An Innovation-Based Endogenous Growth Model with Equilibrium Unemployment

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Abstract

This paper studies the impact of labor market reforms on growth and unemployment. To do this, a no-shirking model of innovation-based growth is constructed, where labor market reforms are mainly aimed at improving job finding rate and increasing unemployment benefit. On a positive standpoint, we find that there always exists a positive relationship between growth and unemployment. Instead, from a normative standpoint, we find that the effectiveness of all labor market interventions crucially depend on how individuals discount future income. When consumers are impatient, we find that improving either labor market performance or unemployment benefit turns out to induce a permanent fall in both the BGP innovation rate and the equilibrium unemployment rate. When consumers are patient, results turn ambiguous with cases in which both growth and unemployment rates might increase.

Jel classification: O33, O34, J6

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1 Introduction

The question of whether growth can destroy employment has been widely debated in economics. According to Aricò (2003), it is possible to gather the existing theoretical studies on growth and unemployment along two distinct research lines. The first line focuses on institutional issues and leads to policy-oriented models (Gordon, 1995; Cahuc and Michel, 1996; Van Schaick and De Groot 1998; Daveri and Tabellini, 2000; and Peretto 2000 among others). The second line focuses on technical change and addresses the issue of growth and unemployment at either micro or macro level (Aghion and Howitt 1994; Acemoglu, 1997; Postel-Vinay, 1998; Cerisier and Postel-Vinay 1998; and Mortensen, 2003). In both cases though, no clear prediction about how growth affects unemployment is gained.

On the empirical side, results are far from being clear and confusion still persists over this issue. Muscatelli and Tirelli (2001), for instance, find negative correlation between growth and unemployment, while Caballero (1993) and Hoon and Phelps (1997) find a positive relationship between changes in unemployment and several measures of productivity growth. In contrast, Bean and Pissarides (1993) find no clear-cut correlation between productivity growth and unemployment whereas Aghion and Howitt (1992) point out that both high and low growth countries experience lower unemployment rate compared with those with intermediate growth experiences.

Thus far, the most fundamental papers have mainly based on the Pissarides' (1990) standard search model. This paper tries to investigate the existence of a possible linkage between growth and unemployment through a dynamic no-shirking model with creative destruction. Although criticized, efficiency-wages models are still considered a good tool to explain altogether real wages rigidity and unemployment.[1] In terms of a growth model though, a characteristic of the original Shapiro and Stiglitz (1984) model is its unrealistic prediction of a fall in the equilibrium unemployment rate in presence of productivity improvements owed to the fact that the no-shirking condition is independent on productivity improvement. One contribution of this paper is to link the no-shirking condition with an endogenous variable measuring the speed of productivity growth.[2] As the creation of more productive production processes pushes long-run growth, innov-

[1] For empirical evaluations of the efficiency-wages models see Danthine and Donaldson (1995) and Gomme (1999).

[2] Phelps (1994) pursue the same objective by introducing optimal saving and a nonlinear utility function into the standard Shapiro and stiglitz's framework. He finds that the accumulation of wealth increases the wage needed for maintaining the equilibrium level of effort unchanged.

ation generates obsolescence of some production units (creative destruction) and forces firms to release workers. The presence of labor market rigidity, modelled through the adoption of a positive (not infinite) rate of job finding, makes innovation to induce unemployment. As in the original Shapiro and Stiglitz (1984) model, we assume that the rate of job finding is exogenously given by institutional factors such as, for instance, labor/product market regulations or local rigidities.

The first contribution combining creative destruction and unemployment is that of Aghion and Howitt (1994). Technically speaking, their study can be split in two parts. The first part addresses the issue of growth and unemployment in a standard search model à la Pissarides (1990) with exogenous aggregate growth. They find that the relationship between growth and unemployment can be either positive or negative depending on the relative strength of two competing effect (namely, capitalization and creative destruction). In the second part they allow for a fully endogenous growth model by integrating their notorious version of vertical innovation process into the basic model. They find that the equilibrium unemployment rate is totally independent on the steady-state rate of growth, in the sense that any change in the frequencies of innovation generates no effect on employment. In contrast to Aghion and Howitt (1994), our model differ in that of predicting a positive relation between growth and unemployment. A permanent increase in the BGP rate of innovation generates a permanent increase in the equilibrium unemployment rate and vice versa. As a result, the positive result of the model is the absence of any possible trade-off between growth and unemployment in the long-run.

The model is solved for the balanced-growth path (BGP) and exploited to study the effect of some unemployment policy on growth and unemployment. Firstly, the paper focuses on the BGP effects of improving labor market performance by mean of an exogenous increase in the rate of job finding. We find that increased job finding rate increases the BGP per capita GDP while has mixed effects on long-run growth. We show that the effectiveness of such a policy on growth crucially depends on the rate at which individuals discount future profits. More specifically, in the presence of low subjective discount rate, improving labor market performance enhances long-run growth via a permanent increase in the BGP innovation rate, and vice versa. In terms of unemployment, results are as well driven by those on innovation and turn out to depends on the size of the subjective discount rate. When the subjective discount rate is relatively high, an increase in the rate of job finding generates a reduction in the BGP rate of unemployment. Unlike, when the subjective discount rate is relatively low, an increase in the rate of job finding has mixed effect on the BGP unemployment rate and a trade-off between growth and unemployment is likely to emerge in the long-run. Secondly, the paper studies what is the impact of an increase of the unemployment benefit. In terms of the growth performance, we find that the comparative statics analysis results are very similar to those of an increase in the exogenous job finding rate. Instead, in terms of unemployment, results are more clear-cut and rule out the existence of a possible trade-off between growth and unemployment.

The paper is organized as follow. Section 2.2 reviews the most important contributions of efficiency-wages unemployment and growth. Section 2.3 setups the model. Section 2.4 displays the main properties of the BGP equilibrium. Section 2.5 presents the comparative statics analysis and provides some possible economic interpretation of the most important results. Finally, Section 2.6 concludes.

2 Efficiency wages in a growing economy

Up to now, only few papers have used efficiency-wages scheme for growth studies. The first approach to growth with efficiency-wages is the paper by Van Schaik and De Groot (1998). In this contribution, the Authors build on the R&D-based growth model of Smulders and Van der Klundert (1995) where efficiency-wage unemployment occurs because effort in one sector of the economy (in the sector producing intermediate goods) is positively related to the relative wage paid in the rest of the economy. In their paper a dual labor market based à la Katz and Summers (1989) is combined with a process of in-house R&D using only high-wage workers. They find that an increase in the elasticity of substitution between product varieties reduces market power, increases long-run growth but reduces labor employment. Moreover, in presence of barriers to entry, they find that removing entry impediments lowers growth and induces mixed effects on equilibrium unemployment.

Meckl (2001) examines the long-run implications of efficiency-wages induced unemployment within a general-equilibrium model of endogenous growth with creative destruction. In his framework, efficiency-wages unemployment occurs because effort in one sector of the economy is positively related to the relative wage paid by that sector in the way proposed by Katz and Summers (1989). The starting point of this study is the oddly prediction of the Van Schaik and De Groot's (1998) model that wage differentials have no effect on labor productivity, which makes the wage structure of the economy in the presence of a homogeneous workforce unexplained by the model. To skip the hurdle, Meckl adopt the Arkelof's (Arkelof (1982) and Arkelof and Yellen (1990)) idea of fairness norm and builds an R&D based-endogenous growth model in the spirit of Aghion and Howitt (1992, 1998). The agent's decision whether to exert or not exert effort is driven by the distance separating the wage rate offered by the firm and a reference wage given by the average wage of the economy. In the long-run, the Author finds that the sign of the correlation between the rate of growth and the rate of unemployment is ambiguous in general, but always strictly related to the sign of the intersectoral wage differentials. More Specifically, the model predicts a positive relationship between growth and unemployment if the research sector is the high wage sector of the economy, while a negative relationship if the research sector is the low wage sector.

