Basic Research and Industrial Development, Patent Protection, and Growth

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Abstract

The empirical analyses show the existence of strong technological links between academic scientific knowledge and industrial innovations that are granted of some form of intellectual property rights (IPR) such as patents. Since the '80s also new universities' basic worth ideas are granted of patent protection to reinforce both the knowledge links and transmission between university and industry. Within a Schumpeterian endogenous growth model this paper allows both basic public ideas and private innovations to be granted of patent protection. It is shown that the market value's fraction of a new industrial valuable innovation appropriated by a firm, and the subsidy of the industrial R&D cost are two equivalent ways to stimulate private R&D effort. However, the higher the market value's fraction appropriated by a innovative firm, the lower the subsidy the positively affect per capita growth. A main policy implication suggests that - once IPR are granted to basic academic ideas - a different regime of patent protection should be set for private and public ideas. In particular, patents should only be granted to very innovative and fundamental academic basic ideas. Finally, a trade-off between the per capita output growth rate and level has found.

Keywords: Intellectual Property Rights, Private and Public R&D, Growth

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1 Introduction

The role of both scientific knowledge and technological innovations to spur the growth performance of countries is emphasized by both academic and non-academic analyses. Both private and public institutions in several developed countries invest large amounts of resources in R&D (see the National Science Foundation reports). Because of knowledge characteristics, firms incurs in large

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fixed R&D costs with a fundamental uncertainty and serendipity for the offsprings of their effort, therefore they need to gain some rents when successful in research. A traditional theoretical argument to guarantee these economic rents consists in granting some form of Intellectual Property Rights (IPR) - such as patents - at new worth innovations. New knowledge is also created in the academy and other publicly funded institutions, and since the '80s the U.S. introduced some form of IPR also for university's ideas with the aim to reinforce the links between academy and industry. In particular, the U.S. adopted several legislative and institutional arrangements to spur the transfer of public basic discoveries and inventions to the industrial sector. Among the several legislative acts the most influential has been the Bayh-Dole Act of the 1980. This Act instituted an uniform federal patent policy for universities and small businesses under which they obtained the rights to any patents resulting from grants and contracts funded by any federal agency and to licence these patents on an exclusive or non-exclusive basis.[1] Based on the belief that legislative arrangements such as the Bayh-Dole Act enhance the technology transfer and the academic contributions to innovation and growth in the U.S., similar legislation is being considered in other OECD countries (OECD, 2002).

The existence of large spillovers of new knowledge university's flow on the industrial sector is well documented.[2]. Narin *et al.* (1997), and McMillan *et al.* (2000) show that, for the U.S. industry, relying in external sources of knowledge centers on public science. In particular, Narin *et al.* (1997) show that during the 1993-1994, 73% of the scientific paper cited by U.S. industrial patents were firm public science sources.[3] Recently, David *et al.* (2000) also find the existence of large and direct spillovers from public R&D to private industry, and the same authors maintain that these spillovers "...often are held to enhance private sector productive capabilities, and, specifically, to encourage applied R&D investments by firms that lead to technological innovations - from which will flow future streams of producer and consumer surpluses."

This paper considers the knowledge links between university and industry within a Schumpeterian ladder quality framework à la Aghion and Howitt (1992, 1998), and Howitt (1999, 2000). The Schumpeterian growth literature considers knowledge accumulation and technological progress as the engines of the economic growth. However, in large part of this strand of literature policy consists into guarantee patent protection for private innovative firms, and to subsidize

[1] Others legislative acts in such direction are the Stevenson-Wydler Technology Innovation Act of the (1980), the Small Business Innovation Development Act (1982), the National Cooperative Research Act (1984), the Federal Technology Transfer Act (1986), the National Cooperative Research and Production Act (1993), the Technology Transfer and Commercialization Act (2000).

[2] See, among others, Mokyr (2002), Nelson (2004), McMillan et al. (2000).

[3] From a theoretical point of view these spillovers are explained both through the intrinsic nature of knowledge as a nonrival input (see Arrow 1962) and the Merton's issue of priority of scientific discoveries. Merton (1973) argued that - within the non market rewards structure - the goal of scientists is to establish the priority of discovery by being the first to announce an advance in knowledge. Therefore, the rewards to priority are the recognition awarded by the scientific community for being the first.

their R&D cost, without any direct public investment in the R&D sector.[4]

This paper distinguishes in the description of the R&D sector in that both private and public ideas can be granted of patent protection, and the spillovers public basic ideas generate on industry are explicitly accounted for. Moreover, as in Aghion and Howitt (1996) and in Cozzi and Galli (2006), the distinction of the basic and development R&D stages is considered.[5] In this set up industry can conduct both the basic and development stage of the R&D activity, while public sector is assumed to only invest in basic research. In fact, the empirical evidence shows that the development stage is the prominent activity of the private research effort,[6] therefore the drastic, yet realistic, assumption that public R&D only consists in basic research programs is introduced. The university's scientific knowledge can be usefully developed by the industry to create new commercial valuable innovative products. Because public basic ideas are granted of patent protection, the industry pays to develop such scientific knowledge. However, in this set up only a successful research firm is assumed to pay for the usefully developed public basic ideas. This assumption is motivated by the empirical analyses. Cohen *et al.* (2002) find that the most important channels to access publicly funded research are publications, conferences, informal interactions rather than more institutional channels such as patents, licenses, and cooperative ventures. These results refer to all industrial sectors, also the high-tech industries.[7] Therefore, a rational R&D firm gains spillovers from public R&D programs through such informal channels. The use of public basic ideas will become evident only in the moment of a patent application, i.e. only for a successful R&D firm. Once a patent application has deposited by a R&D firm it will have to pay for that public basic ideas usefully developed for its innovation. Moreover, the government pays an undifferentiated subsidy to the industrial R&D effort.

It is shown that both a higher subsidy to industrial R&D cost and a lower

[4] The theoretical conclusions are not univocal. Some theoretical and empirical analyses conclude that policy has positive effect on both per capita output growth rate and on per capita output level. The alternative view concludes that policy is ineffective on per-capita output growth rate, even if it can produce positive effects on the per capita output level (see Jones, 1999, Ha and Howitt, 2006).

[5] The Science and Engineering Indicators (SEI, 2006) by the National Science Foundation defines as basic the research aimed "to gain more comprehensive knowledge or understanding of the subject under study without specific application in minds". The development stage is defined as "the systematic use of the knowledge or understanding gained from research directed toward the production of useful materials, devices, systems, or methods, including the design and development of prototypes and processes."

[6] The SEI states: "The federal government, estimated to have found 61,8% of U.S. basic research in 2004, has historically been the primary source of support for basic research...Industry devoted only an estimated 4,8% of its total R&D support to basic research in that year." (SEI 2006, ch.4 p.13). In addition, "The development of new and improved goods, services, and processes is dominated by industry, which performed 90.2% of all U.S. development in 2004." (SEI 2006, ch.4 p.13). See also Pavitt (2001).

