. when*

$$\frac{w_t^M}{r_t} = \frac{w_t^{M,*}}{r_t^*} \tag{33}$$

holds, then absolute factor prices equalize, i.e. $w_t^M = w_t^{M,*}$, $r_t = r_t^*$, *and* $w_t^F = w_t^{F,*}$. *This implies further* $\kappa_t = \kappa_t^*$ *and* $zn_t = zn_t^*$.

Proof. Show first that condition (18) implies $l_{1,t} + l_{1,t}^* > 0$. To this goal assume $l_{1,t} = l_{1,t}^* = 0$. From (32) we have

$$\frac{p_{2,t}}{p_{1,t}} = \frac{1-\theta}{\theta} \frac{a\left[\kappa_t^\alpha m_t + \lambda_t (\kappa_t^*)^\alpha m_t^*\right]}{b\left[1+\lambda_t\right]} \leq \frac{1-\theta}{\theta}\frac{a}{b}\max\{\kappa_t^\alpha(1-zn_t), (\kappa_t^*)^\alpha(1-zn_t^*)\}$$

so that with (18) relative prices fall short of marginal rates of transformation

$$\frac{p_{2,t}}{p_{1,t}} < \frac{a}{b}(\kappa_t^{(*)})^\alpha(1-zn_t^{(*)})$$

for at least one country, a contradiction to profit maximization of firms. Hence, without loss of generality we can assume $l_{1,t} > 0$. This implies $\kappa_t > 0$ and

$$\frac{w_t^M}{r_t} = \frac{1-\alpha}{\alpha}\kappa_t + \frac{b}{a\alpha}\kappa_t^{1-\alpha}$$

We distinguish three cases.

A. $r = r^*$ (33), which implies $w_t^M = w_t^{M,*}$.

B. $r < r^*$ (33), which implies $w_t^M < w_t^{M,*}$. In particular, we have $p_{2,t}b < w_t^{M,*}$ so that $l_{1,t}^* = 1$ and

$$\frac{1-\alpha}{\alpha}\kappa_t + \frac{b}{a\alpha}\kappa_t^{1-\alpha} = \frac{w_t^M}{r_t} = \frac{w_t^{M,*}}{r_t^*} = \frac{1-\alpha}{\alpha}\kappa_t^* + \frac{b}{a\alpha}(\kappa_t^*)^{1-\alpha} \tag{34}$$

This implies $\kappa_t = \kappa_t^*$ and hence $r = r^*$ a contradiction to the assumption.

C. $r > r^*$, which implies $w_t^M > w_t^{M,*}$. In particular, we have $r > p_{2,t}a$ and $w_t^M > p_{2,t}b$ so that $l_{2,t} = k_{2,t} = 0$. This implies that $l_{2,t}^* > 0$ or $k_{2,t}^* > 0$. Notice that $l_{1,t}^* > 0$ implies (34) and hence the statement of the proposition. Thus, the only case left to consider is $l_{2,t}^* = 1$. Now notice that $l_{2,t}^* = 1$ implies $k_{2,t}^* = 0$. To verify this statement, assume $l_{2,t}^* = 1$ and $k_{2,t}^* > 0$, which implies that an atomistic firm in Foreign that hires $\varepsilon\kappa_t$ units of capital and ε units of male labor to produce in the X_1-sector makes positive profits

$$\pi = \varepsilon\left[a\kappa_t^\alpha + b - \kappa_t r_t^* - w_t^{M,*}\right] > \varepsilon\left[a\kappa_t^\alpha + b - \kappa_t r_t - w_t^M\right] = 0$$

This contradicts the no-arbitrage condition. Consequently, we have $l_{2,t}^* = 1$ and $k_{2,t}^* = 0$. This implies

$$p_{1,t}a\alpha\kappa_t^{\alpha-1} = r > r^* = p_{1,t}a\alpha(\kappa_t^*)^{\alpha-1}$$

and

$$p_{1,t}\left[(1-\alpha)a\kappa_t^\alpha + b\right] = w_t^M > w_t^{M,*} \geq p_{1,t}\left[(1-\alpha)a(\kappa_t^*)^\alpha + b\right]$$

The first of the two conditions implies $\kappa_t < \kappa_t^*$ and the second $\kappa_t > \kappa_t^*$. This constitutes a contradiction.

In all three cases we have thus $w_t^M = w_t^{M,*}$ and $r_t = r_t^*$. Since $l_{1,t} + l_{1,t}^* > 0$ and male wage and rental rate of capital are identical, location of production is indeterminate and we can assume wlog that $\kappa_t, \kappa_t^* > 0$. In particular, we have $r_t = p_{1,t}\left[(1-\alpha)a\kappa_t^{1-\alpha}\right] = p_{1,t}\left[(1-\alpha)a(\kappa_t^*)^{1-\alpha}\right]$ or $\kappa_t = \kappa_t^*$. This implies $w_t^F \leq w_t^{F,*}$ and, together with $w_t^M = w_t^{M,*}$, leads to $zn_t \leq zn_t^*$. ■

To determine the conditions for factor price equalization we use the insights of the integrated economy that are summarized in Figure 2. Further, we denote world aggregates with an upper bar and write

$$\bar{k}_t = (k_t + \lambda k_t^*)/(1+\lambda) \qquad \text{and} \qquad \bar{l}_{i,t} = (l_{i,t} + \lambda l_{i,t}^*)/(1+\lambda) \tag{35}$$

By Claim 1 we get $\kappa_t = \kappa_t^* = \bar{\kappa}_t$ and $zn_t = zn_t^* = z\bar{n}_t$ in the case of relative factor price equalization. If factors can freely cross borders, the world economy replicates the equilibrium of the integrated economy by definition. However, if factors are confined to remain within national borders, we have to impose additional conditions on the actual factor distribution in order to obtain factor price equalization. These conditions determine the Factor Price Equalization Set (FPES), which is defined as the partition of factors across countries under which the world equilibrium with costless trade in goods replicates the aggregate output pattern of the integrated economy.

In the following the FPES will be computed by separately considering the three cases that were discussed in the closed economy already ($\bar{k}_t \leq k_L$, $\bar{k}_t \geq k_H$, and $\bar{k}_t \in (k_L, k_H)$). Throughout the computations the aggregate values will be assumed to be constant, e.g. an increase in k_t^* will be accompanied by a corresponding fall of k_t that leaves $\bar{k}_t$ unchanged.

Finally, we assume without loss of generality that the per household stock of capital in Foreign is larger than that in Home, i.e. $k_t \leq k_t^*$.

