Endogenous growth and the role of the educational sector

(provisional draft)

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1. Introduction

What are the factors that determine a country's rate of economic growth in the long run? The growth models of the 1960s assigned a significant role to human capital formation and in particular to education financed by the public sector (Arndt, 1987, ch. 3), but long-run growth was entirely an exogenous process due to technological progress, which was assumed as an unexplained time trend in labor productivity. In recent years, this approach has received renewed attention with the rise of a new line of research that explains the growth rate endogenously, assigning an important role to the private and public sectors in the formation of human capital, at both the aggregate and the individual levels. Within the new growth theory, economists have begun to study the influence of education spending on consumption-saving decisions in models which allow for the possibility of persistent growth (Glomm and Ravikumar, 1997). These recent developments have significant policy implications since public or private expenditures on education may influence long-run growth and social welfare. To the extent that formal schooling is a significant component of human capital investment, the institutions for schooling may be important for economic growth (Gradstein, Justman and Meyer 2005).

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The aim of this paper is to analyze in a simple analytical context the relevance of education financing so that endogenous growth can be modelled even when physical capital accumulation encounters decreasing returns to scale, in the historical perspective as well. Also analyzed will be the importance of fiscal policy as a determinant of long-run economic growth in the form of government debt. We consider the case in which government finances expenditures in the schooling sector by means of tax revenues and public debt.

The paper is structured in three parts. The first part presents the earlier contributions on this topic. We will see that endogenous growth was not possible in the models which flourished in the 1960s because of the hypothesis of labor-augmenting technical progress (Uzawa 1965, Nelson and Phelps 1966, Shell 1969). The second part analyses the new approach proposed in the endogenous growth models where education plays a key role in counteracting the decreasing returns to capital (Glomm and Ravikumar, 1992). The third part considers the role of public debt in education financing. In the context of endogenous growth we shall see that government may foster growth by running deficits in order to finance educational spending (Bräuninger, 2003).

2. Earlier contributions in the models of education and economic growth

In the literature of the 1960s, there was clear awareness that technological progress was to a great extent an endogenous element, but the question set was how to translate this common conviction into analytical terms. Whereas a first group of endogenous growth models sought to link technical change to capital accumulation (the vintage approach), in a second class of models growth was generated from investments in education. Therefore, the earliest attempt at modeling the role of education in economic growth occurred in the 1960s. The scale of the process depended upon the process of knowledge accumulation, and its pace on the social preferences between present and future consumption. We can distinguish three different models of education and economic growth in the earlier approach (Wan, 1971, ch. 6). The feature shared by models of this kind of was that economic growth was considered as an endogenous process resulting from improving labor force productivity.

The first model on the role of education in economic growth was advanced by I. Uzawa (1965). As in the case of K. Arrow's (1962) learning by doing, technical progress took the pure

labor-augmenting form. There are two special assumptions in Uzawa's model. Firstly, labor can be divided into two types: productive L_P and educational, L_E. Only L_P is used as input in production, and there is no reference to the allocation of material wealth to the education sector. Secondly, the rate of increase for labor-augmenting efficiency is a concave, increasing function of the ratio of educators to the labor force. The educational function proposed by Uzawa is:

$$\frac{\dot{A}}{A} = f(\frac{L_E}{E}) \qquad [1]$$

where L_E is the quantity of educators and $L = L_p + L_E$ is the size of the total labor force. It is assumed that the higher the proportion of the labor force employed in education the higher the improvement in labor force efficiency $(f'(L_E / L > 0, f''(L_E / L < 0)$. In other words, the higher the proportion of the total labor force devoted to education, the faster the increase in labor productivity, but this increase is less than the increase in the education population/total population ratio. Uzawa shows that, under this hypothesis, there exists a unique optimal path that tends to the steady state value of macroeconomic variables.

A second model in this strand was developed by R. Nelson and E. Phelps (1966). In their model the role of education is conceived as being primarily that of facilitating the flow of technological information. Nelson and Phelps define education as the process of training the productive actors in the economy to receive and absorb the technological information being transmitted by other sectors in the economy. The notion of the theoretical level of technology at time t, $R(t)$, plays an important role in the model. If all best technological practices re available to all economic agents then $R = A$. When this is not the case, $A < R$. The educational function in this model is:

$$\dot{A} = (R - A) f(h) \qquad [2]$$

where h is a measure of the educational level of society (with $f'(h) > 0)$. The rate of increase of the technology in practice is an increasing function of educational attainment and is

proportional to the "gap" $(R - A)$. Following the Schumpeterian hypothesis that invention does not depend upon other economic variables it can be assumed that R grows at the constant rate ($\dot{R} = \rho R$). Nelson and Phelps show that the long-run equilibrium path of technology is given by:

$$A^* = \left(\frac{f(h)}{f(h) + \rho}\right) R_0 e^{\rho t} \qquad [3]$$

so that the gap between A and R tends to a long-run constant value for constant h. The shortcoming of the model is that the determination of the level of technical education h is left as an exogenous variable.