[7] Cohen *et al.* (2002) find that the pharmaceutical industry more heavily conveys public research knowledge through patents and licenses. However, the same authors maintain that even in this industry informal channels and open science are still more important in conveying public R&D discoveries.

price paid to develop the patented public basic ideas increase the private innovative effort, and the per capita output growth rate, while they reduce the per capita output level. Moreover, it is shown that a lower price paid to develop university basic ideas allows at a low subsidy of greatly spur the industrial innovative effort and the per capita output growth rate. Yet, the subsidy and the price paid to usefully develop public innovations have some fundamental differences. The subsidy of the private R&D cost concerns the certainty aspect of a R&D process, and it is undifferentiated between all the research firms. The price paid for the use of public basic ideas concerns the uncertainty aspect of a R&D process, and it is assumed proportional to the productivity of the research firms. Furthermore, the subsidy and the intellectual appropriation have deep differences from an institutional point of view. The intellectual appropriation of new ideas concerns both public and private research activity, and the policy 'design' of the IPR involves the political, executive, legal, and jurisprudential powers of a country. The subsidy of private research cost does not deeply shape the institutional set-up of a country as the IPR policy 'design' does, and it can also be used as a fine-tuning policy instrument.

These results have a main policy implication. Once IPR are introduced for public ideas, a different regime for private and public innovations should be introduced. Patents to public innovations should be only granted at some radical and very innovative ideas. In fact, the existence of IPR for public innovations generates a further cost for the industry to acquire the licenses from public institutions. This implies that a patent design should guarantee a far larger patentability requirement for public basic ideas than for industrial innovations.[8] In fact, a larger patentability requirement would allow the private firm to appropriate a higher share of a new product's market value. Moreover, this different regime for the public and private sector would not restrict the technological transfer from universities to industrial R&D because of the diffuse use of informal channels to access public ideas (see Cohen *et al.*, 2002). Finally, these results do not rest on the quality of university research. In fact, the rapid increase in academic patenting has provoked new debates about the quality of these patents, and about a industry-biased university research targets induced by the stronger influence of the industry on scientific research.[9]

Recently, a paper by Cozzi and Galli (2007) focuses on the sequential nature of the innovation process within a dynamic general equilibrium framework. Their paper considers a two stage innovation process, and it evaluates the conditions under which IPR should be extended to basic discoveries that does not have an immediate and specific marketable application and a commercial value.

[8]The patentability requirement is a minimum innovation size required to receive a patent. A patent breadth's put restrictions on the products other firms can produce without a license. In particular, the leading breadth limits future innovators by specifying superior products that other firms can not produce (see O'Donoghue, 1998, and O'Donoghue and Zweimuller, 2004). Based on these definitions, the leading breadth could only concern a new marketable product, and it could not apply at a basic innovation that - by its own definition - does not have an immediate and specific market application. However, if the leading breadth also applies at a basic innovation, a lower leading breadth should be granted at public basic innovations.

[9]See D'Este and Patel (2007).

Cozzi and Galli (2007) show that a pro-growth policy consisting into guarantee an intellectual property protection for 'basic half-ideas' could not be at the ground of the reforms undertaken in the U.S. around the '80s. Moreover, Cozzi and Galli (2007) explicitly determine the market value of a 'basic half-idea'. The focus of this paper is on the strength and width of patents granted to public R&D, once patent protection for public innovation already exists in a country. Therefore, this paper complements and reinforces Cozzi and Galli's (2007) contribution. The patent-design literature, on the other hand, addresses the question of how patent policy for private innovations affects the incentives for industrial R&D. Recently, O'Donoghue and Zweimuller (2004) extend the patent-design literature to a general equilibrium framework. Their results show that both a larger patentability requirement and a larger leading breadth spur the private innovative effort, and therefore the per capita output growth rate. However, their work does not consider neither the role of the public R&D investment nor a patent protection for university scientific knowledge.

The paper is organized as follows. Section 2 sets up the model, section 3 describes the general equilibrium and the comparative statics results, section 4 concludes.

2 The Model

This economy is composed of a final good sector, of an intermediate good sector, and of a R&D sector. As in Aghion and Howitt (1992, 1998), competitive firms produce a homogeneous final consumption good by combining a constant input, and a continuum of intermediate goods with heterogeneous productivity. In the intermediate sector the firms produce a continuum of product lines denoted by N_t, at a given time $t \geq 0$. The intermediate sector is characterized by free entry and exit, and by a constant returns to scale technology: workers can be hired by a continuum of firms that produce their intermediate goods on a one to one basis from labor. However, a legally imposed distortion render each of them a local monopoly: this is due to the patent system. According to the standard Schumpeterian approach à la Aghion and Howitt (1992 and 1998), new intermediate goods are patented, and each monopoly is challenged by outsider R&D firms trying to invent and patent a better product and - due to instantaneous price competition - drive the former monopolist out of the market. The mass of the intermediate goods is continuously enlarging thanks to serendipitous imitation as in Howitt (2000).

The R&D sector is composed of both industrial (private firms) and public (university) research activity. Industrial R&D consists into upgrade the quality (or the production process) of any intermediate product line (vertical innovation). As said above, a perfectly enforceable patent law allows the private research firms to gain monopolistic rents for all the effective duration of the patent because - as usual in Schumpeterian growth models with ladder quality innovations - the incumbent monopolist can be replaced by the next innovator along the same product line.[10] This generates the Schumpeterian creative destruction effect. The existence of a perfect stock market channels consumer savings to firms engaged in R&D. Moreover, the government employes skilled workers to obtain basic knowledge and discoveries. A stochastic share of the stock of basic ideas can be usefully developed by industry to find a new commercial valuable innovations and to introduce new intermediate goods. However, the effect basic ideas produce on the productivity of private research firms is heterogeneous between the product lines. According to the legislative acts mentioned in the introduction, academic basic knowledge is granted of intellectual property rights.

2.1 Basic Framework

Let us assume continuous time and unbounded horizon. In this economy a mass $L_t > 0$ of infinitely lived families exists. Each family has an identical preference for non-negative consumption flows represented by the intertemporal utility function $\int_0^\infty e^{-rt} C_t dt$, where C_t is the non negative consumption flow of each household. Moreover, each family is endowed with a unit mass of flow labor time bearing no disutility; $r > 0$ is the common and constant subjective rate of time preference. Population growth - i.e. the growth in the number of households - is constant and equal to $g_L > 0$. The labor market is perfect and the inelastic supply of labor L_t is instantaneously employed by manufacturing firms and by the R&D sector. Capital markets are assumed to be perfect; the linear instantaneous utility implies constant real interest rate always equal to r.

Final output Y_t is produced by perfectly competitive firms combining the fixed factor with a large variety of intermediate goods, that is:

$$Y_t = M^{1-\alpha} \int_0^{N_t} A_{it} x_{it}^{\alpha} di \tag{1}$$

with $0 < \alpha < 1$. x_{it} is the amount of intermediate good i produced and used as an input at a given time $t \geq 0$, A_{it} is the productivity parameter attached at this intermediate good. M is the constant aggregate mass of fixed factor (such as for example, "land, minerals, oils", etc.). $N_t \in [0, \infty)$ denotes the mass of intermediate product lines already invented in the economy at date $t \geq 0$. Because in each sector instantaneous Bertrand competition guarantees that only the most advanced patent holder will be producing, N_t also denotes the mass of active intermediate good industries. The elasticity of substitution between intermediate products is equal to $\varepsilon = \frac{1}{1-\alpha} > 1$.