Case 1: $\bar{k}_t \leq k_L$. In this case we have $\bar{k}_{2,t} = 0$ so that $k_{2,t} = k_{2,t}^* = 0$, which implies

$$k_{1,t}^{(*)} = k_t^{(*)} = \bar{\kappa}_t(l_{1,t}^{(*)} + 1 - z\bar{n}_t) \tag{36}$$

This condition determines the labor allocations in the first sector of either country. Now there are two relevant conditions on the factor distribution. These are, first, that in each country the residual amount of male labor (employed in the second sector) be non-negative, i.e. $l_{2,t}^{(*)} = 1 - l_{1,t}^{(*)} \geq 0$, and second, that the capital stock in each country does not fall short of the level needed to provide female workers with the capital ratio of the integrated economy, i.e. $k_t^{(*)} \geq \bar{\kappa}_t(1 - z\bar{n}_t)$. With the identity (36) the first condition can be formulated as $k_t^{(*)} \leq \bar{\kappa}_t(2 - z\bar{n}_t)$. By $k_t \leq k_t^*$ this inequality is satisfied for k_t whenever it holds for k_t^* while the previous inequality is satisfied for k_t^* whenever it holds for k_t. Hence, the relevant conditions can be summarized as

$$\bar{\kappa}_t(1 - z\bar{n}_t) \leq k_t \quad \text{and} \quad k_t^* \leq \bar{\kappa}_t(2 - z\bar{n}_t)$$

With $\bar{k}_t = (k_t + \lambda k_t^*)/(1 + \lambda)$ and $\bar{k}_t = \bar{\kappa}_t(\bar{l}_{1,t} + 1 - z\bar{n}_t)$ this leads to

$$k_t^* \leq \bar{\kappa}_t \min\left\{2 - z\bar{n}_t, \left[\bar{l}_{1,t}(1 + \lambda)/\lambda + 1 - z\bar{n}_t\right]\right\} \tag{37}$$

(With $\bar{\kappa}_t(\bar{l}_{1,t} + 1 - z\bar{n}_t) = \bar{k}_t$ it is possible to check that the upper bound is larger than $\bar{k}_t$.)

Case 2: $\bar{k}_t \geq k_H$. In this case we have $\bar{l}_{2,t} = 0$ so that $l_{2,t} = l^*_{2,t} = 0$, which implies

$$k^{(*)}_{1,t} = \bar{\kappa}_t(2 - z\bar{n}_t)$$

The only condition on the factor distribution is that capital stocks in both countries do not fall short of these levels, i.e. conditions

$$\bar{\kappa}_t(2 - z\bar{n}_t) \leq k^{(*)}_t$$

By $k_t \leq k^*_t$ this condition is satisfied whenever $\bar{\kappa}_t(2 - z\bar{n}_t) \leq k_t$ holds. With $\bar{k}_t = (k_t + \lambda k^*_t)/(1 + \lambda)$ this can be written as a condition on Foreign's capital stock

$$k^*_t \leq \bar{k}_t \frac{1+\lambda}{\lambda} - \frac{\bar{\kappa}_t}{\lambda}(2 - z\bar{n}_t) \tag{38}$$

(With $\bar{\kappa}_t = \bar{k}_{1,t}/(2 - z\bar{n}_t)$ and $\bar{k}_{1,t} < \bar{k}_t$ it is possible to check that the upper bound is larger than $\bar{k}_t$.)

Case 3: $\bar{k}_t \in (k_L, k_H)$. In this case we have $\bar{l}_{2,t} > 0$ and $\bar{k}_{2,t} > 0$. Now the distribution of two factors (capital and male labor) is to be determined. The conditions to be satisfied are now $k^{(*)}_t \geq \bar{\kappa}_t(l^{(*)}_{1,t} + 1 - z\bar{n}_t)$. Combining them with (35), $\bar{l}_{1,t} = (l_{1,t} + \lambda l^*_{1,t})/(1 + \lambda)$, and $\bar{k}_t = \bar{\kappa}_t(\bar{l}_{1,t} + 1 - z\bar{n}_t)$ leads to

$$\bar{\kappa}_t(l^*_{1,t} + 1 - z\bar{n}_t) \leq k^*_t \leq \frac{1+\lambda}{\lambda}\bar{k}_t - \frac{\bar{\kappa}_t}{\lambda}\left[(1+\lambda)\bar{l}_{1,t} - \lambda l^*_{1,t} + 1 - z\bar{n}_t\right]$$

Now notice that the upper bound and the lower bound are increasing $l^*_{1,t}$. The lower bound is satisfied by setting $l^*_{1,t} = \bar{l}_{1,t}$ since by (35) the inequality $k_t \leq k^*_t$ is equivalent to $\bar{k}_t \leq k^*_t$. The upper bound is maximal at maximal value of $l^*_{1,t}$. To compute this value, observe that sectoral labor force is bound to be positive, i.e. $l^{(*)}_{1,t} \in [0, 1]$. Combining these conditions with (35) this leads to

$$l^*_{1,t} \in [0, 1] \cap [\bar{l}_{1,t}(1 + \lambda)/\lambda - 1/\lambda, \bar{l}_{1,t}(1 + \lambda)/\lambda]$$

In particular, we have $l^*_{1,t} \leq l^*_{1,\max}$ where

$$l^*_{1,\max} = \min\left\{1, \bar{l}_{1,t}(1 + \lambda)/\lambda\right\}$$

With these expression, the relevant condition in on the factor distribution becomes

$$k_t^* \leq \frac{1+\lambda}{\lambda}\bar{k}_t - \frac{\bar{\kappa}_t}{\lambda}\left[(1+\lambda)\bar{l}_{1,t} - \lambda l^*_{1,\max} + 1 - z\bar{n}_t\right] \tag{39}$$

Notice that as $\bar{k}_t \to k_H$ we have $\bar{l}_{1,t} \to 1$ and $l^*_{1,\max} = 1$ so that (39) comprises (38) in the limit. Further, as $\bar{k}_t \to k_L$ we have $\bar{\kappa}_t(\bar{l}_{1,t} + 1 - z\bar{n}_t) \to \bar{k}_t$ so that (39) comprises (37) in the limit.

Given the aggregate state variable $\bar{k}_t$ and the resulting equilibrium of the integrated economy (characterized by $\bar{l}_{1,t}$, $\bar{\kappa}_t$, and $z\bar{n}_t$) the inequalities (37), (38), and (39) reflect the conditions under which factor prices equalize in the three different regimes. Using the graphical representation of the factor price equalization set from Helpman and Krugman (1985), Figure 4 illustrates the FPES as the grey area within the box of all possible factor endowments. Each point in the box represents a unique distribution of factors (male) labor and capital: Home's factor endowments are represented by the distance of such a point to axis, Foreign's simply are the residuals. The constraints (37), (38), and (39) delimit the borders of the FPES.