The third model was developed by K. Shell (1969). It is an educational planning model that requires scarce resources to be devoted to the educational sector in order to increase educational attainment h. This model may be regarded as a synthesis of the Nelson-Phelps and Uzawa models. In this approach the law of the growth of technology in practice is:

$$\frac{\dot{A}}{A} = \left(\frac{R - A}{A}\right) f(h) \qquad [4]$$

But in this case the level of social education, h, is not constant; rather, it is assumed that the educational attainment of the labor force is an increasing function of the proportion of the labor force engaged in education:

$$\dot{h} = g\left[(1 - u)L\right] \qquad [5]$$

where u is the fraction of the labor force to be employed in production. Also in [5] the educational function must satisfy the essential property of diminishing returns with respect to current effort. This means that faster growth can be attained by intensified dosages of educational labor force, yet the law of diminishing returns applies to educational efforts as well as to productive efforts. On this basis Shell identifies a long run equilibrium, actually a saddle-

point, with both the time derivatives of physical capital and technical progress equal to zero, that is $\dot{k} = \dot{h} = 0$ (Shell, 1969).

Summarizing, in the 1960s many authors proposed formal models to shed light on the impact of public financed education on long-run growth. The point of departure was a variation on the Solow-type neoclassical growth model, with education as a vehicle of increasing productivity and economic growth. However, in these models economic growth was not really endogenous, but also depended upon exogenous factors. This was basically because the differential equation that governs the accumulation of education shows diminishing returns: given past education, the marginal effectiveness of new education decreases with the amount of education obtained at a particular time. This condition which guarantees the steady state properties of the models is the one which prevents the economy from obtaining endogenous growth.

3. The macroeconomic role of private expenditure on education in an endogenous growth model

The presence of human capital, in the form of educational expenditure, may relax the constraint of diminishing returns to capital and may give rise to long-term per capita growth in the absence of exogenous technological progress. Hence, the production of human capital may be an alternative to improvements in technology as a mechanism to generate sustained growth. Unlike G. Glomm and B. Ravikumar (1992), who developed the implications of this approach in a overlapping generations framework, we will consider a continuous time model.

Given the usual assumptions, the analysis can use the representative-agent framework, in which the equilibrium derives from the choices of a single household. The household maximises the discounted stream of utility resulting from consumption, C, over an infinite time horizon subject to its budget constraint. The utility function is assumed to be given by $U(C) = C^{1-\gamma}/(1-\gamma)$ and labour is supplied inelastically. The maximization problem can then be written as

$$\underset{C}{Max} \int_0^{+\infty} e^{-\rho t} C^{1-\gamma}/(1-\gamma)dt \qquad [1]$$

subject to

$$Y = C + \dot{K} + \dot{E} \qquad [2]$$

where [2] is the economy's resource constraint and $\dot{E} = Z$ is the private expenditure on education. In equation [2], for the sake of simplicity, we assume that the physical good and education are generated by the same production functions (Barro and Sala-i-Martin, 2004, p. 240). The productive sector is represented by one firm which behaves competitively and which maximises static profit. The firm produces output according to the technology,

$$Y = K^{1-a} E^{a} \qquad [3]$$

The final good in our economy is produced with physical capital and education, where we assume that the household decides about the time that it must spend on education. Additionally, the household uses resources for education in the schooling sector, like expenditures on books and other teaching materials, which are an input to the process of human capital formation. Thus, the input to the schooling sector is composed of time spent for education by the household and of schooling private expenditures. We assume decreasing returns to scale in each input alone and constant returns to scale in both inputs together.