The perfectly competing R&D firms try to achieve and appropriate the next generation of any intermediate good. According to Aghion and Howitt (1998), and Howitt (1999) we consider the leading-edge technology, with an economy-wide leading edge productivity parameter $A_t^{\max}$ that exerts positive

[10] See Aghion and Howitt (1992, 1998), Segerstrom (1998), Howitt (1999). Cozzi (2006) proves that the standard Schumpeterian growth models are compatible with a positive and finite R&D investment by the incumbent monopolistic firms. The analysis of this paper is also compatible with Cozzi's (2006) findings.

R&D spillovers in all intermediate goods. When a new commercial valuable discovery is introduced along an intermediate product line $i \in [0, N_t]$ (a better quality of an intermediate good is introduced) the productivity parameter A_{it} in that sector jumps to $A_t^{\max}$. This specification incarnates Aghion and Howitt's (1998 ch. 3) and Howitt's (1999) inter-sector knowledge spillovers.

The technological frontier $A_t^{\max}$ grows at a rate proportional to the per product line rate of vertical innovations. The Poisson arrival rate of vertical innovations in any product line i is $\lambda_A l_{Ait} \tilde{f}_i(b_t)$. λ_A is a productivity factor, $l_{Ait} = \frac{L_{At}}{N_t}$ is the per product line research labor time, the function $\tilde{f}_i(b_t)$ captures the effect of the per product line stock of basic knowledge b_t on the Poisson arrival rate of a new patentable and commercial valuable idea in the product line i (see the Appendix A, point 1). As the economy develops an increasing number of intermediate goods, an innovation of a given size in any product line will have a smaller impact on the aggregate economy; hence the marginal impact of each innovation on the stock of public knowledge will be $\frac{\sigma}{N_t}$, where $\sigma > 1$ is a proportionality factor. The aggregate flow of vertical innovations is the number of intermediate goods N_t times the expected flow of vertical innovations per industry line. The economy-wide rate of vertical technological progress is described by the following:

$$g_{At} = \frac{\dot{A}_t^{\max}}{A_t^{\max}} = \frac{\sigma}{N_t} \int_0^{N_t} \lambda_A l_{Ait} \tilde{f}_i(b_t)\, di \qquad (2)$$

Notice that the stock of basic knowledge accumulated over time - and not only the flow of new basic discoveries - affects the Poisson arrival rate of industrial innovations. Moreover, the function $\tilde{f}_i(b_t)$ is assumed to be heterogenous between the intermediate product lines,[11] with $\frac{\partial \tilde{f}_i(b_t)}{\partial b_t} \geq 0$. Therefore, a new better quality version of any intermediate product is the result of private innovation that renders marketable and commercial valuable the offsprings of both public and private research effort.

According to this framework, in equilibrium we will observe an ever-evolving intersectoral distribution of the absolute productivity parameters A_{it}, with values ranging from 0 to $A_t^{\max}$. Defining $a \equiv \frac{A_{it}}{A_t^{\max}}$, we can concentrate on the relative intersectoral distribution, that - as shown in Aghion and Howitt (1998, ch. 3) and in Howitt (1999) - converges to the unique stationary distribution of relative productivity parameters - a - characterized by cumulative distribution function $H(a) = a^{\frac{1}{\sigma}}$, with $0 \leq a \leq 1$. Every time a better quality of an intermediate good is introduced into the economy, the absolute distribution will be re-scaled rightward because the technological process rises to $A_t^{\max}$.

The mass of intermediate products grows as a result of serendipitous imitation, not deliberate innovation.[12] Each person has the same propensity to

[11] Basic ideas in some product lines, such as biotechnology, engineering, electronics, etc., can have an immediate market application, so that the spillovers basic ideas produce for the development of new marketable products in such industry lines can be very high. Instead, basic ideas in ares such as economics, literature, antropology, astronomy, etc., can have a far less useful market application, so that their spillovers can be far more low.

[12] See Howitt (2000). In Howitt (1999), and Cozzi and Spinesi (2005) horizontal innovation

imitate $\beta > 0$, thus the aggregate flow of new products is:

$$\dot{N}_t = \beta L_t \tag{3}$$

Sine population grows at the constant rate g_L, the number of workers per product line $\frac{L_t}{N_t}$ converges monotonically to $\frac{g_L}{\beta}$.

2.2 Asset Market, Intermediate Sector, and Industrial R&D

The industrial and public research effort allows to increase the technological frontier. As remarked in the introduction, a public basic idea does not have an immediate and specific commercial application, and it will be developed by the industrial R&D sector to find a new marketable intermediate product. Therefore, the commercial value of a new intermediate product is given by the firm's expected stock market value that monopolizes the commercialization of the new intermediate product. Let V_t be the expected stock market value of a new intermediate product with maximum productivity $A_t^{\max}$. Moreover, a R&D firm incurs a cost for the patented public basic ideas usefully developed, and this cost is here described as a fraction of the commercial value of a new intermediate product. The fraction of the market value respectively appropriated by a R&D firm and by a public research unit that have contributed to introduce the new intermediate product is described as a Nash-bargaining solution between these two forces. The bargaining process is described as taking place between a representative firm and a public institution from which the rights on the patented basic innovations are acquired.[13] Let V_t^p be the expected stock market value of a new intermediate product appropriated by a private R&D firm, and let V_t^b be the expected stock market value of a new intermediate product appropriated by the public institution, with $V_t = V_t^p + V_t^b$. The expected stock market value appropriated by private and public innovators is the solution to the following:

$$\begin{aligned} & \max_{V_t^p, V_t^b} \left(V_t^p\right)^{\phi_i} \left(V_t^b\right)^{1-\phi_i} \\ s.t.\ V_t &= V_t^p + V_t^b \end{aligned} \tag{4}$$

The solution to this problem gives $V_t^p = \phi_i V_t$, and $V_t^b = (1 - \phi_i) V_t$. The parameter $\phi_i \in (0, 1)$ represents the institutional set-up in which the bargaining process takes place. ϕ_i indicates that a private R&D firm pays to use an array of basic ideas which are granted of some form of IPR. The existence of laws such as the Bayh- Dole Act - and of other legislative arrangements - heavily

is motivated by the same profit seeking objectives as quality improving innovation. The results of this paper are not qualitatively affected by this specification for horizontal innovation.

[13] Because of the macroeconomic framework of this paper, the bargaining process is here described with a centralized set-up. A representative firm bargains with a centralized public institution to buy the rights on a stock of basic ideas per unit of time.

contribute to determine the value of the parameter ϕ_i in the economy. [14] The parameter ϕ_i is heterogeneous between the product lines. This heterogeneity is explained because of the heterogeneous spillovers between the product lines. In fact, when basic ideas generate a high spillover on the product line i - i.e. there is a high value for function $\tilde{f}_i(b_t)$ - each firm targeting the industry line i is willing to pay a higher price to develop such patented basic ideas, therefore ϕ_i will be correspondingly lower. This implies that when the same basic idea can be usefully developed by more than one product line, the government will license that patented idea at the product line that gain a higher spillover, also because it has a higher Poisson arrival rate of innovations and a lower average interarrival time between two valuable ideas. Moreover, because of the symmetry of the private R&D firms within a product line, each firm in any line i is willing to pay exactly the same price of any other firm in the same intermediate line.