The top panel depicts the case $\bar{k}_t < k_L$ (condition (37) applies) where only labor is used in X_2-production. This means that a country without any capital can trade with a relatively capital abundant country and the efficient use of factors of the integrated economy is still granted. Necessary condition, however, is that the capital-less country is not too big, i.e. its male labor force does not exceed the male labor allocation to the second sector in the integrated economy.

The middle panel illustrates the case $\bar{k}_t \in (k_L, k_H)$ (condition (38) applies), where both factors - capital and labor - are employed in X_2-production in the integrated economy. Accordingly, moderately sized countries either without any capital endowment or else with a negligible labor force can be part of the efficient two-country world economy that replicates the integrated economy.

The bottom panel $\bar{k}_t > k_H$ (condition (39) applies), where only capital is used in X_2-production in the integrated economy. In this case, a country with a negligible labor force can be part of the efficient world economy. A country without any capital, however, cannot.

If the conditions for factor price equalization are satisfied and Claim 1 applies we can draw some conclusions concerning labor income of the economy. To this aim recall that we assume $k_t \leq k_t^*$, which implies the inequalitites $k_t \leq \bar{k}_t \leq k_t^*$. Now, Claim

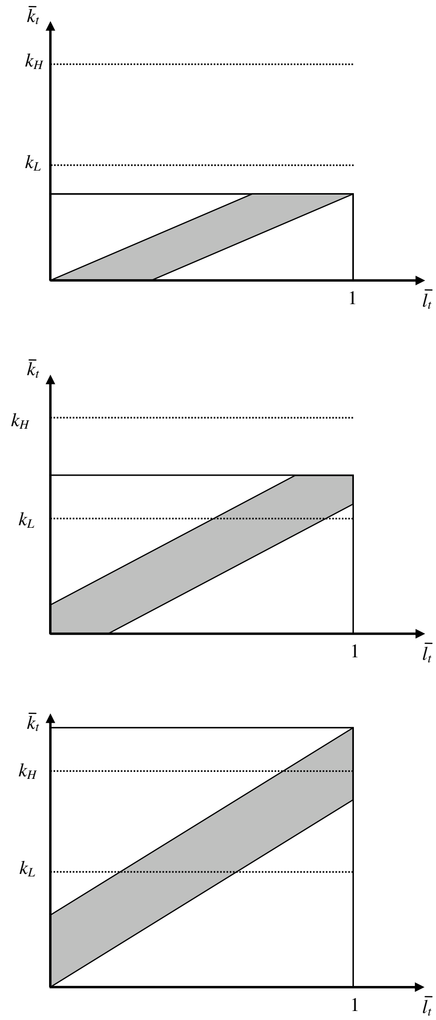

Figure 4: The Factor Price Equalization Sets for $\bar{k}_t < k_L$ (top panel), $\bar{k}_t \in (k_L, k_H)$ (middle panel)) and $\bar{k}_t > k_H$ (bottom panel).

1 shows that within the factor price equalization set, the ratio of capital to mental labor in the first sector equalizes in both countries and must therefore coincide with the one of the integrated economy $\kappa_t = \kappa_t^* = \bar{\kappa}_t$. Since κ is non-decreasing in the per household capital stock this implies that trade reduces κ in the capital rich country (Foreign) relative to autarky while trade increases κ_t in the capital scarce country (Home). Denoting autarky variables with the superscript ^{A} we can write

$$\kappa_t^A \leq \bar{\kappa}_t \leq \kappa_t^{A,*}$$

Further, since wages (12) - (14) and female labor force participation (19) are non-decreasing in κ this implies that trade weakly increases total labor income in the capital scarce country relative to autarky while it weakly reduces it in the capital rich country. By (8) labor income equals total savings, which means that the following period's per household capital stock and consequentially trade increases (reduces) total savings in capital scarce (rich) countries.

Finally, as male and female wages as well as female labor force participation is identical in both countries labor earning and thus savings per household are identical by (8). This implies that, following a period of factor price equalization, per household capital stocks are identical in the two countries for all consecutive period. Thus, all per household variables of in the two economies trivially coincide in all times following a period factor price equalization.

Both observations together imply that, conditional on factor price equalization in the first period, trade increases (reduces) per household capital stock persistently in the capital scarce (rich) country.

The above reflections show that in the present model where the motives of international specialization come from differences in factor endowments all economic action that stems from international trade is shut down after a period of factor price equalization. This observation directs our interest to the periods where factor prices do not equalize. The second claim treats this case.

Claim 2 *Assume that*

$$\frac{w_t^M}{r_t} < \frac{w_t^{M,*}}{r_t^*} \tag{40}$$

holds in the world equilibrium with free trade. Then

(i) $k_{2,t} = 0$.

(ii) $l_{1,t} > 0 \quad \Rightarrow \quad l_{2,t}^* = 0$.

(iii) $l_{1,t} + l_{1,t}^* > 0$.

(iv) $l_{1,t}^* > 0$.

(v) $\kappa_t < \kappa_t^*$.

(vi) $r_t > r_t^*$, $w_t^M \leq w_t^{M,*}$, *and* $w_t^F < w_t^{F,*}$.

(vii) $zn_t > zn_t^*$.

(viii) $k_t < k_t^*$.

(ix) $s_t \leq s_t^*$.

Proof. (i) Suppose $k_{2,t} > 0$. This implies

$$r_t = p_{2,t}a \leq r_t^*$$

so that with (40)

$$w_t^M < w_t^{M,*}$$

The lower bound on male wages is $p_{2,t}b \leq w_t^M$ implying $p_{2,t}b < w_t^{M,*}$ and hence $l_{2,t}^* = 0$. We have thus $k_{2,t} > 0$ and $l_{1,t}^* > 0$. The latter inequality implies $k_{1,t}^* > 0$ so that, with $p_{1,t}\left[(1-\alpha)a\alpha\kappa_t^\alpha + b\right] \leq w_t^M$ we get

$$\frac{1-\alpha}{\alpha}\kappa_t + \frac{b}{a\alpha}\kappa_t^{1-\alpha} \leq \frac{w_t^M}{r_t} < \frac{w_t^{M,*}}{r_t^*} = \frac{1-\alpha}{\alpha}\kappa_t^* + \frac{b}{a\alpha}(\kappa_t^*)^{1-\alpha}$$

and with $r_t \leq r_t^*$

$$p_{1,t}a\alpha\kappa_t^{\alpha-1} \leq p_{1,t}a\alpha(\kappa_t^*)^{\alpha-1}$$

The first of these two equations implies $\kappa_t < \kappa_t^*$ while the second leads to $\kappa_t \geq \kappa_t^*$, which constitutes a contradiction.