Meckl (2002) analyzes the relation between long-run growth and unemployment induced by efficiency wages and how the emergence of a growth–unemployment trade-off is linked to intersectoral wage differentials. He finds that both growth and unemployment depends on the intersectoral allocation of labor that the relation between the growth and equilibrium unemployment is ambiguous in general. He also find that the importance of wage differentials for the growth-unemployment trade-off is preserved when differentiating between skilled and unskilled labor and that higher minimum wages for unskilled labor raise growth and unskilled unemployment, while possibly reducing unemployment of skilled labor.

Mendez (2002) uses an efficiency-wages mechanism to explain several economic issues related to growth and wage inequality. In particular, the paper tries to provide a theoretical explanation to both the rising within-group wage inequality and the volatility of earnings. The model used is based on a dual labor market, that differs from the aforementioned papers owed to the presence of a non-homogeneous workforce. The paper demonstrates that unbiased innovation can contribute to explain within group wage inequality but says nothing on equilibrium unemployment.

Brecher et al. (2002) studies the long-run relationship between unemployment and growth through an dynamical efficiency-wage model with labor-augmenting technical progress. In order to keep the analysis as general as possible, the Authors assume that the rate of productivity growth is either exogenous or endogenous, and find that in both cases the key results of the Shapiro and Stiglitz (1983) analysis without growth are preserved. They find that an exogenous increase in the growth rate may raise the rate of efficiency-wage unemployment, while a permanent raise in the labor force may reduce the unemployment rate in the endogenous-growth case. In presence of endogenous growth, the paper shows that an increase in unemployment benefits raises the unemployment rate and lowers the rate of growth.

Palokangas (2003) studies the impact of improving labor market competition on economic growth and unemployment. In doing so, he presents an R&D-based endogenous growth model in which unemployment is caused by efficiency wages and union-firm bargaining. He shows that the effectiveness of labor market intervention are strictly related to the strength of union. When unions are initially very strong, he finds that regulation increases only the workers' profit share and has no impact on employment and growth. Instead, when union power is initially low, labour market regulation decreases employment and current consumption but fosters economic growth. The mechanism leading the positive impact on the growth performance relies on the firms' need to escape the raise in the labor cost through R&D-based productivity gains.

Finally, Nakajima (2006) examine the relationship between indeterminacy and unemployment insurance. The framework adopted is that of the one-sector neoclassical growth model except that the workers effort is imperfectly observable by firms. He shows that the less unemployment insurance is, the more likely equilibrium is to be indeterminate. In terms of growth and unemployment though, the paper is silent and does not bring the reader to a clear conclusion. In fact, reading between the lines of the main propositions of the paper, no clear-cut result emerges. The interesting aspects of the Nakajima's contributions lays on the extension of the Alexopoulos (2004) version of the efficiency-wages model to a growing, although exogenously, economy.

3 The model economy

The model is set in continuous time. The production side of the economy consists of three sectors: a final-good sector, an intermediate-goods sector, and a research sector. The final-good sector produces a single homogeneous output that is used only for consumption. The intermediate sector consists of a continuum of industries, indexed by $i \in [0, 1]$, each one producing with vertically differentiated production process. The final-good output is the numeraire of the model.

Technological progress entails improvement in labor productivity, which in turn rises total factor productivity in final output sector. In each industry i

firms are distinguished by the vintage of technology they use to produce the ith intermediate. We denote the vintage of technology used by the ith industry at time t by $j\,(i,t)$, with higher values of the index j denoting more efficient production technology. To learn how to introduce the $j+1$ vintage, firms participate in innovative R&D races. In general, the discoverer of the $j+1$ vintage gets an everlasting patent protection that makes him to be the unique allowed to freely use it.

Labor is the only factor used in all production activities as well as research. In contrast to standard Schumpeterian literature, we assume the presence of a detecting technology that prevents firms to monitoring workers' and researchers' effort perfectly. Over time, GDP improves as innovations push upwardly the technology frontier. We focus on the steady-state equilibrium.

3.1 Consumers and workers

At each instant of time, the economy is populated by a continuum $N(t)$ of identical, infinitely-lived consumers/workers who dislike putting effort. Population grows over time at an exogenous rate n, so that at each point in time t, the size of total population is $N(t) = N\,(0)\,e^{nt}$. Consumers are either employed or unemployed. When employed, workers can decide to either exert effort or not depending on the level of the current wage offered by firms. We assume that firms cannot perfectly monitoring workers' effort and that monitoring technology is such that there exists an exogenous probability $q \in [0,1]$ that a worker engaged in shirking is caught and fired. To avoid firing, workers must exert effort but they also derive some disutility. We model effort ε as a discrete variable depending of whether the individuals decide to exert effort ($\varepsilon = 1$) or not ($\varepsilon = 0$).

Individuals are allowed to save or borrow in the financial market. To simplify the model, we assume that individuals are risk neutral with instantaneous utility linear function of consumption c and effort $\varepsilon\chi$, where $\chi > 0$ is a measure of the individual's disutility in terms of final-output units. Denoting expectation at time 0 by E_0, the representative consumer/worker intertemporal utility function reads:

$$U\,(c,\varepsilon) \equiv E_0 \int_0^\infty e^{-\rho t}\,[c\,(t) - \varepsilon\chi]\,dt \tag{1}$$

where $\rho > n$ denotes the subjective discount rate.

The intertemporal maximization problem consists of maximizing [1] subject to intertemporal budget constraint:

$$A(t) + Z(t) = \int_t^{\infty} c(\tau)\, e^{-[R(\tau)-R(t)]} d\tau \tag{2}$$

where $A(t) \equiv \int_t^{\infty} a(\tau)\, e^{-[R(\tau)-R(t)]} d\tau$ is the present value of the representative consumer/worker's financial assets at time t and $Z(t) \equiv \int_t^{\infty} z(\tau)\, e^{-[R(\tau)-R(t)]} d\tau$ is the present value of labor income at time t, with a and z denoting the consumer/worker's financial assets and labor income respectively, and $R(\tau) \equiv \int_0^{\tau} r(s)$ ds is the cumulative interest rate up to time τ (with $R'(\tau) = r(\tau)$)

Labor income z depends on worker's status. It might include either the current wage rate w when employed or the unemployment benefits b when unemployed, i.e., $z = \{w, b\}$.[3] Let now assume that z evolves over time according to two independent Poisson processes, q_w and q_b, governing, respectively, the process of job destruction and that of job finding. Following Wälde (1999) and Sonnewald and Wälde (2006), the evolution of labor income z can then be represented by the following stochastic differential equation:

$$dz = -(w-b)\, dq_w + (w-b)\, dq_b \tag{3}$$

where $(w-b)$ represent the finite jump-term of each Poisson process.