An equilibrium allocation is defined as an allocation such that the firm maximizes profit and the household solves [1] subject to [2]. The current-value Hamiltonian is written as

$$H = \frac{C^{1-\gamma}}{1-\gamma} + \lambda(K^{1-a} E^{a} - C - Z) + \vartheta Z \qquad [4]$$

where λ and ϑ are shadow prices associated with $\dot{K}$ and $\dot{E}$.The first-order conditions can be obtained by setting the derivatives of H with respect to C, Z to 0 and equating $(\dot{\lambda} - \lambda\rho)$ to $-\partial H / \partial K$ and $(\dot{\vartheta} - \vartheta\rho)$ to $-\partial H / \partial E$, respectively, allowing for the budget constraint. If we simplify these conditions, we obtain the familiar result for the growth rate of consumption:

$$\frac{\dot{C}}{C} = \frac{1}{\gamma}((1-a)K^{-a}E^{a} - \rho) \qquad [5]$$

The second condition is that the marginal product of physical capital must equal the marginal product of education:

$$(1-a)K^{-a}E^{a} = aK^{1-a}E^{a-1} \qquad [6]$$

This condition implies that the ratio of educational expenditure on physical capital is constant and given by

$$\frac{E}{K} = \frac{a}{1-a} \qquad [7]$$

This result for E/K implies that the rate of return to physical capital is constant because the production function in equation [3] exhibits constant returns with respect to broad capital, K and E. Therefore, diminishing returns do not apply when E/K is constant, that is, when capital and expenditure on education grow at the same rate.

If E/K is constant, the rate of growth of consumption is also constant and becomes:

$$g = \frac{\dot{C}}{C} = \frac{1}{\gamma}((1-a)(\frac{a}{1-a})^{a} - \rho) \qquad [8]$$

where we substituted for E/K from equation [7]. We assume that the parameters are such that $g > 0$. We obtain a balanced growth path in which all endogenous variables grow at the same rate, i.e., $\dot{C}/C$, $\dot{K}/K$, $\dot{E}/E$ and the resource constraint of the economy is fulfilled.

This simple model of the private financing of education expenditure, in which physical good and education are generated by the same production functions, indicates the way in which the recent endogenous growth literature has addressed the problem of analyzing the role of the

educational sector in order to avoid the difficulties encountered in the 1960s. There are two main differences with respect the earlier literature. From the economic point of view, education is now considered to be an autonomous production factor and not a parameter that increases labor force productivity. From the analytical point of view, endogenous growth is made possible by the absence of diminishing returns in the true factor accumulated, E/K. In other words, expenditure on schooling is the element that offsets the negative effect on economic growth of diminishing returns to physical capital. In the next section we will consider the more realistic case in which education is provided by the government.

4. The macroeconomic role of public expenditure on education in an endogenous growth model

In this section we consider the more common case in which education is provided by the government. We focus on the case of pure public education, excluding all private acquisition of education, and assume that public schooling is financed by a proportional tax determined by majority voting among parents, where t denotes the tax rate. Taxes are used to provide all children with a uniform level of schooling, which – assuming a balanced budget for the time being – equals

$$E = tY \qquad [9]$$

Public education severs the link between individual household income and the level of schooling. This is the principal feature that distinguishes public education from privately financed education. The resource constraint becomes the following, $\dot{K} = (1-t)Y - C$. In this case the Hamiltonian expression is

$$H = \frac{C^{1-\gamma}}{1-\gamma} + \lambda((1-t)K^{1-a}E^{a} - C) \qquad [10]$$

The evolution of the growth rate in consumption is modified in the following way:

$$g = \frac{\dot{C}}{C} = \frac{1}{\gamma}((1-a)(1-t)(t)^{\frac{a}{1-a}} - \rho) \qquad [11]$$

Following equation [11], growth-maximizing depends on the tax rate t. Different amounts of public expenditure have two effects on the growth rate g (Barro, 1990). An increase in t reduces g, but an increase in E/Y raises output, which raises g. We can verify that the latter force dominates when the government is small, and the former dominates when the government is large. To maximize the growth rate, the government sets its share on education expenditure equal to a, the elasticity to income with respect to expenditure on education.

It is worthwhile to compare the equilibrium path income for the two educational systems, abstracting from distributional issues. The comparison is facilitated by considering the fact that the equations representing the growth rates have the same structure. From equations [8] and [11] the evolution of the growth rates ratio in both economies is similar to the capital accumulation equation in the Cass-Koopmans model. From a direct comparison we are able to conclude that the private education financing produces a higher growth rate if

$$(\frac{a}{1-a})^a > (1-t)(t)^{\frac{a}{1-a}} \qquad [12]$$

In the last equation the left term is constant, while the right term depends upon the tax rate chosen by the representative agent. As in the case considered by Glomm and Ravikumar (1992) the private education economy has a higher growth rate for usual values of parameters a and t. The main reason why the evolution of income in the public education economy is different from that in the private education economy is that the resources devoted to education are different in the two economies. Consequently, the representative agent in the private education economy is better off than its counterpart in the public education economy. By optimizing, it can do better and attain a higher level of utility. Also in the previous case, the key assumption is that it is possible to counter the decreasing returns to capital through investment in education.