(ii) Assume $l_{1,t} > 0$ and $l_{2,t}^* > 0$. This implies $w_t^{M,*} = p_{2,t}b \leq w_t^M$ and, by (40), $r_t > r_t^*$. Thus, an atomistic firm in Foreign that hires $\varepsilon\kappa_t$ units of capital and ε units of male labor to produce in the X_1-sector makes positive profits

$$\pi = \varepsilon\left[a\kappa_t^\alpha + b - \kappa_t r_t^* - w_t^{M,*}\right] > \varepsilon\left[a\kappa_t^\alpha + b - \kappa_t r_t - w_t^M\right] = 0$$

which contradicts the no-arbitrage condition. This proves (ii).

(iii) Assume $l_{1,t} = l_{1,t}^* = 0$. From (32) we have

$$\frac{p_{2,t}}{p_{1,t}} = \frac{1-\theta}{\theta}\frac{a\left[\kappa_t^\alpha m_t + \lambda_t(\kappa_t^*)^\alpha m_t^*\right]}{b\left[1+\lambda_t\right]} \leq \frac{1-\theta}{\theta}\frac{a}{b}\max\{\kappa_t^\alpha(1-zn_t), (\kappa_t^*)^\alpha(1-zn_t^*)\}$$

so that with (18) relative prices fall short of marginal rates of transformation

$$\frac{p_{2,t}}{p_{1,t}} < \frac{a}{b}(\kappa_t^{(*)})^\alpha(1-zn_t^{(*)})$$

for at least one country, a contradiction to profit maximization of firms.

(iv) If $l_{1,t} > 0$ the statement follows from (ii). If $l_{1,t} = 0$ it follows from (iii).

(v) From (i) we have $r_t = p_{1,t} a\alpha\kappa_t^{\alpha-1}$; from (iv) we have $w_t^{M,*}/r_t^* = \frac{1-\alpha}{\alpha}\kappa_t^* + \frac{b}{a\alpha}(\kappa_t^*)^{1-\alpha}$. Since further $w_t^M \geq (1-\alpha)a\alpha\kappa_t^\alpha + b$ equation (40) leads to

$$\frac{1-\alpha}{\alpha}\kappa_t + \frac{b}{a\alpha}\kappa_t^{1-\alpha} < \frac{1-\alpha}{\alpha}\kappa_t^* + \frac{b}{a\alpha}(\kappa_t^*)^{1-\alpha}$$

and thus $\kappa_t < \kappa_t^*$.

(vi) $r_t > r_t^*$ follows directly from (v) and $r_t^{(*)} = p_{1,t} a\alpha(\kappa_t^{(*)})^{\alpha-1}$. If $l_{1,t} = 0$ we get $w_t^M = p_{2,t} b \leq w_t^{M,*}$ immediately. If $l_{1,t} > 0$, $w_t^M < w_t^{M,*}$ follows from (v). $w_t^F < w_t^{F,*}$ follows from (v).

(vii) Note that for constant prices w_t^M/w_t^F is decreasing in κ_t:

$$\frac{w_t^M}{w_t^F} = \frac{\max\left\{p_{1,t}\left[(1-\alpha)a\kappa_t^\alpha + b\right], p_{2,t} b\right\}}{p_{1,t}(1-\alpha)a\kappa_t^\alpha}$$

With (v) this implies $w_t^M/w_t^F > w_t^{M,*}/w_t^{F,*}$ and (vii) follows from equation (9).

(viii) Statements (ii) and (iv) imply $l_{1,t} \leq l_{1,t}^*$ so that with (vii) we get $m_t = l_{1,t} + 1 - zn_t \leq l_{1,t}^* + 1 - zn_t^* = m_t^*$. Using now (i) and (v) leads to

$$\frac{k_t}{m_t^*} \leq \frac{k_t}{m_t} = \kappa_t < \kappa_t^* \leq \frac{k_t^*}{m_t^*}$$

which proves the statement.

(ix) For $zn_t > 0$ we have with (9)

$$s_t = \left(w_t^M + w_t^F\right)(1-\gamma) < \left(w_t^{M,*} + w_t^{F,*}\right)(1-\gamma) \leq s_t^*$$

for $zn_t = 0$ (i) implies $l_{1,t} > 0$ and $w_t^M < w_t^{M,*}$. This leads to

$$s_t = w_t^M < w_t^{M,*} \leq s_t^*$$

■

Notice that together Claims 1 and 2 (viii) imply that whenever two countries trade the ratio of male wage and rental rate (weakly) lower in the capital rich country, i.e.

$$k_t < k_t^* \qquad \Rightarrow \qquad \frac{w_t^M}{r_t} \leq \frac{w_t^{M,*}}{r_t^*} \tag{41}$$

Claim 1 implies that, unless the closed economy, a country with an initially lower per household capital stock, can catch up with a capital rich country in finite time under international trade. At the same time Claim 2 (ix) means that, just as in the closed economy, there is no true leapfrogging, i.e. a country that is capital scare initially does not enjoy a higher capital stock than a country that is initially relatively capital abundant.

In the following we will compare the key variables from the autarky and the trade equilibrium. To this aim we denote all variables from the autarky equilibrium with a superscript ^{A} (e.g. κ_t^A), all others are variables of the free trade equilibrium (e.g. κ_t).

Definition *For $k_t < k_L$ define $g(k_t)$ as the capital stock $g(k_t) > k_t$ for which prices equalize in two closed economies one endowed with k_t, the other with $g(k_t)$, i.e. g* is implicitly defined by

$$\frac{p_{2,t}^A(k_t)}{p_{1,t}^A(k_t)} = \frac{p_{2,t}^A(g(k_t))}{p_{1,t}^A(g(k_t))}$$

Further, let g^{-1} be the inverse of g so that $g^{-1}(g(k)) = k$ for all $k \in [0, k_L)$.

With Figure 3, the definition of $\hat{k}$ implies that the pair $(k_t, g(k_t))$ lies on a horizontal line that crosses the price schedule twice in the corresponding price levels.

With these notations we can formulate the following Claim.