Applying Aghion and Howitt's (1992 and 1998) methods, the intermediate good i production level that maximizes the monopolist profits at time t is:

$$x_{it} = M\left(\frac{\alpha^2 A_{it}}{w_t}\right)^{\frac{1}{1-\alpha}},$$

because the distribution of relative productivities is unchanging, we do not classify the sectors by their index i but by their relative productivity $a \equiv \frac{A_{it}}{A_t^{\max}}$. Defining the productivity-adjusted real wage as $\omega_t \equiv \frac{w_t}{A_t^{\max}}$, and normalizing the fixed factor to one (that is positing $M = 1$), the instantaneous labor demand function for a product line i with relative productivity a at date t is rewritten as:

$$\tilde{x}_{it}\left(\frac{\omega_t}{a}\right) = \left(\frac{\alpha^2 a}{\omega_t}\right)^{\frac{1}{1-\alpha}} \tag{5}$$

where $\tilde{x}_{it}\left(\frac{\omega_t}{a}\right)$ is a labor demand function for the manufacturing firm. The labor force employed in the intermediate sector negatively depends on the productivity-adjusted real wage.

The R&D is a perfectly competitive sector, with free entry and exit. Each vertical R&D firm targeting an intermediate product i chooses its R&D intensity to maximize $\phi_i V_t \lambda_A l_{Ajt} \tilde{f}_i(b_t) - (1-s) w_t l_{Ajt}$, where l_{Ajt} is the labor time flow employed by the vertical R&D firm j at time t, s is the subsidy to private research. Rational individuals and firms know they will appropriate a fraction ϕ_i of the expected stock market value of a patentable and commercial valuable idea in the product line i. The solution to the above problem is $\phi_i V_t \lambda_A \tilde{f}_i(b_t) = (1-s) w_t$. Notice that, the per product line basic stock knowledge b_t is taken as given by each individual and firm. Because each R&D firm can invest in any product line, the same first order condition for a maximum profit must hold along any product line $k \neq i$, that is $\phi_k V_t \lambda_A \tilde{f}_k(b_t) = (1-s) w_t$.

[14] In this setting, the tightness, the width and the ease of the intellectual property rights regime - as measured by ϕ_i - has studied. A low value of ϕ_i can indicate that it is extremely easy for public R&D to obtain patent grants for any basic innovation. This implies that private firms must pay to also use basic discoveries that have a very low innovative power.

In equilibrium, each R&D firm must be indifferent to invest in any intermediate product line. The industry lines that benefit from a higher spillovers - i.e. have a higher $\tilde{f}_i(b)$ - will pay a higher price to use the patented basic ideas and therefore they will appropriate of a lower share ϕ_i of the expected stock market value V_t. This implies the following no-arbitrage equation between the existing industry lines:

$$\phi_i V_t \lambda_A \tilde{f}_i(b_t) = \phi_k V_t \lambda_A \tilde{f}_k(b_t) \tag{6}$$

that implies $\tilde{f}_i(b_t) = \tilde{f}_k(b_t)\frac{\phi_i}{\phi_k}$.

We will focus on the symmetric steady state, that is $x_{it} = x_t$, $l_{Ait} = l_{At}$, etc., for every intermediate product line i.[15] In the multisector economy the R&D arbitrage condition is similar to Aghion-Howitt (1998, ch.3, Appendix):

$$\begin{aligned} \frac{(1-s)}{\phi_i \lambda_A \tilde{f}_i(b_t)} w_t &= A_t^{\max} \int_0^\infty e^{-(r+g_A/\sigma)\tau} \tilde{\pi}\left(\omega e^{g_A \tau}\right) d\tau = \\ &= A_t^{\max} \int_0^\infty e^{-(r+g_A/\sigma)\tau} \frac{1-\alpha}{\alpha} \omega_t \tilde{x}(\omega) e^{-\frac{\alpha}{1-\alpha} g_A \tau} d\tau \end{aligned} \tag{7}$$

On the left hand side of eq. (7) the probability of appropriating the new innovation by the author has been considered. On the right hand side of eq. (7) the discount rate $(r + g_A/\sigma)$, and the profit flows $A_t^{\max}\tilde{\pi}\left(\omega e^{g_A\tau}\right)$ accruing to a successful innovator from date t to infinity have been considered.

2.3 Public R&D

Population differs in the basic research ability, while there are no quality differences among workers employed in industrial R&D and in manufacturing. Let us $G(\theta)$ be the distribution of the 'basic research ability' θ, with θ taking value on $\left[0, \bar{\theta}\right]$, and $\bar{\theta} < 1$. The usual properties $G'(\theta) > 0$, $G(0) = 0$, $G\left(\bar{\theta}\right) = 1$ apply. Since each worker must be indifferent between manufacturing and vertical research activity, it will be $w_t = \frac{\phi_i \lambda_A \tilde{f}_i(b_t)}{(1-s)} V_t$. The additional no-arbitrage condition between improving/manufacturing and basic research effort can be written as:

$$w_t = \frac{\phi_i \lambda_A \tilde{f}_i(b_t)}{(1-s)} V_t = \lambda_B \varphi \left[E\left(\frac{A_{it}}{A_t^{\max}}\right)^{-1} \right] \theta_0 w_t = \tilde{\lambda}_B \theta_0 w_t \tag{8}$$

where the function $\varphi\left[E\left(\frac{A_{it}}{A_t^{\max}}\right)^{-1}\right] = \varphi(1+\sigma)$ represents the spillovers from vertical innovation to basic research productivity, and $\tilde{\lambda}_B \equiv \varphi(1+\sigma)\lambda_B$

[15] As proven by Cozzi (2005), Howitt's (1999) model admits a continuum of symmetric balanced growth paths.

is a productivity parameter of each public researcher.[16] On the left hand side of eq. (8) w_t indicates the wage of a worker in the industrial sector - manufacturing and R&D - which in equilibrium must be equal to the expected returns of improving the quality of an intermediate product $\frac{\phi_i \lambda_A \tilde{f}_i(b_t)}{(1-s)} V_t$. The right hand side of the last part of eq. (8) indicates the expected flows return to be employed in basic research. Because $\theta \in \left[0, \bar{\theta}\right]$, with $\bar{\theta} \leq 1$, the gross salary paid by the public sector is lower than the wage paid by the industrial R&D laboratory. However, the productivity term $\tilde{\lambda}_B$ allows to increase the perceived salary of a public researcher.[17]

Let us denote θ_0 the threshold value of the 'basic research ability' that satisfies equality (8): θ_0 ability researchers are indifferent between trying to improve the quality of one of the existing intermediate goods, to be employed in basic research, and to be employed in the intermediate sector. The higher the basic research talent an individual is endowed with, the higher the gain to be employed in basic research programs. The no-arbitrage equation (8) determines the threshold ability value

$$\theta_0 = \frac{1}{\tilde{\lambda}_B}, \tag{9}$$

which is constant along the BGP.[18] Each individual endowed of a research ability $\theta > \theta_0$ will find it profitable to be employed in basic research programs. Hence, in such an economy, for $\theta > \theta_0$, $\left[1 - G\left(\theta_0\right)\right] L_t$ individuals will choose to be employed in basic research programs. Instead, the individuals endowed with an ability $\theta \leq \theta_0$, that is $G\left(\theta_0\right) L_t$, will decide either to introduce a better quality of the existing intermediate goods, or to work in the manufacturing sector. A policy that affects the productivity of basic research effort also affects the threshold ability parameter θ_0. This in turn changes the population employed in basic research programs, and therefore the per product line stock of basic knowledge. This implies that the institutional set-up can affect in different ways the interplay between public and private research effort.