Poisson process, q_w, measures how often a consumer/worker leaves the status of employed owed to job destruction. It takes place at an exogenous rate $I + (1-\varepsilon)\, q$, where I denotes the rate of job destruction related to the economy-wide process of innovation (i.e., creative destruction), and $(1-\varepsilon)\, q$ is that related to the firms' monitoring technology. Similarly, Poisson process, q_b, measures how often a consumer/worker leaves the status of unemployed owed to job creation. It takes place at an exogenous rate $\mu > 0$ reflecting all the institutional characteristics of the labor market such as, for instance, the employment protection legislation, labor/product market regulations or local rigidities. Observe that μ can also be interpreted as the probability of finding a job at each instant of time and $1/\mu$ as the expected duration of the unemployment status.

3.1.1 The value function

In this section we resort to the stochastic dynamic programming to solve the consumer's intertemporal optimization program. The state space is described by a and z. From [2] it is easy to verify that wealth evolves according to:

[3] Throughout the analysis we exclude tax distortion by assuming lump-sum taxation to finance unemployment benefit. Moreover, in order to index the unemployment benefit, we also assume that in the BGP it increases at the same rate as productivity.

$$da = [(r - n)\, a + z - c]\, dt \tag{4}$$

As a result, the program consists of maximizing [1] subject to [3] and [4]. Defining the value function by $V(a, z)$, the Bellman equation takes the following structure[4]:

$$\rho V(a, z) = \max_{c,\varepsilon} \left\{ \begin{array}{c} c - \varepsilon\chi + V_a [(r - n)\, a + z - c] + \\ + [V(a, z + b - w) - V(a, z)] [I(t) + (1 - \varepsilon)\, q] \\ + [V(a, z + w - b) - V(a, z)]\, \mu \end{array} \right\} \tag{5}$$

The first-order condition reads:

$$V_a = 1 \tag{6}$$

The economic rationale of [6] is the following. Current utility from a marginal increase in consumption, i.e., $u_c(c, \varepsilon)$, must equal future utility from a marginal increase in wealth measured by the costate variable V_a. With linear preferences, the current utility of an additional unit of consumption equals one and [6] follows straightforwardly.

Eq. [6] makes consumption a function of the state variable, $c = c(a)$. As a result, replacing control variable by their optimal values $c(a)$, the maximized Bellman equation reads:

$$\begin{aligned} \rho V(a, z) \;\; = \;\; & c(a) - \varepsilon\chi + V_a [(r - n)\, a + z - c(a)] \\ & + [V(a, z + b - w) - V(a, z)] [I(t) + (1 - \varepsilon)\, q] + \\ & + [V(a, z + w - b) - V(a, z)]\, \mu \end{aligned} \tag{7}$$

Finally, in order to avoid bang-bang solution for the intertemporal consumption problem, risk-neutral preferences implies $r = \rho$

3.1.2 The no-shirking condition and the asset equations

Each worker selects its effort level to maximize his expected life-time utility. This means comparison of the utility from shirking with the utility of not shirking. Define the present-discounted value of the expected income stream of an employed

[4]The details of the analytical derivation of the Bellman equation are collected in Appendix A.

worker by $W \equiv V(a, w)$ and the present-discounted value of the expected income stream of an unemployed worker by $U \equiv V(a, b)$. For an employed worker the probability of getting a job is nil (i.e., $\mu = 0$) whereas that of loosing a job is positive and equal to $I + (1 - \varepsilon) q$. Plugging $r = \rho$ and first order condition [6] into [7], the present-discounted value of the expected income stream of an employed worker (regardless of her tendency to shirk) can be described by the asset equation:

$$\rho W = w - \varepsilon\chi + (\rho - n) a + [I + q(1 - \varepsilon)](U - W) \tag{8}$$

Define W_S as the expected life-time utility of a shirking worker (i.e., Eq. [8] when $\varepsilon = 0$), and W_{NS} as the expected life-time utility of a non-shirking worker (i.e., Eq. [8] when $\varepsilon = 1$). The worker will choose to shirk or not to shirk if and only if $W_{NS} \geq W_S$. This is the famous no-shirking condition (NSC) after Shapiro and Stiglitz (1984) which says that the worker will choose to exert effort if and only if the wage rate is set high enough to discourage workers from shirking. In order to get the effort-enhancing wage rate of the economy, we compare the worker lifetime utility for $\varepsilon = 1$ and $\varepsilon = 0$ and obtain the following no-shirking condition (NSC):

$$w \geq \rho U - (\rho - n) a + (\rho + I + q) \chi / q \tag{9}$$

The economic intuition behind NSC [9] is easy gained. In presence of a penalty associated with being caught shirking, individuals will choose not shirking if and only if the ongoing wage rate is sufficiently high to induce workers to exert effort. In such a situation, unemployment acts as a self-disciplining device: if after being fired an individual could immediately be hired by another firm, then $U = W_S$ should hold and $W_{NS} \geq W_S$ should never be satisfied. As a result, variable U is a key variable leading firms' wage setting behavior.

Let's now analyze the case of an unemployed worker. For an unemployed worker the probability of loosing a job is nil (i.e., $I + q(1 - \varepsilon) = 0$) while that of finding a job is positive and equals μ. The present-discounted value of the income stream can then be described by the following asset equation:

$$\rho U = b + (\rho - n) a + \mu (W - U) \tag{10}$$

Finally, comparison of the lifetime utility of a non-shirking worker (i.e., Eq. [8] with $\varepsilon = 1$) with that of an unemployed worker (Eq. [10]) gives the following aggregate NSC[5]:

$$w \geq \hat{w} \equiv b + \chi + (\rho + I + \mu) \frac{\chi}{q} \tag{11}$$

As usual for no-shirking models, aggregate NSC [11] can be taken as a measure of labor supply. Observe that the effort-enhancing wage rate, $\hat{w}$, is higher: (i) the higher the unemployment benefit, b, (ii) the higher the disutility of effort in terms of consumption, χ, (iii) the higher the rate of time preference ρ, (iv) the less efficient the monitoring technology, q, and (v) the higher the job turnover (the sum of job finding μ and job destruction I).

3.2 Production

Final output is produced under perfect competition. At every instant in time t, final output producers ensemble all the available intermediate goods according to the following constant returns to scale production function.

$$Y(t) \equiv \exp\left\{ \int_0^1 \ln\left[x(i,t)\right] di \right\} \tag{12}$$

where $Y(t)$ denotes the final output (or GDP) and $x(i,t)$ is the quantity of the ith intermediate used to produce the final output.

Each final output producer will choose $x(i,t)$ to minimize costs. The solution of this minimization problem leads to demand function

$$p(i,t) = \frac{Y(t) P(t)}{x(i,t)} \tag{13}$$

where $P(t) \equiv \exp\left\{ \int_0^1 \ln\left[p(i,t)\right] di \right\}$ is the aggregate price index and $p(i,t)$ is the price of the ith intermediate at time t. Since we let the final-output ot be the numeraire, in the remainder of the paper $P(t) = 1$ (i.e., $\int_0^1 \ln\left[p(i,t)\right] di = 0$) always holds.

In the intermediate sectors, production technology is CRS and uses labor as primary input according to:

$$x(i,t) = \lambda^{j(i,t)} L_X(i,t) \tag{14}$$

[5] To get Eq. [11], solve [8] (when $\varepsilon = 1$) and [10] for W and U. Then, plug U into [9] and solve the resulting expression for w. The result is right Eq. [11].

where $\lambda > 1$ is the innovation-jump separating two consecutive technology vintages, and $L_X(i,t)$ is the amount of workers hired by the ith industry at time t.