Claim 3 *For capital scarce countries exporting the mental-labor-intensive good (X_1), trade decreases the ratio κ_t while for capital abundant countries exporting the mental-labor-intensive good (X_1), trade increases the ratio κ_t. More precisely, we state*

$$\begin{array}{lllll}
(i) & k_t < k_L \;\; \textit{and} \;\; k_t^* \in (k_t, g(k_t)) & \Rightarrow & \kappa_t^A \leq \kappa_t \\
(ii) & k_t < k_L \;\; \textit{and} \;\; k_t^* \notin (k_t, g(k_t)) & \Rightarrow & \kappa_t^A \geq \kappa_t \\
(iii) & k_t \in [k_L, k_M] & \Rightarrow & \kappa_t^A \geq \kappa_t \\
(iv) & k_t \in [k_M, k_H] & \Rightarrow & \kappa_t^A \leq \kappa_t \\
(v) & k_t > k_H \;\; \textit{and} \;\; k_t^* \in (g^{-1}(k_t), k_t) & \Rightarrow & \kappa_t^A \geq \kappa_t \\
(vi) & k_t > k_H \;\; \textit{and} \;\; k_t^* \notin (g^{-1}(k_t), k_t) & \Rightarrow & \kappa_t^A \leq \kappa_t
\end{array}$$

where k_M is defined by

$$k_M = \kappa_M \max\left\{1, (1-\gamma)2 - \gamma\frac{b}{(1-\alpha)a\kappa_M^\alpha}\right\} \tag{42}$$

and κ_M solves (20)-(21).

Proof. See appendix. ■

A quick look at Figure 3, which represents relative prices as a function of the capital stock shoes that autarky prices $p^A_{2,t}/p^A_{1,t}$ in Home are higher than in Foreign in the cases (ii), (iii), (iv), and (vi) so that Home imports X_2 and exports X_1 in these cases. If its capital stock k_t falls short of the threshold k_M, this means that trade weakly reduces its capital-mental labor ratio in the first sector (κ_t). In, instead, its capital stock k_t exceeds the threshold k_M trade increases it. In cases (i) and (v) autarky prices $p^A_{2,t}/p^A_{1,t}$ in Home are lower than in Foreign, Home imports X_1 and exports X_2 and the effect of trade on κ_t is reversed.

The intuition of the finding in Claim 3 is the following. A capital poor country produces X_2 using primarily the factor physical (male) labor. Now suppose the X_2-sector in this country contracts because it engages in trade and imports X_2. This means that male workers are driven out of the X_2-sector and into the X_1-sector, thus reducing the capital labor share κ_t in the first sector. If instead the X_2-sector expands because the country exports X_2, male workers leave the first sector and the ratio κ_t increases.

A contrary effect happens in a capital scarce country that produces X_2 using primarily capital. If such a country imports X_2, its X_2-sector contracts and capital is reallocated from the second to the first sector, increasing κ_t. If the country exports X_2 instead, its X_2-sector expands and capital is shifted from the first to the second sector, decreasing the ratio κ_t.

Notice that the additional effect of changing female labor participation does not alter this picture. This is because female labor force participation reacts to the gender wage gap and thus to changes in κ_t and, being subject to this determinant cannot overturn the original effect.

Claim 3 establishes statements concerning the effect of trade on the ratio κ_t in different constellations of trade partners. Under some parameter restrictions these translate directly into statements on female labor force participation (FLFP). The relevant conditions are, first, that advantages for male labor is not too small

$$b \geq (1-\alpha)a \tag{43}$$

and second, that the weight on children exceeds a minimum[13]

$$\gamma \geq \bar{\gamma} = \frac{\alpha^{2\alpha}}{(1-\alpha)^{\alpha}(1+\alpha)^{1+\alpha} + 2\alpha^{2\alpha}} \tag{44}$$

Under these assumptions we can formulate the following

Proposition 1 *Assume (43) and (44) hold. Then, if $k_t < k_M$ and Home exports the mental-labor-intensive good X_1, trade decreases FLFP, $(1 - zn_t)$. Conversely, if $k_t > k_M$ and Home exports the mental-labor-intensive good (X_1), trade increases FLFP. In particular,*

$$\begin{array}{rlcl}
(i) & k_t < k_L \ \ \textit{and} \ \ k_t^* \in (k_t, g(k_t)) & \Rightarrow & zn_t^A \geq zn_t \\
(ii) & k_t < k_L \ \ \textit{and} \ \ k_t^* \notin (k_t, g(k_t)) & \Rightarrow & zn_t^A \leq zn_t \\
(iii) & k_t \in [k_L, k_M] & \Rightarrow & zn_t^A \leq zn_t \\
(iv) & k_t \in [k_M, k_H] & \Rightarrow & zn_t^A \geq zn_t \\
(v) & k_t > k_H \ \ \textit{and} \ \ k_t^* \in (g^{-1}(k_t), k_t) & \Rightarrow & zn_t^A \leq zn_t \\
(vi) & k_t > k_H \ \ \textit{and} \ \ k_t^* \notin (g^{-1}(k_t), k_t) & \Rightarrow & zn_t^A \geq zn_t
\end{array}$$

where k_M is defined by (42).

Proof. See appendix. ■

For the cases (i) and (iv) - (vi) Proposition 1 has direct implications on wages and, by consequence, on capital accumulation of Home's and Foreign's economies. In particular, under the conditions of (i), (vi), and (v) trade increases male and female wages in Home, decreases Home's the gender gap and per household savings - i.e. capital accumulation - rises. Conversely, under the conditions (iv) trade decreases wages, widens the gender gap and depresses per household capital accumulation.[14] To see this, use (8) and (9), which lead to

$$k_{t+1} = z\frac{s_t}{zn_t} = \begin{cases} zw_t^M & \text{if } \ zn_t = 1 \\ z\frac{1-\gamma}{\gamma}w_t^F & \text{if } \ zn_t < 1 \end{cases} \tag{45}$$

In case (i) applies Home exports X_2 and $p_{2,t}^A \leq p_{2,t}$ so that $(w_t^M)^A = p_{2,t}^A b \leq p_{2,t} b \leq w_t^M$. By Claim 3 (i) trade closes the gender gap so that $(w_t^F)^A \leq w_t^F$ as well. Thus, by (45) trade increases capital accumulation.

[13] $\alpha \in (0,1)$ implies $\bar{\gamma} < 1/2$; for $\alpha = 1/2$ the thresholds is $\bar{\gamma} = 0.218$, for $\alpha = 1/3$ it is $\bar{\gamma} = 0.215$.
[14] Under conditions (ii) and (iii) the impact is ambiguous.

In case (iv) or (vi) applies, Home exports X_1 and $p_{1,t}^A \leq p_{1,t}$. As $l_{1,t}, l_{1,t}^A > 0$ and, by Claim 3, $\kappa_t^A \leq \kappa_t$ this means that male and female wages increase and the gender gap closes. Again, by (45) trade increases capital accumulation.