The government conducts basic research programs to accumulate basic knowledge P_t according to the following dynamic law

[16] $\varphi(\cdot)$ can be any positive function of the average relative productivity $E\left[\left(\frac{A_{it}}{A_t^{\max}}\right)\right] = (1+\sigma)^{-1}$.

[17] See Aghion et. al (2005), and the Science and Engineering Indicators (2004, 2006) for the empirical evidence of an average higher wage gained in the private R&D laboratories than in nonprofit/government R&D laboratories. The parameter $\tilde{\lambda}_B$ can also positively depend on public expenditures. To fix ideas, the public expenditures affect the quality of laboratories equipment, because higher public expenditures allow to have higher quality of equipment. This affect the expected and perceived gain of a public researcher. Moreover, a public researcher often obtains some financial support from a variety of institutions above their contractual gross salary. These elements contribute to increase the expected gain of a public researcher (see D'Este and Patel, 2007).

[18] Restriction $\frac{1}{\tilde{\lambda}_B} < \bar{\theta} < 1$ is assumed to hold.

$$\dot{P}_t = \tilde{\lambda}_B \left[\int_{\theta_0}^{\bar{\theta}} \theta dG(\theta) \right] L_t = \tilde{\lambda}_B m(\theta_0) L_t \tag{10}$$

where $m(\theta_0) L_t \equiv \left[\int_{\theta_0}^{\bar{\theta}} \theta dG(\theta) \right] L_t$ is the average conditioned cumulated basic research effort, $\tilde{\lambda}_B$ is the productivity of each researcher engaged in basic research programs. Eq. (10) implies that the stock of basic knowledge P_t grows at the same rate as the population growth rate g_L.

2.4 Labor Market Equilibrium

Each researcher endogenously decides to allocate her research labor time to inventive or to manufacturing activity.

Plugging these results in the manufacturing/vertical R&D arbitrage condition (7), and solving the integral yields:

$$\frac{(1-s)}{\phi_i \lambda_A \tilde{f}_i(b_t)} = \frac{\frac{1-\alpha}{\alpha} \tilde{x}(\omega_t)}{r + \frac{g_{At}}{\sigma} + \frac{\alpha}{1-\alpha} g_{At}} \tag{11}$$

Solving the above equation for $\tilde{x}(\omega)$, the labor force employed in the production of the top quality intermediate good is obtained:

$$\tilde{x}(\omega_t) = \frac{(1-s)}{\phi_i \lambda_A \tilde{f}_i(b_t)} \left(r + \frac{g_{At}}{\sigma} + \frac{\alpha}{1-\alpha} g_{At} \right) \frac{\alpha}{1-\alpha} \tag{12}$$

from which, by inverting eq. (12), it is possible to determine the productivity-adjusted real wage ω_t.

The labor market clearing condition for manufacturing and vertical innovation is:

$$G(\theta_0) L_t = N_t l_{At} + N_t \int_0^1 \tilde{x}(\omega/a) h(a) da = N_t l_{At} + \frac{N_t \tilde{x}(\omega_t)}{1 + \frac{\sigma}{1-\alpha}} \tag{13}$$

where $\tilde{x}(\omega/a)$ is the labor demand function of a product line with relative productivity parameter a at the date t, and $h(a)$ is the density function of the cumulative distribution function $H(a)$.

The labor market clearing condition for basic research programs is:

$$[1 - G(\theta_0)] L_t = L_{Bt} \tag{14}$$

which is a constant fraction of the population because the threshold ability parameter θ_0 is constant along the BGP.

From eq. (1), and reclassifying intermediate goods by their relative productivities, the aggregate GDP can be written as (see Aghion and Howitt 1998, ch. 3, and Howitt 1999):

$$Y_t = A_t^{\max} N_t \int_0^1 a\tilde{x}\left(\omega/a\right)^{\alpha} h\left(a\right) da =$$
$$= A_t^{\max} N_t \int_0^1 \frac{1}{\sigma} a^{\frac{1}{\sigma}} \left(\frac{\alpha^2 a}{\omega_t}\right)^{\frac{\alpha}{1-\alpha}} da = \frac{N_t A_t^{\max}\left(\frac{\alpha^2}{\omega_t}\right)^{\frac{\alpha}{1-\alpha}}}{\left(1+\frac{\sigma}{1-\alpha}\right)} \tag{15}$$

Notice that, in the light of eq.s (15) and (1), the productivity-adjusted fixed factor rent is:

$$\frac{re}{A_t^{\max}} = (1-\alpha)\frac{Y_t}{MA_t^{\max}} = (1-\alpha)\frac{N_t\left(\frac{\alpha^2}{\omega_t}\right)^{\frac{\alpha}{1-\alpha}}}{\left(1+\frac{\sigma}{1-\alpha}\right)} \tag{16}$$

Therefore, the fixed factor rent increases in the number of intermediate goods, simply because they complement it in the production of the final good; and it decreases in the productivity-adjusted real wage.

3 General Equilibrium

The economy has a unique rational expectation equilibrium on which rational individuals instantaneously jump on. From now onward the time index is eliminated for the sake of notational simplicity.

Let us consider the law of motion of the basic knowledge (10), along the BGP it is obtained:

$$p \equiv \frac{P}{N} = \frac{m\left(\theta_0\right)\tilde{\lambda}_B}{\beta} \tag{17}$$

From eq. (2) the productivity growth rate becomes (see the Appendix A, point 2):

$$g_A = \frac{\dot{A}^{\max}}{A^{\max}} = \frac{\sigma}{N}\int_0^{N_t} \lambda_A l_{Ait} \tilde{f}_i\left(b\right) di = \sigma\lambda_A l_A \tilde{F}\left(b,\Theta\right) \tag{18}$$

Therefore, the labor demand in eq. (12) for the top quality intermediate good becomes:

$$\tilde{x}\left(\omega\right) = \frac{(1-s)}{\phi_i\lambda_A\tilde{f}_i\left(b\right)}\left(r + \frac{g_A}{\sigma} + \frac{\alpha}{1-\alpha}g_A\right)\frac{\alpha}{1-\alpha} \tag{19}$$