The main differences between the two approaches spring from the distributional implications in the long period. If the financial markets are imperfect, in the private scheme only wealthy households can afford the schooling expenditure, reducing in this way the accumulation of human capital and economic growth (Galor and Zeira, 1993). By contrast, public education, by offering an equal supply of resources to all households, can help foster economic growth. In this context, in which the public expenditure for education has positive effects as regards the marginal product of capital, the public debt can be used as an instrument to promote economic growth.

5. The public debt for education financing in an endogenous growth model

In this section we consider the effects of public policy on the balanced growth rate if we allow for public debt. In this case an increase in government schooling expenditure may be financed by additional debt. An increase in government expenditure on schooling does not necessarily go along with a decline in spending on public infrastructures, but instead may be financed by additional debt. However, we impose a certain restriction on the government's behaviour.

5.1 The government

Human capital in the economy is produced in the schooling sector, where we assume that the government decides on the time that the household must spend in education. We consider the simple case in which the government has revenues from issuing government bonds that it then uses for public spending in the schooling sector and for interest payments on the public debt. Thus, the period budget constraint of the government is given by

$$\dot{B} = rB + E - T \qquad [13]$$

with T denoting tax revenue and $(T - E)$ denoting the primary surplus. As usual, the primary surplus is defined as tax revenue minus government spending exclusive of interest payments.

Further, the government fixes the education expenditure so that it is a positive linear function of the debt-GDP ratio. The motivation for this rule is that it guarantees the sustainability of public debt. Public debt at time zero must equal future present-value surpluses and rules out that the government plays a Ponzi game (Blanchard and Fisher, 1989, ch. 2). The primary surplus ratio can thus be written as

$$T - E = \phi Y \qquad [14]$$

The parameter ϕ determines whether the level of the expenditure on schooling rises or falls with an increase in national income. Using [14], the differential equation describing the evolution of public debt can be written as

$$\dot{B} = rB - \phi Y \qquad [15]$$

Solving this differential equation and multiplying both sides by e^{-rt}, to get the present value of public debt, yields

$$e^{-rt}B(t) = B(0) - \phi \int_0^{+\infty} e^{(r-g)t} dt \qquad [16]$$

with $B(0)$ being the public debt at time $t = 0$, and g the balance growth rate at which the national income grows in the long period. Note that for $r < g$ the intertemporal budget constraint is irrelevant because in this case the economy is dynamically inefficient, implying that the government can play a Ponzi game. Therefore, the only interesting case is $r > g$. If the government increases public debt, for whatever reason, it must raise the primary surplus so that fiscal policy remains sustainable. This, however, means that more resources must be used for the debt service, implying that the government has less scope for other types of spending, e.g., on schooling as in our case.

5.2 The household and the productive sector

The utility function is given by equation [1]. The only difference is that the assets are now denoted by $S = B + K$ which are equal to public debt, B, and physical capital , K, and t is the income tax. The economy budget constraint becomes $\dot{S} = (1-t)Y - C$. Using the first order conditions, the growth rate of consumption is derived as

$$g = \frac{\dot{C}}{C} = \frac{1}{\gamma}((1-a)(1-t)(\frac{E}{K})^{a} - \rho) \qquad [17]$$

This condition is the same as the one that characterizes the public scheme of education financing.

5.3 The balanced growth path

An equilibrium allocation is defined as an allocation such that the firm maximises profits, implying that factors prices equal their marginal products, the household solves [1] subject to budget constraint, and the budget constraint of government is fulfilled. To find the differential equation describing the evolution of capital, i.e. the economy resource constraint, we note that $\dot{K} + \dot{B} = Y - C - T$,

$$\dot{K} + rB - \phi Y = Y - C - T \qquad [18]$$

Using the equilibrium condition, $Y = K^{1-a}E^{a}$ and $r = (1-a)K^{-a}E^{a}$, the economy-wide resource constraint is

$$\frac{\dot{K}}{K} = -(1-a)K^{-a}E^{a}\frac{B}{K} + \phi\frac{K^{1-a}E^{a}}{K} + \frac{K^{1-a}E^{a}}{K} - \frac{C}{K} - t\frac{K^{1-a}E^{a}}{K} \qquad [19]$$