In case (v) applies, $k_t^* < k_t$ holds so that by Claim 2 (iv) $l_{1,t} > 0$. As $p_{1,t}^A \geq p_{1,t}$ and - by Claim 3 (v) - $\kappa_t^A \geq \kappa$ this implies $(w_t^M)^A \geq w_t^M$. Again by $\kappa_t^A \geq \kappa$ the gender gap widens so that $(w_t^F)^A \geq w_t^F$ as well. Hence, by (45) in this case trade increases capital accumulation.

3 Concluding Remarks

This paper reveals the impact of trade on household's tradeoff between fertility and female labor force participation. Moreover, our theory suggests that international trade has affected the evolution of economies asymmetrically. Initial differences in capital labor ratios across countries and factor intensity across sectors determine specialization patterns which can further intensify asymmetries of capital labor ratios. A novel feature of our model is that due to differences in factors substitution across sectors capital intensity switches as economies develop. This feature produces a non-monotonic relation between the specialization pattern and the stock of capital of the trade partner.

As a result of international specialization some countries will specialize in production of goods which is particularly suitable for female workers. Somewhat surprisingly, the effect of the expansion of this "female sector" on female labor force participation is ambiguous. The reason is that, by general equilibrium forces, trade makes the importing sector contract and factors reallocate to the exporting sector. If capital is the main factor reallocated to the "female sector", female wages increase and female labor participation rises. If, however, primarily male workers enter the "female sector", this leads to a depression of wages and the exit female labor. Which of both effects prevails is determinant of the capital endowment of the country in question.

APPENDIX

Proof of Claim 3 (i) - (vi). Notice first that the preference structure (3) and technologies (1) and (2) domestic demand for X_2 is decreasing while domestic supply of X_2 is increasing in the relative price $p_{2,t}/p_{1,t}$ so that domestic excess supply (excess demand) of X_2 is increasing (decreasing) in $p_{2,t}/p_{1,t}$. Consequently, the following inequality of autarky price ratios

$$\frac{p_{2,t}^{A}}{p_{1,t}^{A}} < \frac{p_{2,t}^{*,A}}{p_{1,t}^{*,A}} \tag{46}$$

implies that Home exports X_2. The argument regarding excess demand and supply implies further that, as long as (46) holds, relative world prices are bounded by relative autarky prices, i.e.

$$p_{2,t}^{A}/p_{1,t}^{A} \leq p_{2,t}/p_{1,t} \leq p_{2,t}^{*,A}/p_{1,t}^{*,A}$$

Since final good price normalization (6) leads to $p_{2,t} = \theta^{\theta}(1-\theta)^{1-\theta}\left(p_{2,t}/p_{1,t}\right)^{\theta}$ and $p_{1,t} = \theta^{\theta}(1-\theta)^{1-\theta}\left(p_{2,t}/p_{1,t}\right)^{\theta-1}$, the latter inequalities imply $p_{1,t}^{A} \geq p_{1,t}$ and $p_{2,t}^{A} \leq p_{2,t}$. Together, we conclude

$$\frac{p_{2,t}^{A}}{p_{1,t}^{A}} < \frac{p_{2,t}^{*,A}}{p_{1,t}^{*,A}} \quad \Rightarrow \quad p_{1,t}^{A} \geq p_{1,t},\ \ p_{2,t}^{A} \leq p_{2,t},\ \text{ and Home exports } X_2. \tag{47}$$

$$\frac{p_{2,t}^{A}}{p_{1,t}^{A}} > \frac{p_{2,t}^{*,A}}{p_{1,t}^{*,A}} \quad \Rightarrow \quad p_{1,t}^{A} \leq p_{1,t},\ \ p_{2,t}^{A} \geq p_{2,t},\ \text{ and Home exports } X_1. \tag{48}$$

Proof of (i). Note that assumption $k_t < k_L$ implies $k_{1,t}^{A} = k_t$ and $l_{2,t}^{A} > 0$. By (20) and (21) (see also Figure 3) condition $k_t^{*} \in (k_t, g(k_t))$ implies that inequality (46) holds and (47) applies. In case that factor prices equalize we have $k_t < \bar{k}_t < k_t^{*}$ so that $\kappa_t^{A} \leq \bar{\kappa}_t = \kappa_t$ follows.

In case that factor prices do not equalize Claim 2 (i) applies and $k_{2,t} = 0$. As Home exports X_2 this implies $l_{2,t} > 0$ so that

$$w_t^{M} = p_{2,t}b \geq p_{2,t}^{A}b = (w_t^{M})^{A}$$

We distinguish the two cases $l_{1,t} > 0$ and $l_{1,t} = 0$. If $l_{1,t} > 0$ we have

$$p_{1,t}\left[(1-\alpha)a\kappa_t^{\alpha} + b\right] = w_t^{M} \geq (w_t^{M})^{A} = p_{1,t}^{A}\left[(1-\alpha)a(\kappa_t^{A})^{\alpha} + b\right]$$

so that, since $p_{1,t}^A \geq p_{1,t}$, we conclude $\kappa_t^A \leq \kappa_t$.

If instead $l_{1,t} = 0$ we have (again by Claim 2 (i)) $k_{1,t} = k_t$ so that

$$\kappa_t = \frac{k_t}{1 - zn_t} > \frac{k_t}{l_{1,t}^A + 1 - zn_t^A} = \kappa_t^A$$

This proves (i).

Proof of (ii). As in (i) assumption $k_t < k_L$ implies $k_{1,t}^A = k_t$ and $l_{2,t}^A > 0$. Condition $k_t^* \notin (k_t, g(k_t))$ implies

$$\frac{p_{2,t}^A}{p_{1,t}^A} > \frac{p_{2,t}^{*,A}}{p_{1,t}^{*,A}}$$

and (48) applies. As Home exports X_1 we have $l_{1,t} > 0$ by condition (18) and we distinguish the two cases $l_{1,t} < 1$ and $l_{1,t} = 1$.

If $l_{1,t} < 1$ we have $l_{2,t} > 0$ and

$$w_t^M = p_{2,t} b \leq p_{2,t}^A b = (w_t^M)^A$$

which implies

$$p_{1,t} \left[(1-\alpha) a \kappa_t^\alpha + b\right] = w_t^M \leq (w_t^M)^A = p_{1,t}^A \left[(1-\alpha) a (\kappa_t^A)^\alpha + b\right]$$

so that (as $p_{1,t}^A \leq p_{1,t}$) $\kappa_t^A \geq \kappa_t$.

If $l_{1,t} = 1$ we have $l_{2,t} = 0$ and

$$\kappa_t = \frac{k_t}{2 - zn_t} < \frac{k_t}{l_{1,t}^A + 1 - zn_t^A} = \kappa_t^A$$

which proves (ii).