According to [14], technological progress entails improvement in labor input, which in turn rises total factor productivity in final outputs sector. As in the Shapiro and Stiglitz's (1984) framework, we assume that workers contribute one unit of labor if they do not shirk, or nothing otherwise. We also assume that when innovative firms offer a wage packages given by [11], they do not need to wait for finding a worker; i.e., the time required for finding a worker is nil.

Once an innovation occurs, the discoverer of the most productive process become the market leader by displacing the previous incumbent. Each leader compete à la Bertrand with unequal marginal cost depending on the technology index and the efficiency wage $\hat{w}$. Producing one unit of output implies a marginal cost of $\hat{w}(t)/\lambda^{j(i,t)}$. As the quality leader of each industry i operates with a downward-sloping price schedule given by [13], Bertrand competition leads each technological leader to maximize her profit by limit pricing all the followers present in the market. This is obtained by setting a price equal to the marginal cost of the most efficient follower:[6]

$$p(i,t) = \frac{\lambda \hat{w}(t)}{\lambda^{j(i,t)}} \tag{15}$$

By setting a price equal to [15], each quality leader captures the entire industry market and will perform a flow of sells equal to $x(i,t) = \lambda^{j(i,t)-1} Y(t)/\hat{w}(t)$.[7] According to [15], all leaders have the same profit flow given by

$$\pi(t) = \left(1 - \frac{1}{\lambda}\right) Y(t) \tag{16}$$

whereas all followers cannot do better than exit the industry.

[6] Observe that the limit pricing behavior summarized by [15] is source of asymmetry in the size of firms. As will be verified later on in the paper, such a characteristic does not affect the symmetric structure of the BGP equilibrium.

[7] It is worth-noting that no-shirking wage rate $\hat{w}$ is not constant over time but grows at the same rate as average productivity. As the chosen numeraire of the model is the final good and $\int_0^1 \ln[p(i,t)]\, di = 0$ must hold at each instant t, taking logs and integrating between 0 and 1 Eq. [15], we get $\int_0^1 \ln[\lambda \hat{w}(t)]\, di - \int_0^1 \ln\left[\lambda^{j(i,t)}\right] di = 0$, which in turn can be rewritten as $\ln[\lambda \hat{w}(t)] = It \ln \lambda$. At each instant t (not only in the BGP equilibrium) efficiency-wage $\hat{w}(t)$ grows at rate $I \ln \lambda$.

3.3 R&D Races

In each industry i there is free entry into R&D race and all firms have the same R&D technology. The instantaneous probability of innovation of a firm by time $t + dt$ is $I(i,t)\,dt$ whereas the probability of not having innovated is $1 - I(i,t)\,dt$. In contrast to standard Schumpeterian growth models (Grossman and Helpman, 1991; Aghion and Howitt, 1992), we assume that R&D difficulty increases over time according to a difficulty function $K(t)$ and that the industry-wide instantaneous probability of innovation at time t is:

$$I(i,t) = \frac{L_R(i,t)}{aK(i,t)} \tag{17}$$

where $L_R(i,t)$ is the industry-wide R&D employment at time t, $K(i,t)$ is a measure of the R&D difficulty of industry i, and $a > 0$ is an exogenous parameter measuring the productivity of R&D workers.

The $K(i,t)$ expression in the denominator of [17] is not new to the endogenous growth literature and is intended to escape the "scale effect" critique of Jones (1995). We assume that as the economy grows, $K(i,t)$ increases over time and innovating becomes more difficult. More specifically, throughout the paper we assume that R&D difficulty does not change between industries and equals $K(i,t) = \kappa N(t)$, where $\kappa > 0$ is an exogenous parameter governing R&D difficulty growth.[8]

Let $v(i,t)$ denote the expected discounted profit for winning a R&D race in industry i at time t. Each R&D firm chooses its labor input to maximize its expected profits. Perfect competition then implies the following free entry condition to hold

$$v(i,t) \begin{cases} \leq a\kappa N(t)\,\hat{w}(t) & \text{if} \quad I(i,t) = 0 \\ = a\kappa N(t)\,\hat{w}(t) & \text{if} \quad I(i,t) > 0 \end{cases} \tag{18}$$

When [18] holds with equality, $I(i,t) > 0$ and firms are globally indifferent concerning how much labor to devote to R&D.

To finance R&D, firms issue equity claims on the flow of profits generated by the innovation. Claims on particular firms are risky assets which current valuation is given by the stock market. At each point in time t, then, investors

[8] The specification adopted for $K(t)$ can be justified by saying that R&D difficulty is proportional to the size of global market because of the existence of organizational costs related to product distribution (Dinopoulos and Segerstrom (1999)) or the existence of costs to protect firm's intangible assets from misappropriations (Dinopoulos and Syropoulos (2006)).

must solve a portfolio allocation problem among shares in a variety of profit-maximizing firms and among riskless bonds. As there is a continuum of industries and the returns to engaging in R&D races are independently distributed across firms and industries, the risk attached to every single equity is idiosyncratic and each investor can completely diversify away risk by holding a diversified portfolio of stocks. Thus, over a time interval dt, the shareholder receives a dividend $\pi\left(i,t\right)dt$, and the value of the firm appreciates by $\dot{v}\left(i,t\right)dt$ in each industry. Because each quality leader is targeted by firms conducting innovative R&D, the shareholder suffers a loss of $v(i,t)$ if further innovation occurs. This event occurs with probability $I(i,t)dt$, whereas no innovation occurs with probability $1-I(i,t)dt$. Efficient financial markets make the expected rate of return from holding a stock of a quality leader equals to the riskless rate of return $r(t)dt$ that can be obtained through complete diversification. As a result, no arbitrage condition into capital market requires:

$$\frac{\pi\left(i,t\right)}{v\left(i,t\right)}+\frac{\dot{v}\left(i,t\right)}{v\left(i,t\right)}=\rho+I\left(i,t\right) \tag{19}$$

Plugging [16] into [19] and combining the result with [18] yields

$$\hat{w}\left(t\right)a\kappa=\frac{\left(1-1/\lambda\right)y\left(t\right)}{\rho+I-n} \tag{20}$$

where $y\left(t\right)\equiv Y\left(t\right)/N\left(t\right)$ denotes the level of per capita GDP at time t, and where, based on free-entry [18], we have used $\frac{\dot{v}(i,t)}{v(i,t)}=\frac{\dot{L}(t)}{L(t)}=n$.

According to [20], the profits earned by each leader $\left(1-1/\lambda\right)y\left(t\right)$ are appropriately discounted using the interest rate $\rho-n>0$ plus the instantaneous probability $I\left(t\right)$ of being driven out of business by further innovation. As Eq. [20] holds for every i, in the rest of the paper we focus on a symmetric structure in which both R&D difficulty and innovation rate do not vary across industries.