The description of the economy is completed by equation [17] giving the evolution of consumption, and by the differential equation describing the path of public debt. Thus, we have the following three-dimensional differential system with appropriate initial conditions and the usual transversality conditions

$$\frac{\dot{K}}{K} = -(1-a)(\frac{E}{K})^a \frac{B}{K} + (\phi - t)(\frac{E}{K})^a + (\frac{E}{K})^a - \frac{C}{K} \qquad [20]$$

$$\frac{\dot{C}}{C} = \frac{1}{\gamma}((1-a)(1-t)(\frac{E}{K})^a - \rho) \qquad [21]$$

$$\frac{\dot{B}}{B} = (1-a)(\frac{E}{K})^a - \phi(\frac{E}{K})^a \frac{K}{B} \qquad [22]$$

The rates of growth tend to zero if the decline in the marginal product of physical capital, caused by a rising capital stock, is not made up of an increase in schooling expenditure. However, if the stock of public education is sufficiently large for the marginal productivity of private capital not to converge on ρ in the long run, we observe endogenous growth. In this case the r.h.s. of the system is always positive.

Defining $c = C/K$, $b = B/K$ and $e = E/K$, the new system of differential equations is given by $\dot{c}/c = \dot{C}/C - \dot{K}/K$, $\dot{b}/b = \dot{B}/B - \dot{K}/K$ and $\dot{e}/e = \dot{E}/E - \dot{K}/K$, leading to

$$\frac{\dot{c}}{c} = \frac{1}{\gamma}((1-t)(1-a)e^a - \rho) + (1-a)e^a b + (\phi - t)e^a + e^a - c \qquad [23]$$

$$\frac{\dot{b}}{b} = (1-a)e^a - \phi b^{-1}e^a + (1-a)e^a b + (\phi - t)e^a + e^a - c \qquad [24]$$

A stationary point of this system then corresponds to a balanced growth path of the original system where all variables grow at the same rate. A solution of $\dot{c} = \dot{b} = 0$ with respect to c, b gives a balanced growth path for the model an the corresponding ratios, b^*, c^*. We thus obtain c for $\dot{c}/c = 0$,

$$c = \frac{1}{\gamma}((1-t)(1-a)e^a - \rho) + (1-a)e^a b + (\phi - t)e^a + e^a \qquad [25]$$

Inserting this value in equation [24], and setting the l.h.s. equal to zero we obtain

$$b^* = \frac{-\phi(e^*)^a}{\frac{1}{\gamma}((1-a)e^a - \rho) - (1-a)e^a} = \frac{\phi(e^*)^a}{r-g} \qquad [26]$$

In this model the level of public deficit, $b*$, becomes an endogenous variable, and it can be positive or negative. However a positive value of government debt is more realistic, since most real economies are characterized by public debt. If $\phi > 0$ the primary surplus rises as national income increases, and sustained growth with positive public debt is feasible. The government becomes a net debtor in order to provide the level of schooling necessary for growth. If the public expenditure has a productive nature, as it does in this case, the government must not conduct a too strict budgetary policy. This holds because with an overly strict budgetary policy the government does not invest enough in the formation of human capital, which is the source of economic growth in this framework. But whilst the government may run deficits, it must also increase the primary surplus as public debt rises so that the path of public debt remains sustainable.

6 **Conclusions**

The paper has considered the influence of schooling expenditure on long-run growth as input in the production function. This topic was addressed in the earlier literature during the 1960s, but the final result suffered from a severe shortcoming due to the fact that education was considered as a vehicle to improve labor-force efficiency. In this kind of perspective the economic growth remained an exogenous process. The recent literature has addressed the problem of the role of education in economic growth in a different way so as to obtain sustained growth. On this approach, schooling expenditure can be considered a factor that counteracts any diminishing returns to capital, the key element of endogenous growth theory. Per capita growth is feasible because the decline of the marginal product of physical capital, brought about by increases in that stock, is made up of investment in schooling.

The government finances expenditures in the schooling sector by the tax revenue and by public deficit. In this case the main result is that there is a level of public debt compatible with economic growth. The government may run deficits but it must increase the primary surplus as public debt rises so that the path of public debt remains sustainable.

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