Proof of (iii). For $k_t^* \in [k_L, k_H]$ prices under autarky and under trade equalize, there is no trade and the statement follows trivially. Thus, assume $k_t^* \notin [k_L, k_H]$, which implies $p_{2,t}^A / p_{1,t}^A > p_{2,t}^{*,A} / p_{1,t}^{*,A}$ and (48) applies. Now consider four different cases.

First, $l_{2,t}, k_{2,t} > 0$, which implies that (20) and (21) hold and thus $\kappa_t = \kappa_t^A$.

Second, $l_{2,t} > 0$ and $k_{2,t} = 0$, which implies

$$(1-\alpha)\frac{a}{b}\kappa_t^\alpha + 1 = \frac{p_{2,t}}{p_{1,t}} \leq \frac{p_{2,t}^A}{p_{1,t}^A} = (1-\alpha)\frac{a}{b}(\kappa_t^A)^\alpha + 1$$

so that $\kappa_t \leq \kappa_t^A$.

Third, $l_{2,t} = 0$ and $k_{2,t} > 0$, which implies

$$\alpha\kappa_t^{\alpha-1} = \frac{p_{2,t}}{p_{1,t}} \leq \frac{p_{2,t}^A}{p_{1,t}^A} = \alpha(\kappa_t^A)^{\alpha-1}$$

so that $\kappa_t \geq \kappa_t^A$. This, however, leads to

$$\frac{k_t}{l_{1,t} + 1 - zn_t} > \frac{k_{1,t}}{l_{1,t} + 1 - zn_t} = \kappa_t \geq \kappa_t^A$$

so that with (42) $k_t > k_M$, contradicting the assumption $k_t \in [k_L, k_M]$.

Fourth, $l_{2,t} = k_{2,t} = 0$, which leads to

$$\kappa_t = \frac{k_t}{2 - zn_t} \leq \frac{k_M}{2 - zn_t} \leq \frac{k_M}{2 - zn_M} = \kappa_t^A$$

where the second inequality follows from (42) and

$$zn_M = \min\left\{\gamma\left(2 + b/((1-\alpha)a(\kappa_t^A)^\alpha)\right), 1\right\}$$

Proof of (iv). For $k_t^* \in [k_L, k_H]$ prices under autarky and under trade equalize, there is no trade and the statement follows trivially. Thus, assume $k_t^* \notin [k_L, k_H]$, which implies $p_{2,t}^A/p_{1,t}^A > p_{2,t}^{*,A}/p_{1,t}^{*,A}$ and (48) applies. Again, we consider four different cases.

First, $l_{2,t}, k_{2,t} > 0$, which implies that (20) and (21) hold and thus $\kappa_t = \kappa_t^A$.

Second, $l_{2,t} > 0$ and $k_{2,t} = 0$, which implies

$$(1-\alpha)\frac{a}{b}\kappa_t^\alpha + 1 = \frac{p_{2,t}}{p_{1,t}} \leq \frac{p_{2,t}^A}{p_{1,t}^A} = (1-\alpha)\frac{a}{b}(\kappa_t^A)^\alpha + 1$$

so that $\kappa_t \leq \kappa_t^A$. This, however, leads to

$$\frac{k_t}{2 - zn_t} < \frac{k_t}{l_{1,t} + 1 - zn_t} = \kappa_t \leq \kappa_t^A$$

so that with (42) $k_t < k_M$, contradicting the assumption $k_t \in [k_M, k_H]$.

Third, $l_{2,t} = 0$ and $k_{2,t} > 0$, which implies

$$\alpha\kappa_t^{\alpha-1} = \frac{p_{2,t}}{p_{1,t}} \leq \frac{p_{2,t}^A}{p_{1,t}^A} = \alpha(\kappa_t^A)^{\alpha-1}$$

so that $\kappa_t \geq \kappa_t^A$.

Fourth, $l_{2,t} = k_{2,t} = 0$, which leads to

$$\kappa_t = \frac{k_t}{2 - zn_t} \geq \frac{k_M}{2 - zn_t} \geq \frac{k_M}{2 - zn_M} = \kappa_t^A$$

where the second inequality follows from (42) and

$$zn_M = \min\left\{\gamma\left(2 + b/((1-\alpha)a(\kappa_t^A)^\alpha)\right), 1\right\}$$

Proof of (v). Note that $k_t^* \in (g^{-1}(k_t), k_t)$ implies $k_t^* < k_t$ and $p_{2,t}^A/p_{1,t}^A < p_{2,t}^{*,A}/p_{1,t}^{*,A}$ so that (47) applies.

In case that factor prices equalize we have $k_t^* < \bar{k}_t < k_t$ so that $\kappa_t = \bar{\kappa}_t \leq \kappa_t^A$. Thus consider the where case factor prices do not equalize. Since Foreign exports X_1 we have $l_{1,t}^* > 0$ by (18) so that, with Claim 2 (ii) we conclude $l_{1,t} = 1$ (by $k_t^* < k_t$ Home and Foreign switch roles in Claim 2). As Home exports X_2 this implies $k_{2,t} > 0$ and hence with (47)

$$r_t^A = p_{2,t}^A a \leq p_{2,t} a = r_t$$

At the same time $k_{1,t}, k_{1,t}^A > 0$ so that

$$r_t^A = p_{1,t}^A \alpha a(\kappa_t^A)^{\alpha-1} \leq r_t = p_{1,t}\alpha a\kappa_t^{\alpha-1}$$

With $p_{1,t}^A \geq p_{1,t}$ from (47) this implies $\kappa_t \leq \kappa_t^A$.

Proof of (vi). Note that $k_t^* \notin (g^{-1}(k_t), k_t)$ implies $p_{2,t}^A/p_{1,t}^A > p_{2,t}^{*,A}/p_{1,t}^{*,A}$. We distinguish the two cases $k_{2,t} = 0$ and $k_{2,t} > 0$.

If $k_{2,t} = 0$ we have

$$\kappa_t \geq \frac{k_t}{2 - zn_t} > \frac{k_{1,t}^A}{2 - zn_t^A} = \kappa_t^A$$

If instead $k_{2,t} > 0$ we have

$$\frac{p_{2,t}^A}{p_{1,t}^A} = \alpha(\kappa_t^A)^{\alpha-1} > \frac{p_{2,t}}{p_{1,t}} = \alpha\kappa_t^{\alpha-1}$$

so that $\kappa_t^A < \kappa_t$, which proves (vi). ■

Proof of Proposition 1. We use superscript ^{A} for autarky variables while plain variables stand for those of the trade equilibrium. For all cases where $l_{1,t} > 0$ equation (9) is becomes (19). This is the case whenever Home exports X_1, so the statement statement follows from Claim 3 in the cases (ii), (iii), (iv), and (vi).