3.4 The Economic growth

This model identifies two possible sources of growth: productivity and manufacturing employment. Using [13] together with [15], each representative manufacturing quality-leader employs the same measure of workers $L_X=Y/\lambda\hat{w}$. Plugging [14] into [12], the aggregate production function boils down to

$$\ln Y\left(t\right)=\ln\left(L_x\right)+\int_0^1\ln\left[\lambda^{j(i,t)}\right]di \tag{21}$$

Since $j(i,t)$ jumps up to $j(i,t)+1$ when an innovation occurs and the innovation rate I does not change across industries and over time, the growth performance of the economy is completely described by the time evolution of the two terms on the right-hand side of [21]. Following Grossman and Helpman (1991), we know that the average productivity can be rewritten as $\int_0^1 \ln\left[\lambda^{j(i,t)}\right] di = It\log\lambda$. Differentiation of [21] with respect to time gives:

$$\gamma_Y \equiv \frac{\dot{Y}}{Y} = \frac{\dot{L}_x}{L_x} + I\ln\lambda \tag{22}$$

According to [21], the growth rate of real GDP, γ_Y, equals the growth rate of manufacturing employment $\dot{L}_x/L_x$ plus a term that is proportional to the overall innovation rate of the economy, I.

3.5 The labor market

The labor market is perfectly competitive. Defining the aggregate level of employment by L, the flow of workers in the unemployment pool is given by IL whereas that in the employment pool (both manufacturing and R&D) is given by $\mu(N-L)$. Accordingly, out of the BGP the differential equation governing how unemployment evolves over time reads:

$$\dot{u} = I(1-u) - \mu u$$

where $u \equiv (N-L)/N$ stands for the current rate of unemployment of the economy.

In the balanced-growth path equilibrium, the flow in equals the flow out (i.e., $\dot{u}=0$) and the unemployment rate is given by:

$$u = \frac{I}{I+\mu} \tag{23}$$

According to Eq. [23], the model rules out the presence of a trade-off between growth and unemployment. Indeed, given μ, a permanent increase in the BGP rate of innovation I tends to increase both the BGP growth rate of real GDP, γ_Y (via Eq. [22]), and the BGP unemployment rate, u (via Eq. [23]).[9] This result differs from that of Aghion and Howitt (1994) who find that a permanent

[9] Cerisier and Postel-Vinay (1998) find the same result for the case of search model. With respect to Mortensen and Pissarides (1995) and Aghion and Howitt (1994), in their model the creative destruction effect always dominates the capitalization effect with the result that a positive relationship between growth and unemployment always emerges in the long-run.

increase in the frequency of innovation leaves equilibrium unemployment rate unaffected.[10]

In order to get the aggregate labor demand we proceeds as follow. From the previous section we know that each representative manufacturing quality-leader employs the same measure of workers $L_X = Y/\lambda\hat{w}$, while the representative R&D firm employs the same measure of researchers $aI\kappa N(t)$. With a unit measure of industries, aggregate labor demand reads:

$$1 = \frac{y}{\lambda\hat{w}}\frac{1}{1-u} + \frac{aI\kappa}{1-u} \tag{24}$$

Eq. [11] and [24] completely describe the labor market. To see this more in depth, we substitute [23] for μ into [11] and get:[11]

$$\hat{w} = b + \chi + \left(\rho + \frac{I}{u}\right)\frac{\chi}{q} \tag{25}$$

Eq. [25] is downward-sloping and Eq. [24] is upward-sloping in $(u, \hat{w})$ space. Given y and I, the intersection of these two curves gives the equilibrium pair of wage and unemployment rate (see Figure 1).

Notice that as in Shapiro and Stiglitz (1984), this model is inconsistent with full employment. To see that, suppose that $\mu = +\infty$; i.e., each fired worker is instantaneously hired by another firm in either manufacturing or R&D. In such a situation, workers are encouraged to shirk thereby forcing firms to set a wage rate equal to infinity and reducing their labor demand. On the other hand, from the point of view of firms (regardless of whether it operates in manufacturing or R&D) there is no gain in offering wages lower than [11]. If a firm offered a lower wage, workers would stop providing effort and start shirking, with the result of damaging the firm through a drop in its overall productivity.

4 The Balanced-growth path equilibrium

In this section we analyze the BGP equilibrium properties of the model. In the BGP the going wage rate is such that no worker choose to shirk, product markets

[10] For the sake of truth, this result is akin to that of Aghion and Howitt for the sub-case of exogenous growth with iso-elastic preferences. In presence of innovation-based growth though, their results completely differs from ours.

[11] Observe that as $\hat{w}$ at the same rate as average productivity, both b and χ grow at the same rate $I \ln \gamma$. As it will be cleared later on, this result makes the share of employment in manufacturing (first term on the right-hand side of [24]) constant and allows us to study the BGP properties of the model.

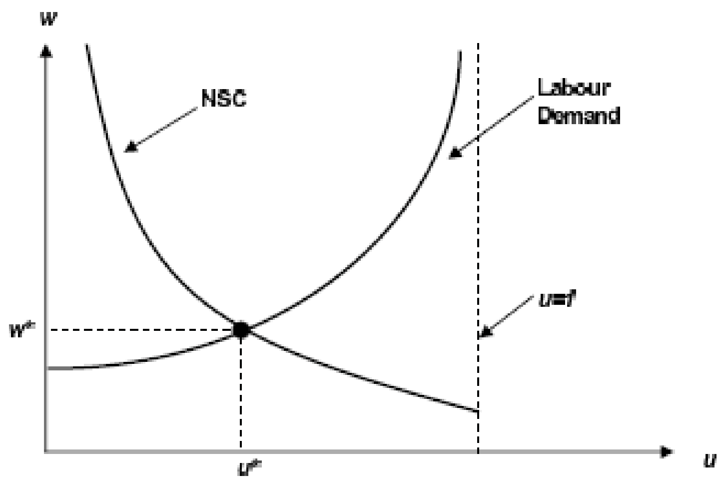

Figure 1: The equilibrium unemployment

clear and firms invest in R&D. As the presence of moral hazard makes the going wage rate to be higher than that of perfect competition, the model predicts a positive unemployment rate in the BGP given by [23].

In the BGP the allocation of all resources to various activities remains fixed over time; this implies that manufacturing employment must grow at the same rate as population n (i.e., $\dot{L}_x/L_x = n$) and that the growth rate of per capita GDP y is also constant and completely pinned down by the BGP rate of innovation

$$\frac{\dot{y}}{y} = \gamma_Y - n = I \ln \lambda \tag{26}$$

The BGP equilibrium system consists of four equations: aggregate NSC [11], no-arbitrage/ research equation [20], aggregate rate of unemployment, [23], aggregate labor demand [24], and the differential equation governing the growth rate of GDP, [26]. The five endogenous variables are: the equilibrium effort-enhancing wage rate $\hat{w}$, the level of per capita GDP y, the BGP rate of unemployment u, the growth rate of GDP γ_Y, and the BGP rate of innovation I.

In order to reduce the dimension of the system, we proceed as follows. First, we plug [11] and [23] into [20] and obtain the BGP research equation:

$$\left[b + \chi + (\rho + I + \mu)\frac{\chi}{q}\right] a\kappa = \frac{\left(1 - \frac{1}{\lambda}\right) y}{\rho + I - n} \tag{27}$$

The right-hand side of [27] is related to the benefit (in terms of expected discounted profits) from innovating while the left-hand side is related to the cost of innovation. The benefit from innovating increases when y increases (the per capita GDP increases), when ρ decreases (future profits are discounted less), and when I decreases (the quality leader is less threatened by further innovation). The cost of innovation increases when both a and κ increase (innovative R&D becomes relatively more difficult or the productivity of individual researcher decreases), when χ and b increase (the disutility of working in terms of consumption as well as the opportunity cost of being exerting effort increase), when μ and I increase (labor turnover increase), and when q decreases (monitoring technology becomes less efficient in caching shirkers).