Let us consider both eq.s (19) and (13), along the rational expectation equilibrium, a positive and finite value for the per product line industrial research effort exists:

$$\bar{l}_A = \frac{\frac{L}{N}G\left(\theta_0\right) - r\frac{(1-s)}{\phi_i\lambda_A\tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)}\frac{\alpha}{1-\alpha}}{\left[1+\frac{(1-s)}{\phi_i\lambda_A\tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)}\frac{\alpha}{1-\alpha}\left(\frac{1}{\sigma}+\frac{\alpha}{1-\alpha}\right)\sigma\lambda_A F\left(\tilde{b},\Theta\right)\right]} \tag{20}$$

From eq. (20), in order to have a positive R&D effort the following condition must hold along the BGP (this condition is similar to condition V in Howitt, 1999 and condition A in Cozzi and Spinesi, 2006)

$$g_L > \frac{\beta}{G(\theta_0)} \frac{r(1-s)}{\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)} \frac{\alpha}{1-\alpha} \tag{C}$$

The per capita output is:

$$\frac{Y}{L} = \frac{\frac{N}{L} A^{\max}\left(\frac{\alpha^2}{\omega}\right)^{\frac{\alpha}{1-\alpha}}}{\left(1+\frac{\sigma}{1-\alpha}\right)} = \frac{\frac{\beta}{g_L} A^{\max}\left(\frac{\alpha^2}{\omega}\right)^{\frac{\alpha}{1-\alpha}}}{\left(1+\frac{\sigma}{1-\alpha}\right)} \tag{21}$$

where eq. (15) has been used. Therefore, the per capita output growth rate is equal to the technological frontier growth rate:

$$g_{Y/L} = g_A = \sigma \lambda_A l_A \tilde{F}(b, \Theta) \tag{22}$$

In the light of eq.s from (17) to (22) the following can be stated:

Proposition 1 *Along the rational expectation BGP, a constant fraction of population is employed in the intermediate sector, in the industrial and public research sector. Along the BGP, an increase either in the parameter ϕ_i proportional in all product lines i or in the subsidy s positively affects the per capita output growth rate, and negatively affects the per capita output level.*

Proof. *See Appendix B* ■

Proposition 1 compares the IPR policy design for both industrial and public R&D with an undifferentiated subsidy of the industrial research cost. It is found that the intellectual appropriation of a new commercial valuable ideas by the industrial firms - as summarized by the parameter ϕ_i - and the subsidy of the industrial R&D cost have the same qualitative effect on the growth performance of a country. Yet, some fundamental differences between these two elements exist. The policy 'design' of the patent system for both industrial and public ideas concerns the uncertainty aspect of a R&D process, and it involves the political, executive, jurisprudential authorities of a country. Therefore, this policy 'design' strongly shapes the institutional set-up and the environment in which both private and public R&D operate. The subsidy to private R&D does not shape the institutional set-up of the economy as the policy 'design' of the intellectual property rights does, it only directly affects the private R&D costs, and therefore the certainty aspect of a R&D process. Moreover, the subsidy can be also managed in short time horizon.

Given these fundamental differences between these two elements, their effect on the per capita output growth rate and level can be different in magnitude, as the following states:

Proposition 2 *Whenever condition (C1) holds along the BGP - i.e.* $s \geq 1 - \phi_{\min}$ *- a larger subsidy would produce a higher per capita output growth rate and a lower per capita output level than what could be obtained from a higher value of the parameter* ϕ_i *in each product line* $i \in [0, N_t]$.

Proof. *See Appendix C* ■

Condition (C1) implies that a marginal increase in the subsidy greatly spurs the private research effort along all the product lines, and the per capita output growth rate. Yet, a marginal increase in the subsidy to private R&D firms magnifies the trade off between the per capita output growth rate and the per capita output level. Notice that the higher $\phi_{\min}$ the lower the threshold subsidy that greatly affects the per capita output growth rate, and the per capita output level.

It seems noteworthy to recall that ϕ_i is a measure of the strength, tightness and ease of IPR granted to public innovations. Therefore, proposition 2 has a main policy implication. Once some form of IPR are introduced for public ideas, two different regimes of IPR should be provided for public and private innovations. In this framework the different regime concerns a patent protection against future innovators, and therefore it refers to both patentability requirement and to patent breadth.[19] O'Donoghue and Zweimuller (2004) show that both a larger patentability requirement and leading breadth better off the growth performance of a country. However, in this setting patents are also granted to public innovations, and this represents a further cost for the industrial R&D firms. Therefore, a larger patentability requirement for a public basic idea would reduce the industrial R&D cost and would better off the growth performance of a country. In fact, once patents are granted to only very innovative and fundamental public basic ideas, condition (C1) more easily holds because of a higher value of $\phi_{\min}$. This in turn implies that also a low subsidy to industrial R&D can greatly spur the innovative effort of the economy and its per capita output growth rate.

4 Conclusions

The existence of strong knowledge links and transmission between industry and academy is widely documented. In particular, the existence of large and positive scientific knowledge spillovers from university to industry is also well documented. Moreover, since the '80s the U.S. and then other OECD countries introduced some form of IPR also for university's ideas with the aim to reinforce the links between academy and industry.

[19] Because the leading breadth limits future innovators by specifying superior products that other firms can not produce, it could not apply at a basic idea that does not have an immediate and specific market application. However, to be as general as possible, also a leading breadth for non marketable basic ideas could be considered. In this case, the results suggest that a larger leading breadth for a public basic innovation generates a cost of licences for both universities and private firms, and this discourages the R&D effort. Therefore, a lower leading breadth should be set for public basic innovations.

This paper investigates on the macroeconomic implications of an institutionally imposed IPR regime for both academic and industrial ideas. To this aim a Schumpeterian growth model à la Aghion and Howitt (1992, 1998) and à la Howitt (1999, 2000) is adopted. In the R&D sector, public research programs generate basic ideas that do not have an immediate commercial value and application. The industrial R&D firms develops useful academic ideas to introduce new valuable product innovations on the market. However, a successful R&D firm only appropriates a fraction of the commercial value of a new patented product because it pays for the licenses of public basic ideas that have been usefully developed. Moreover, private R&D firms obtain a subsidy for their research costs.

This paper shows that the intellectual property rights and the subsidy to private R&D costs are two alternative ways to finance private research effort. In fact, both a higher fraction of a patented product's market value appropriated by a firm and a higher subsidy to R&D cost spur the industrial innovative effort. This in turn generates a trade off between the per capita output growth rate and level, by increasing the former and reducing the second. Moreover, a higher fraction of a patented product's market value appropriated by the industrial firm allows at a low subsidy to greatly spur the innovative effort and the per capita output growth rate. This result seems remarkable because the policy design of the intellectual property rights strongly shapes the institutional set-up of the economy, and it concerns the uncertainty aspect of a R&D process. While the subsidy to industrial R&D cost does not have an institutional 'weight' as the intellectual property rights has, and it concerns the certainty aspect of a R&D process. The results of the paper generate a remarkable policy implication. This paper suggests the introduction of a different regime of IPR for industrial and public innovations. In fact, although patents granted to basic academic ideas can spur the technological transfer from universities to industry, they also represent a cost for the private firms. According to these considerations policy should provide IPR to only fundamental and radical public ideas in order to limit the cost incurred by private firm to benefit from the use of basic discoveries.