In the case (v) we have $k_t > k_t^*$. Under factor price equalization we have $\kappa_t^A \geq \bar{\kappa}_t \geq (\kappa_t^*)^A$ so that $zn_t^A \leq z\bar{n}_t = zn_t$. When factor prices do not equalize we have $l_{1,t} > 0$ by Claim 2 (iv) and the statement follows with Claim 3 (v).

In case (i) we have $k_t < k_t^*$. Under factor price equalization we have $\kappa_t^A \leq \bar{\kappa}_t \leq (\kappa_t^*)^A$ so that $zn_t^A \geq z\bar{n}_t = zn_t$. Thus, we are left with the case where $l_{1,t} = 0$ and factor prices do not equalize. By Claim 2 (i) we have $k_{1,t} = k_t$.

Show first that under $l_{1,t} = 0$ the gender gap w_t^M/w_t^F is increasing in world prices $\pi_t = p_{2,t}/p_{1,t}$. Assume not, i.e. w_t^M/w_t^F is decreasing in π_t. Since $w_t^M = p_{2,t}b = \theta^\theta(1-\theta)^{1-\theta}\pi_t^\theta$ is increasing in π_t this implies that $w_t^F = p_{1,t}(1-\alpha)(a/b)\kappa_t^\alpha = \theta^\theta(1-\theta)^{1-\theta}\pi_t^{\theta-1}(1-\alpha)(a/b)\kappa_t^\alpha$ is increasing so that $\kappa_t = k_t/(1-zn_t)$ is increasing and zn_t is increasing in π_t. This, by (9) implies that w_t^M/w_t^F is increasing in π_t, a contradiction.

Hence, w_t^M/w_t^F under $l_{1,t} = 0$ is locally maximal at maximal π_t, which is reached at κ_M that solves (20) and (21) (see Figure 3.), i.e.

$$\pi^M = (1-\alpha)\frac{a}{b}(\kappa_t^M)^\alpha + 1$$

Now assume that in the trade equilibrium with maximal relative prices (i) does not hold, i.e. $zn_t^A < zn_t$. This is equivalent to $(w_t^M/w_t^F)^A < w_t^M/w_t^F$ or

$$\frac{\pi^M}{\pi^A} > \left(\frac{\kappa_t}{\kappa_t^A}\right)^\alpha$$

As $\kappa_t^M > \kappa_t^A$ the expression to the left has the upper bound

$$\left(\frac{\kappa_t^M}{\kappa_t^A}\right)^\alpha > \frac{(1-\alpha)\frac{a}{b}(\kappa_t^M)^\alpha + 1}{(1-\alpha)\frac{a}{b}(\kappa_t^A)^\alpha + 1} = \frac{\pi^M}{\pi^A}$$

To establish a lower bound on the expression to the right combine (17), (21), and

(24) to derive

$$
\begin{aligned}
\frac{m_t^A}{1-zn_{1,t}^A} &= \frac{1}{1-zn_{1,t}^A}\frac{(2-zn_{1,t}^A)\frac{zn_{1,t}^A-\gamma}{zn_{1,t}^A-2\gamma}+\frac{1-\theta}{\theta}}{\frac{1-\theta}{\theta}\left[\frac{1}{1-\alpha}\frac{\gamma}{zn_{1,t}^A-2\gamma}+1\right]+1} \\
&> \frac{1-\alpha}{1-zn_{1,t}^A}\frac{(2-zn_{1,t}^A)\frac{zn_{1,t}^A-\gamma}{zn_{1,t}^A-2\gamma}+\frac{1-\theta}{\theta}}{\frac{1-\theta}{\theta}\frac{zn_{1,t}^A-\gamma}{zn_{1,t}^A-2\gamma}+1} > \frac{1-\alpha}{1-zn_{1,t}^A}
\end{aligned}
$$

where the last inequality holds since $\theta > 1/2$ by (18). This implies

$$
(1-\alpha)\frac{a}{b}(\kappa_t^A)^\alpha + 1 = \frac{1-\theta}{\theta}\frac{(a/b)(\kappa_t^A)^\alpha m_t^A + l_{1,t}}{(1-l_{1,t})} > \frac{1-\theta}{\theta}\frac{(a/b)(\kappa_t^A)^\alpha m_t^A + l_{1,t}}{(1-l_{1,t})}
$$

Combining upper and lower bound delivers

$$
\left(\frac{\kappa_t^M}{\kappa_t^A}\right)^\alpha = \left(\frac{m_t^A}{1-zn_t}\right)^\alpha > \left(\frac{m_t^A}{1-zn_t^A}\right)^\alpha > \left(\frac{1-\alpha}{1-zn_{1,t}^A}\right)^\alpha
$$

or, with (24)

$$
\kappa_t^M > \kappa_t^A\frac{1-\alpha}{1-zn_{1,t}^A} = \frac{1-\alpha}{\frac{1-2\gamma}{\kappa_t^A} - \gamma\frac{b/a}{1-\alpha}(\kappa_t^A)^{1-\alpha}}
$$

maximizing the denominator over κ_t^A leads to

$$
\kappa_t^M > \frac{(1-\alpha)(1+\alpha)}{\alpha(1-2\gamma)}\left((1+\alpha)\frac{b/a}{1-\alpha}\frac{\gamma}{1-2\gamma}\right)^{1/\alpha}
$$

Finally, κ_t^M (17), (21) establish an upper bound on

$$
\kappa_t^M = \left[\frac{\alpha}{1+(1-\alpha)a/b(\kappa_t^M)^\alpha}\right]^{1/(1-\alpha)} < \alpha^{1/(1-\alpha)}
$$

so that a necessary condition for $(w_t^M/w_t^F)^A > w_t^M/w_t^F$ to hold is

$$
\begin{aligned}
\alpha^{\alpha/(1-\alpha)} &> \left(\frac{(1-\alpha)(1+\alpha)}{\alpha(1-2\gamma)}\right)^\alpha (1+\alpha)\frac{b/a}{1-\alpha}\frac{\gamma}{1-2\gamma} \\
&> \left(\frac{(1-\alpha)(1+\alpha)}{\alpha(1-2\gamma)}\right)^\alpha (1+\alpha)\frac{\gamma}{1-2\gamma}
\end{aligned}
$$

implying

$$
\alpha^{\alpha(2-\alpha)/(1-\alpha)} > (1-\alpha)^\alpha(1+\alpha)^{\alpha+1}\frac{\gamma}{1-2\gamma}
$$

This contradicts (44) and proves the Proposition. ∎

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