Next, we plug [11] and [23] into [24] and get BGP labor market constraint:

$$\frac{\mu}{I+\mu} = aI\kappa + \frac{y}{\lambda\left[b+\chi+(\rho+I+\mu)\,\frac{\chi}{q}\right]} \tag{28}$$

Eq. [28] has a natural economic interpretation. The left-hand side is the employment rate of the economy while the two terms on the right-hand side are, respectively, the share of the employment in R&D and the share of employment in manufacturing. The manufacturing employment share increases when y increases (the per capita GDP increases), when λ decreases (the productivity gain from innovating decreases), when χ and b decrease (the disutility of working as well as the opportunity cost of exerting effort decrease), when μ and I decrease (labor turnover decrease), and when q increases (monitoring technology improves). The R&D employment share increases when I increases (BGP R&D intensity increases), and when both a and κ increase (R&D becomes relatively more difficult or the productivity of individual researcher decreases).

As shown in Figure 2.2, Eq. [27] (RE-curve in the graph) is downward-sloping while [28] (LM-curve in the graph) is upward-sloping in (I, y) space.[12] In order to ensure the existence of a positive BGP equilibrium, we impose the following restriction on the subjective discount rate:

$$\rho < \tilde{\rho}_1 \equiv n + \frac{\lambda - 1}{a\kappa}$$

implying that the vertical intercept of the RE-curve is higher than that of the LM-curve.

The BGP equilibrium consists of the intersection of the curves in the positive orthant (point A in Figure 2.2).

[12] See Appendix A for an analytical demonstration.

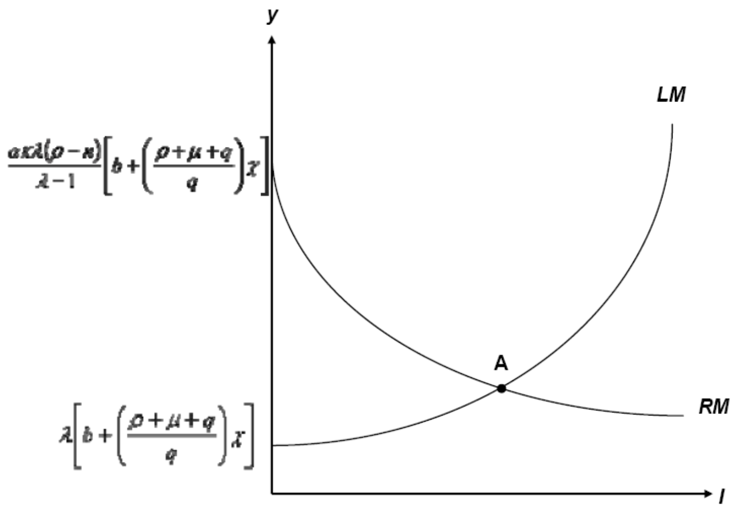

Figure 2: The BGP equilibrium

Proposition 1 *When the subjective discount rate is not too high (i.e., when $\rho < n + (\lambda - 1)/a\kappa$ holds), there exists a BGP equilibrium in which per capita GDP y, innovation rate I , and unemployment rate u are positive and constant, and where individuals have no incentive to shirk.*

Proof. *See Appendix B.* ■

According to Proposition 3, a BGP equilibrium exists only in the case in which individuals' subjective discount rate is not too high. In the remainder of the paper we will show some comparative statics properties of the model by restricting our attention to the special case in which $\rho < \tilde{\rho}_1$ holds.

5 Improving labor market performance

In this section we restrict attention to analyzing the BGP effects of improving the job market performance. An increase in the job market performance can be seen through an increase in the rate of job finding μ. The comparative statics results are displayed by Figure 2.3. Starting from BGP A, a permanent increase in μ shifts the BGP labor market constraint, LM, to the right and the BGP research equation, RE, to the left. The new BGP equilibrium is now represented by point B (see Figure 2.3).

As improving job market performance has a positive impact on per capita

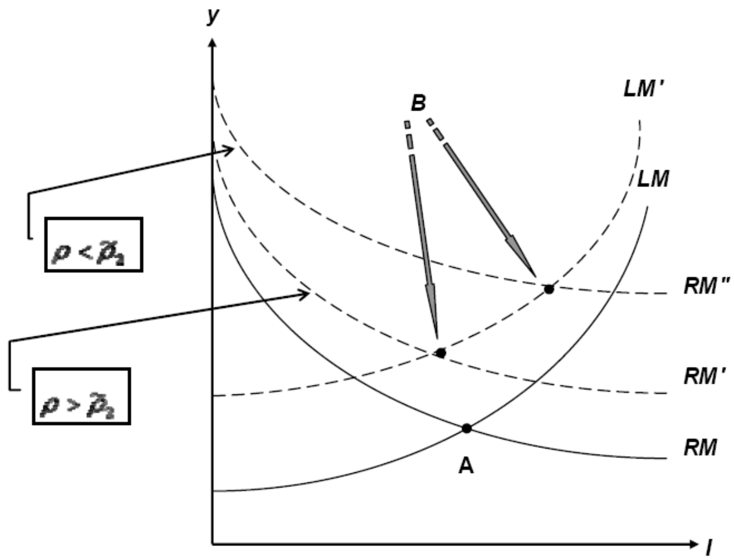

Figure 3: The BGP comparative statics

GDP, the final effect on the BGP rate of innovation is ambiguous and depends on whether the LM curve is able of shifting more than the RE curve. In Appendix B.1 we show that this result crucially depends on the size of the subjective discount rate ρ. In order to see that more in depth, define[13]

$$\tilde{\rho}_2 \equiv n - \lambda I + \frac{(\lambda - 1)\,\mu}{a\kappa\,(\mu + I)}$$

When $\rho < \tilde{\rho}_2$ holds, an increase in μ increases the BGP growth rate of the economy γ_Y via a permanent increase in the BGP rate of innovation I. When $\rho > \tilde{\rho}_2$ holds, an increase in μ decreases the BGP growth rate of the economy γ_Y via a permanent fall in the BGP innovation rate I.

Proposition 2 *For the economy in the BGP, an increase in the rate of job finding μ has a positive impact on BGP per capita GDP y and an ambiguous impact on the BGP growth rate of the economy γ_Y, that depends on the size of the subjective discount rate ρ. In particular, (i) when $\rho \in (n, \tilde{\rho}_2)$ an increase in μ increases the BGP growth rate of the economy γ_Y, and (ii) when $\rho \in (\tilde{\rho}_2, \tilde{\rho}_1)$ an increase in μ decreases the BGP growth rate of the economy γ_Y*

Proof. *See Appendix C.1* ■

[13] Observe that threshold $\tilde{\rho}_2$ is never larger that $\tilde{\rho}_1$ when $I > 0$.

The economic intuition behind Proposition 4 is not difficult to grasp. An increase in μ reduces unemployment duration with the result that fired workers can more easily be rehired when caught shirking. This forces firms to offer higher wages in order to discourage workers from shirking. The raised efficiency wage $\hat{w}$ has a twofold effect on R&D effort. On the one hand, increased $\hat{w}$ has a negative impact on R&D investment owed to a permanent increase in the cost of innovation (left-hand side of [27]). On the other, an increase in $\hat{w}$ makes employed workers to enjoy an increased stream of current income which results, in presence of risk-neutral preferences, in a quick increase in the level of per capita consumption. This has a positive impact on R&D via a permanent increase in the expected benefit of innovation (right-hand side of [27]). However, an equilibrium with a positive innovation rate requires that free-entry [18] must hold with equality. The BGP innovation rate must then adjust in order to warrant the equilibrium between the cost and the benefit of R&D (the denominator of the ratio on the right-hand side of [20]).