Appendix A

1. In this part the Poisson arrival rate of vertical innovations has obtained. Let us consider the per product line stock of basic ideas at a given time $t \geq 0$, $p_t \equiv \frac{P_t}{N_t}$. In this framework only a fraction of academic basic ideas can be usefully developed by industry to find new commercial valuable intermediate products. This fraction is assumed to be a continuous random variable denoted by $b_t \in \left[0, \bar{b}\right]$, with $\bar{b} \leq \frac{m(\theta_0)\tilde{\lambda}_B}{\beta}$, and cumulative distribution function (cdf) $\Omega(\cdot)$ common for all $i \in [0, N_t]$. Private R&D firms gain a positive spillovers from basic ideas that have commercial valuable applications. These spillovers are measured through the function $f_i(\cdot)$ in the product line i. The spillovers functions $f_i(\cdot)$ for each $i \in [0, N_t]$ are assumed bounded above and constants over time. Therefore the expected value of spillovers from commercial valuable basic ideas in the product line i is:

$$\tilde{f}_i(b_t) \equiv E\left[f_i(b_t)\right] = \int_0^{\bar{b}} f_i(b_t)\, d\Omega(b_t) \tag{A1}$$

Therefore the expected Poisson arrival rate of vertical innovation along a product line i is:

$$\begin{array}{c} \int_0^{\bar{b}} \lambda_A l_{Ait} f_i(b_t)\, d\Omega(b_t)\, di = \\ = \lambda_A l_{Ait} \int_0^{\bar{b}} f_i(b_t)\, d\Omega(b_t) = \lambda_A l_{Ait} \tilde{f}_i(b_t) \end{array} \tag{A2}$$

2. The no-arbitrage equation in vertical R&D between the product lines imply $\tilde{f}_i(b) = \frac{\phi_k}{\phi_i}\tilde{f}_k(b)$. Moreover, at a given time $t \geq 0$ in the economy there exist a continuum N_t of product lines. Therefore the technological frontier growth rate along the symmetric BGP can be rewritten as:

$$\begin{array}{c} g_{At} = \frac{\sigma}{N_t}\int_0^{N_t} \lambda_A l_{Ait}\tilde{f}_i(b)\, di = \frac{\sigma}{N_t}\lambda_A l_{At}\int_0^{N_t} \frac{\phi_k}{\phi_i}\tilde{f}_k(b)\, dk = \\ = \frac{\sigma}{N_t}\lambda_A l_{At}\tilde{F}(b,\Theta)\, N_t = \sigma\lambda_A l_{At}\tilde{F}(b,\Theta) \end{array} \tag{A3}$$

where $\tilde{F}(b,\Theta)$ summarizes the average per product line spillovers. This function is bounded above; $\Theta > 0$ is a proportional factor that also summarizes the shares ϕ_k for each k. Therefore, all the heterogeneities in $\tilde{f}_k(b)$ and in ϕ_k are captured through the function $\tilde{F}(b,\Theta)$ which is constant over time because b, $\tilde{f}_k(b)$, and ϕ_k are all constant along the BGP. Therefore, along the BGP the technological frontier growth rate is constant. Moreover, because each average spillovers function $\tilde{f}_i(b)$ is increasing in b, also $\tilde{F}(b,\Theta)$ is increasing in b. Q.E.D.

3. By following the same steps as in Aghion and Howitt (1998), the profit flow of any monopolistic firm that manufactures an intermediate product i with productivity A_{it} is

$$\pi_{it} = A_t^{\max}\frac{1-\alpha}{\alpha}\omega_t\left(\frac{\alpha^2}{\omega_t}\right)^{\frac{1}{1-\alpha}} a^{\frac{1}{1-\alpha}} = A_t^{\max}\tilde{\pi}(\omega)\, a^{\frac{1}{1-\alpha}}$$

where $\omega_t \equiv \frac{w_t}{A_t^{\max}}$ is the productivity-adjusted real wage, $\tilde{\pi}(\omega)$ is the profit flow of the intermediate good with the maximum productivity parameter $A_t^{\max}$.

The expected stock market value of the last successful R&D firm that has productivity $A_t^{\max}$ is described by the eq. (5) in the text. The expected stock market value of an intermediate product i with absolute productivity A_{it} and relative productivity $\frac{A_{it}}{A_t^{\max}}$ is $V_{it} = V_t a^{\frac{1}{1-\alpha}}$. Therefore, the cumulative expected stock market value of all manufacturing monopolies at a given time $t \geq 0$ is:

$$\int_0^{N_t} V_{it} di = N_t \int_0^1 V_{it} dH(a) = N_t V_t \int_0^1 a^{\frac{1}{1-\alpha}} dH(a) = \frac{N_t A_t^{\max} V_t}{1 + \frac{\sigma}{1-\alpha}} \qquad \text{(A3)}$$

Appendix B

The first part of this Appendix proves the effect of a higher intellectual appropriation parameter ϕ_i for each product line on the economic performance of a country, $\phi_i \in (0, 1)$ is assumed. The marginal change in the appropriation parameter is assumed to happen proportionally in all the product lines, so that the ratio $\frac{\phi_i}{\phi_k}$ is constant. This proportional change in all the appropriation parameter ϕ_i has interpreted as a lower strength of IPR for public basic ideas. By calculating a marginal change in all parameters ϕ_i, the change has denoted with a generic parameter ϕ. The second part analyses the effect of a higher R&D subsidy.

1. Let us consider the eq. (20). By simply differentiation along the BGP the following is obtained:

$$\frac{\partial l_A}{\partial \phi} = \Delta^{-2} \left\{ \frac{r(1-s)\frac{\alpha}{1-\alpha}\lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)}{\left[\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)\right]^2} \Delta - \left[\frac{L}{N} G(\theta_0) - r \frac{(1-s)}{\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)} \frac{\alpha}{1-\alpha}\right] * \right.$$
$$\left. \left[\frac{(1-s)\frac{\alpha}{1-\alpha}\left(\frac{1}{\sigma}+\frac{\alpha}{1-\alpha}\right)\sigma\lambda_A \frac{\partial \tilde{F}(b,\Theta)}{\partial \phi} \phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right) - (1-s)\frac{\alpha}{1-\alpha}\left(\frac{1}{\sigma}+\frac{\alpha}{1-\alpha}\right)\sigma\lambda_A \tilde{F}(b,\Theta)\lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)}{\left[\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)\right]^2}\right]\right\} \qquad \text{(B1)}$$

where $\Delta \equiv \left[1 + \frac{(1-s)}{\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)} \frac{\alpha}{1-\alpha} \left(\frac{1}{\sigma} + \frac{\alpha}{1-\alpha}\right) \sigma \lambda_A \tilde{F}(b, \Theta)\right] > 0$, and condition (C) holds. Because the spillovers are assumed constant in this analysis, then $\frac{\partial F(\tilde{b},\Theta)}{\partial \phi} = \frac{\partial\left[\int_0^{N_t} \frac{\phi_k}{\phi_i} \tilde{f}_k(b) dk\right]}{\partial \phi} = 0$, because we impose a proportional marginal change in each appropriation parameter ϕ_i such that $\frac{\phi_k}{\phi_i}$ remains constant. In fact:

$\frac{\partial \tilde{F}(b,\Theta)}{\partial \phi} = \frac{\partial\left[\int_0^{N_t} \frac{\phi_k}{\phi_i} \tilde{f}_k(b) dk\right]}{\partial \phi} = \frac{\partial\left[\frac{1}{\phi_i} \int_0^{N_t} \phi_k \tilde{f}_k(b) dk\right]}{\partial \phi} = -\frac{1}{\phi_i^2} \int_0^{N_t} \phi_k \tilde{f}_k(b)\, dk + \frac{1}{\phi_i} \int_0^{N_t} \tilde{f}_k(b)\, dk =$

$= \frac{1}{\phi_i}\left[\int_0^{N_t} \tilde{f}_k(b)\, dk - \frac{1}{\phi_i}\int_0^{N_t} \phi_k \tilde{f}_k(b)\, dk\right] = \frac{1}{\phi_i}\left[\int_0^{N_t} \tilde{f}_k(b)\, dk - \int_0^{N_t} \tilde{f}_k(b)\, dk\right] = 0$ Q.E.D.

In eq. (B1) this variation is simply denoted with $\frac{\partial \Theta}{\partial \phi_i}$ because $\Theta > 0$ summarizes all the ratios $\frac{\phi_k}{\phi_i}$ in the integral $\int_0^{N_t} \frac{\phi_k}{\phi_i} \tilde{f}_k(b)\, dk$. This implies that eq. (B1) is strictly positive, i.e. $\frac{\partial l_A}{\partial \phi} > 0$. Therefore, along the BGP, a higher intellectual appropriation parameter ϕ increases the per product line private R&D effort.

In order to determine the effects of a higher intellectual appropriation parameter ϕ on the market demand for any existing intermediate good, we use the labor market clearing condition:

$$L = G(\theta_0) L + [1 - G(\theta_0)] L = N\bar{l}_A + \frac{N\tilde{x}(\omega)}{1 + \frac{\sigma}{1-\alpha}} + N l_B \tag{B2}$$

where $l_B = \frac{L_B}{N} = [1 - G(\theta_0)] \frac{L}{N}$ denotes the per product line basic research effort. From eq. (8) a constant threshold ability parameter θ_0 is obtained. Therefore - along the new BGP with a higher intellectual appropriation parameter ϕ - the per product line basic research effort $[1 - G(\theta_0)] \frac{L}{N}$ is constant and equal to $[1 - G(\theta_0)] \frac{g_L}{\beta}$. Moreover, eq. (B1) proves that, along the new BGP, the per product line vertical research effort is higher. Therefore, eq. (B2) necessarily implies a lower market demand $\tilde{x}\left(\frac{\omega}{a}\right)$ for each existing intermediate good. Finally, from eq. (21), it immediately follows that a higher appropriation parameter ϕ determines a lower per capita output level. Q.E.D.

The positive effect of a change in the appropriation parameter ϕ on the per capita output growth rate is easily proven:

$$\frac{\partial g_{Y/L}}{\partial \phi} = \sigma \lambda_A \frac{\partial \bar{l}_A}{\partial \phi} \tilde{F}(b, \Theta) > 0 \tag{B3}$$

where the inequality follows from eq. (B1). Q.E.D.

2. This part analyses the effect of a change in the subsidy to private research effort s on the economic performance of the economy; $s \in (0, 1)$ is assumed. From eq. (20) the following is obtained:

$$\frac{\partial l_A}{\partial s} = \Delta^{-2} \left\{ \frac{r \frac{\alpha}{1-\alpha}}{\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)} \Delta - \left[\frac{L}{N} G(\theta_0) - r \frac{(1-s)}{\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)} \frac{\alpha}{1-\alpha} \right] * \left[- \frac{\frac{\alpha}{1-\alpha}\left(\frac{1}{\sigma}+\frac{\alpha}{1-\alpha}\right)\sigma \lambda_A \tilde{F}(b,\Theta)}{\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)} \right] \right\} > 0 \tag{B4}$$

Therefore, along the BGP a positive relationship between the subsidy to private research effort s and the per product line private R&D labor time $\bar{l}_A$ is proven. Q.E.D.

In order to determine the effects of a higher subsidy s on the market demand for any existing intermediate good, the labor market clearing condition (B2) is used. As proven above, along the BGP, the per product line basic research effort is constant and equal to $[1 - G(\theta_0)] \frac{g_L}{\beta}$. Moreover, eq. (B4) proves that - along the new BGP with a higher subsidy to private R&D firms - the per product line vertical research effort is higher. Therefore, eq. (B2) necessarily implies a lower market demand $\tilde{x}\left(\frac{\omega}{a}\right)$ for each existing intermediate good. Finally, from eq. (21), it immediately follows that a higher subsidy s determines a lower per capita output level. Q.E.D.

The positive effect of a change in the subsidy to private research effort s on the per capita output growth rate is easily proven:

$$\frac{\partial g_{Y/L}}{\partial s} = \sigma\lambda_A \frac{\partial \bar{l}_A}{\partial \phi} \tilde{F}(b, \Theta) > 0 \tag{B5}$$

where the inequality follows from eq. (B4). Q.E.D.

Appendix C

This Appendix compares the effect of a proportional marginal change in the appropriation parameter ϕ_i in all product lines i with the effect of a marginal change in the subsidy s. Along a new BGP with a larger value of either ϕ_i or s determines a higher per product line private innovation effort and a higher per capita output growth rate. In order to compare the magnitude of these effects it suffices to consider the eq.s (B1) and (B4). Whenever the following condition is satisfied $\frac{\partial \bar{l}_A}{\partial s} \geq \frac{\partial \bar{l}_A}{\partial \phi}$, an increase in the subsidy generates the same economic effects as an increase in the appropriation parameter, but the former are higher in magnitude. Let us define $\phi_{\min} \equiv \min \{\phi_i\}_{i=0}^{N}$. Therefore $\phi_{\min}$ is the product line that appropriates the lowest market value among the existing product lines. From eq.s (B1) and (B4), it immediately follows that $\frac{\partial \bar{l}_A}{\partial s} \geq \frac{\partial \bar{l}_A}{\partial \phi}$ if and only if

$$\begin{aligned} &\left[1 - \frac{1-s}{\phi_i}\right] * \left\{ \frac{r \frac{\alpha}{1-\alpha} \Delta}{\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)} + \right. \\ &+ \left. \left[\frac{L}{N} G(\theta_0) - r \frac{(1-s)}{\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)} \frac{\alpha}{1-\alpha}\right] \frac{\frac{\alpha}{1-\alpha}\left(\frac{1}{\sigma}+\frac{\alpha}{1-\alpha}\right)\sigma\lambda_A \tilde{F}(b,\Theta)}{\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)} \right\} \geq 0 \end{aligned}$$

that is always true whenever

$$s \geq 1 - \phi_{\min} \tag{C1}$$

Q.E.D.

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