Obviously, the direction of such a change is not unique and turns out to depend on the size of the discount factor. When ρ is relatively small (i.e., $\rho < \tilde{\rho}_2$), the effective discount rate is too low and the expected benefit exceeds the cost of innovation. The rate of innovation has then to raise in order to re-establish equilibrium into free-entry condition [18]. Unlike, when ρ is relatively large (i.e., $\rho > \tilde{\rho}_2$) and the effective discount rate is too high, the expected cost exceeds the benefit of innovation. The rate of innovation has then to fall in order to re-establish the equilibrium into free-entry condition [18].

As far as the BGP unemployment rate is concerned, from [23] it is easy to see that an increase in the job-finding rate affects u both directly, through parameter μ, and indirectly through the BGP rate of creative destruction, I. Differentiation of [23] with respect to μ gives:

$$\frac{du}{d\mu} = \frac{1}{(I+\mu)^2}\left(\frac{dI}{d\mu}\mu - I\right) = \frac{I}{(I+\mu)^2}(\eta - 1)$$

where $\eta \equiv (dI/d\mu)(\mu/I)$ denotes the elasticity of I with respect to μ.

As is easy to check, an increase in μ has a mixed effect on the equilibrium unemployment rate depending on whether η is either lower or larger than one. Indeed, when the economy is impatient (i.e., $\rho > \tilde{\rho}_2$) and $dI/d\mu < 0$, $\eta < 0$ and the economy shifts towards a new BGP equilibrium with a lower unemployment rate. Unlike, when the economy is patient (i.e., $\rho < \tilde{\rho}_2$) and $dI/d\mu > 0$, the final effect of an increase in the rate of job finding is ambiguous and depends on the size of η; i.e., on whether elasticity η is either lower or larger than one.

Proposition 3 *For the economy in the BGP, improving labor market performance has mixed effects on the equilibrium unemployment rate that depends on both the size of the subjective discount rate ρ and how BGP innovation rate I reacts to changes in μ.*

Proof. *See Appendix C.1* ■

The economic intuition of Proposition 5 is not difficult to grasp. When $\eta > 1$, improving the labor market performance induces a large increase in the BGP rate of creative destruction. This reduces the life-span of the firm by more than the duration of unemployment. As a result, the flow in the unemployment pool, $I(1-u)$, is stronger than that in the employment pool, μu, so that equilibrium unemployment raises in the new BGP. Unlike, when $\eta < 1$ improving the labor market performance induces a small increase in the BGP rate of creative destruction. The reduction in the expected life-span of firms is now smaller than the duration of unemployment. As a consequence, the flow in the unemployment pool, $I(1-u)$, is not able to overwhelm that in the employment pool, μu, leading the economy toward a new BGP with a reduced equilibrium unemployment rate.

6 Unemployment benefit

Let's now analyze the BGP effects of a permanent increase in the unemployment benefit, b. Graphically, this comparative statics exercise leads to the same scenario described by Figure 2.3. Starting from the BGP equilibrium, A, a permanent increase in μ shifts the BGP labor market constraint, LM, to the left and the BGP research equation, RE, to the right. In the new BGP equilibrium, B, per capita BGP y have risen, whereas the rate of innovation can either increase or decrease depending, once again, on whether the subjective discount rate ρ is larger or lower than the threshold $\tilde{\rho}_2$.[14]

Proposition 4 *For the economy in the BGP, an increase in the unemployment benefit b has a positive impact on BGP per capita GDP y and an ambiguous impact on the BGP growth rate of the economy γ_Y. More specifically, (i) when $\rho \in (n, \tilde{\rho}_2)$ an increase in b leads to an increase in the BGP growth rate of the*

[14] The analitical details are collected by Appendix C.2.

economy γ_Y, and (ii) when $\rho \in (\tilde{\rho}_2, \tilde{\rho}_1)$ an increase in b leads to a decrease in the BGP growth rate of the economy γ_Y

Proof. *See Appendix C.2* ■

The economic explanation of the results showed by Proposition 6 is easy gained and relies on the existence of moral hazard in job relationships. Given Eq. [11], an increase in b generates a permanent increase in the effort-enhancing wage rate, $\hat{w}$. In the presence of inefficient monitoring technology, an increase in b represents an increase in the workers outside-option; in order to maintain the workers' incentive to exert effort, firms cannot do better than offering an higher wage rate, thereby raising both the cost and the benefit of R&D. The remaining explanation goes back over that of the previous section, so we skip it and focus on the BGP unemployment rate.

Differentiation of [23] with respect to μ gives:

$$\frac{du}{d\mu} = \frac{\mu}{(I+\mu)^2}\frac{dI}{db}$$

As is easy checkable, the only impact affecting equilibrium unemployment rate is that coming from changes in the BGP innovation rate. Indeed, whatever the effect of increasing the unemployment benefit b on the BGP innovation rate I is, the final impact on the equilibrium unemployment rate is always strictly related to. This means that when consumers are impatient, higher unemployment benefits have a positive impact on growth and unemployment; on contrary, when consumers are patient, higher unemployment benefit have a negative impact on growth and unemployment.

7 Conclusion

In this paper we addressed the issue of growth and unemployment through an innovation-based endogenous growth model with efficiency-wages. On a positive standpoint, we pointed out that there always exists a positive relationship between growth and unemployment, whereas, from a normative standpoint, we showed how labor market reforms can lead to mixed results.

Specifically, reforms aimed at improving the labor market performance, mainly in terms of either improving job finding rate or increasing unemployment benefit, crucially depend on how individuals discount future income. For the case of an impatient economy, we found that improving either labor market performance or unemployment benefit turns out to induce a permanent fall in both the BGP innovation rate and the equilibrium unemployment rate.

For the case of a patient economy, improving both labor market performance and unemployment induces a permanent increase in the BGP innovation rate but has mixed effects on the equilibrium unemployment rate. For reforms aimed at improving job market performance, the final effect on employment depends on the elasticity of innovation with respect to changes in the job finding rate. Specifically, when the response of the innovation rate to such reforms is high, the equilibrium unemployment rate turns out to increase in the new BGP. On contrary, when the response is low, the economy turns out to converge towards a new BGP with a lower equilibrium unemployment rate. For reforms aimed at improving unemployed worker's income, the final effect on equilibrium unemployment follows that on the innovation rate and is strictly related to the size of the rate of impatience of the economy.

Appendix A
Derivation of the Bellman Equation

This Appendix provides the analytical details of the derivation of the Bellman equation. Define the value function by $V(a,z)$. Based on Sennewald and Wälde (2006) and Sennewald (2007), the Bellman equation takes the following structure:

$$\rho V(a,z) = \max_{c,\varepsilon} \left\{ c(t) - \varepsilon\chi + \frac{1}{dt} E_t [dV(a,z)] \right\} \tag{A.1}$$

The change of $V(a,z)$ is given by

$$\begin{aligned} dV(a,z) = V_a (ra + z - c)\, dt \qquad & + [V(a, z+b-w) - V(a,z)]\, dq_w + \\ & [V(a, z+w-b) - V(a,z)]\, dq_b \end{aligned}$$

where $V_i \equiv \partial V(a,z)/\partial i$ with $i = \{a, z\}$.

According to [A.1], in the presence of two independent Poisson processes the change of $V(a,z)$ can be split in two terms: the "normal term" (first term on the right-hand side) and the "jump terms" (last two terms on the right-hand side).

Forming expectations on [A.1] gives:

$$\begin{aligned} E_t [dV(a,z)] \quad = \quad & V_a (ra + z - c)\, dt + \\ & + [V(a, z+b-w) - V(a,z)] [I(t) + (1-\varepsilon) q]\, dt + \\ & + [V(a, z+w-b) - V(a,z)]\, \mu dt \end{aligned} \tag{A.2}$$

Insertion of [A.2] into the Bellman equation [A.1] gives [5].

Appendix B
The steady-state equilibrium

Rewrite the BGP equilibrium system as

$$y - \left[b + \chi + (\rho + I + \mu) \frac{\chi}{q} \right] \frac{a\kappa\lambda(\rho + I - n)}{\lambda - 1} = 0$$

and

$$y - \lambda \left[b + \chi + (\rho + I + \mu) \frac{\chi}{q} \right] \frac{\mu - (\mu + I)\, a\kappa I}{(\mu + I)} = 0$$

Differentiation with respect to y and i gives:

$$\left.\frac{dy}{dI}\right|_{Labor} = a\kappa\lambda \frac{(2\rho + 2I - n + q + \mu)\,\chi + bq}{q\,(\lambda - 1)} > 0$$

and

$$\left.\frac{dy}{dI}\right|_{R\&D} = -\frac{\lambda\left\{bq\mu + \mu\,(\rho + q)\,\chi + a\kappa q\,(I + \mu)^2\,[b + (\rho + q + 2I + \mu)\,\chi/q]\right\}}{q\,(I + \mu)^2} < 0$$

respectively.

Labor market/NSC [27] is an upward-sloping curve and the no-arbitrage equation [28] is downward-sloping in (y, I) space. A BGP equilibrium exists if and only if $\rho < n + \frac{\lambda - 1}{a\kappa}$; i.e., if the vertical intercept of the LM curve is lower than that of the RE curve.

Appendix C
Comparative Statics

C.1. Labor market frictions

Rewrite Eqs.[11], [20] and [24] as follows

$$F^1\,(y, I, \hat{w}; \mu, b) \equiv \hat{w} - \left[b + \chi + (\rho + I + \mu)\,\frac{\chi}{q}\right]$$

$$F^2\,(y, I, \hat{w}; \mu, b) \equiv \hat{w}a\kappa - \frac{(1 - 1/\lambda)\,y\,(t)}{\rho + I - n}$$

$$F^3\,(y, I, \hat{w}; \mu, b) \equiv 1 - \left(\frac{y}{\lambda\hat{w}} + aI\kappa\right)\left(\frac{\mu + I}{\mu}\right)$$

Differentiation of with respect to the three endogenous variables $\{y, I, w\}$ and the finding rate μ yields:

$$\Gamma \cdot \begin{bmatrix} \frac{dy}{d\mu} \\ \frac{dI}{d\mu} \\ \frac{dw}{d\mu} \end{bmatrix} = \begin{bmatrix} \frac{\chi}{q} \\ 0 \\ \frac{wI\lambda}{(I+\mu)^2} \end{bmatrix}$$

where

$$
\Gamma \equiv \begin{bmatrix} 0 & -\frac{\chi}{q} & 1 \\ 1 & -\frac{aw\kappa\lambda}{\lambda-1} & -\frac{a\kappa\lambda(\rho+I-n)}{\lambda-1} \\ 1 & -\frac{w\lambda\left[\mu+a\kappa(I+\mu)^2\right]}{(I+\mu)^2} & -\lambda\left(1-aI\kappa-\frac{I}{I+\mu}\right) \end{bmatrix}
$$

The determinant of the Jacobian reads:

$$
|\Gamma| = \frac{\lambda}{q}\left\{\left(\frac{\mu}{\mu+I}\right)\left(\frac{qw}{\mu-I}-\chi\right)+\frac{a\kappa\left[qw\lambda+(\rho+I\lambda-n)\,\chi\right]}{\lambda-1}\right\}
$$

As in the BGP $w > (I+\mu)\,\chi/q$, otherwise workers would shirk, the determinant of the Jacobian is positive; i.e. $|\Gamma| > 0$.

Using Cramer rule yields:

$$
\frac{dy}{d\mu} = \frac{1}{|\Gamma|}\frac{aw\kappa\lambda^2\left\{wIq+(I+\mu)\left[I+\mu+(I+a\kappa I+a\kappa\mu)(\rho-n)\right]\chi\right\}}{q\,(\lambda-1)\,(I+\mu)^2} > 0
$$

$$
\frac{dI}{d\mu} = \frac{1}{|\Gamma|}\frac{\lambda\left\{qIw\,(\lambda-1)+\chi\,(I+\mu)\left[\mu\,(\lambda-1)-a\kappa\,(I+\mu)\,(\rho+I\lambda-n)\right]\right\}}{q\,(\lambda-1)\,(I+\mu)^2} \gtreqless 0
$$

and

$$
\frac{dw}{d\mu} = \frac{1}{|\Gamma|}\frac{w\lambda\chi\left\{(\lambda-1)+a\kappa\lambda\,(I+\mu)\right\}}{q\,(\lambda-1)\,(I+\mu)} > 0
$$

The sign of $\frac{dI}{d\mu}$ is not clear-cut and turns out to depend on the sign of the term in the squared brackets. Solving for ρ gives:

$$
\frac{dI}{d\mu} \gtreqless 0 \;\Leftrightarrow\; \rho \lesseqgtr \tilde{\rho}_2 \equiv n - I\lambda + \frac{(\lambda-1)\,\mu}{a\kappa\,(I+\mu)}
$$

For ρ very small (or far from $\tilde{\rho}_2$) the impact on I is positive and vice versa.

C.2. Unemployment benefit

Differentiation with respect to the three endogenous variables $\{y, I, w\}$ and the parameter measuring the unemployment benefit b yields:

$$
\Gamma \cdot \begin{bmatrix} \frac{dy}{db} \\ \frac{dI}{db} \\ \frac{dw}{db} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
$$

Using Cramer rule yields:

$$\frac{dy}{db} = \frac{1}{|\Gamma|} \frac{aw\kappa\lambda^2 \left[a\kappa (I+\mu)^2 (\rho - n) + \mu (\rho + 2I + \mu - n)\right]}{(\lambda - 1)(I+\mu)^2} > 0$$

$$\frac{dI}{db} = \frac{1}{|\Gamma|} \frac{\lambda \left[(\lambda - 1)\mu + a\kappa (I+\mu)(\rho + I\lambda - n)\right]}{(\lambda - 1)(I+\mu)} \gtreqless 0$$

and

$$\frac{dw}{db} = \frac{1}{|\Gamma|} \frac{w\lambda \left[(\lambda - 1)\mu + a\kappa\lambda (I+\mu)^2\right]}{(\lambda - 1)(I+\mu)^2} > 0$$

Once again, the sign of $\frac{dI}{d\mu}$ is ambiguous and related to subjective discount rate ρ. Indeed, it is easy to check that $\frac{dI}{d\mu} \gtreqless 0$ if and only if $\rho \lesseqgtr \tilde{\rho}$.

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