Education and Poverty in a Solow Growth Model*

Thomas Bassetti†

Abstract

In this paper, we study the effect of education on economic growth. In particular, we show how education may generate nonlinearities in the process of economic growth. In our model, the assumption of a non constant human capital obsolescence rate causes non constant returns to education in the production of human capital. This will allow us to identify the conditions under which a poverty trap may arise in a Solow growth model. As we will see, a sufficient investment in education may help poor countries to escape from this trap. Subsequently, we will conduct some econometric analyses to prove that, at the aggregate level, our theoretical conclusions are confirmed by evidence.

Keywords: Economic growth, education, poverty trap.
JEL: E13, I2, O4.

*For helpful comments, I am grateful to W. Baumol, W. Brock, D. Fiaschi, A. M. Lavezzi, and seminar participants at the University of Pisa in Pisa. The usual disclaimer applies.

†Dipartimento di Scienze Economiche, Università di Pisa, Via Ridolfi 10, 56100 Pisa Italy, e-mail: bassetti@ec.unipi.it.

1 Introduction

In macroeconomic literature, education is considered one of the most important inputs to produce human capital, which can be defined as: "the stock of accumulated skills and experience that make workers more productive," [Stiglitz and Boadway (1994)]. Traditionally, human capital does not play an explicit role in the exogenous growth theory, while it is central in the endogenous growth theory.[1] Nevertheless, the exogenous growth model has been recently extended to the inclusion of human capital by Mankiw, Romer and Weil (hereafter MRW) (1992). By using a cross-country analysis, they show that data are fairly consistent with a Solow model augmented to take into account human capital as a factor of production. They obtain a rather satisfactory estimate of the aggregate production function. In this framework, education does not produce externalities at the aggregate level. That is, education appears as a private input, which is remunerated according to its marginal product. Barro and Sala-i-Martin (1995), through an extensive test on cross-country data, show that the MRW model can explain several empirical facts.

A common assumption in the theory of economic growth is that human capital is produced under constant returns to scale, using education as a single input. Yet, there is no compelling evidence to support this assumption. On the contrary, an increasing number of analyses shows that the production of human capital exhibits increasing returns to scale for low levels of education and decreasing returns to scale for high levels of education. For instance, Krueger and Lindahl (2001) find evidence in favor of an inverted-U shaped relationship between the stock of human capital and the GDP growth rate. By comparing different regression models, they find that the best fitting of the data is provided by a regression model that considers a quadratic form for education. The inverted-U pattern suggests that social returns to schooling are increasing only for countries with a low level of education (below 7.5 average years of schooling), while for countries with high levels of education (above 7.5 average years of schooling) social returns to schooling are decreasing. By analyzing the effect of human capital in an open economy, Isaksson (2002) confirms the Krueger and Lindahl's result concerning the existence of a nonlinear relationship between education and economic growth. By introducing a measure for trade openness, Isaksson finds also important interaction effects between education and trade openness when education enters in a nonlinear fashion.

At the micro level, Trostel (2004) obtains the same results. By using an international microdataset to estimate an aggregate Mincerian equation [from Mincer (1974)], he shows that the production function of human capital displays increasing (private) returns at low levels of education and decreasing (private) returns at high levels of education. Since these nonlinearities occur primarily within countries, Trostel concludes that these nonlinearities could be a direct consequence of the process of human capital accumulation. In other words, he shows that human capital production function has a cubic shape, that is, the typical production function used in microeconomics courses.

In this paper, we aim to provide an exogenous growth model able to explain Krueger and Lindhal's results as well as Trostel's results. To do this, we introduce a possible mechanism to explain the nonlinear relationship between education and human capital and then between education and economic growth. Here, as in standard literature, education is a fundamental input to produce human capital. Nevertheless, with respect to standard theory, we do not assume a priori the existence of constant returns to scale in producing human capital through education. In particular, we study the effect of a non constant depreciation rate of human capital on GDP

[1] A few examples of growth models in which human capital plays a central role in economic growth are: Nelson and Phelps (1966), Lucas (1988, 1990), Romer (1986, 1990), King and Rebelo (1990), Grossman and Helpman (1991).

growth.

Following studies such as Rosen (1976), McPherson and Winston (1983) and Neumann and Weiss (1995), we assume that the depreciation rate of human capital is positively related to the level of education attained by an individual.[2] As we will see, given this hypothesis, education may generate a nonlinear process of human capital accumulation. We will then use this result to show how the process of human capital accumulation may cause the existence of multiple equilibria, and consequently the existence of a poverty trap, in a Solow growth model.[3] There we will see how investing in education is a possible way out from a situation of persistent poverty.

In the second part of the paper, we use a cross-section of 78 countries to conduct an econometric analysis. The aim of this analysis is to show that, at the aggregate level, returns to education are not constant. To see this fact, we estimate the impact of education on total factor productivity (TFP) growth by using parametric and semiparametric techniques. We will show that evidence is consistent with our theoretical conclusions.

The rest of the paper is organized as follows: Section 2 contains our modified Solow growth model; Section 3 contains our econometric analysis; Section 4 illustrates the main conclusions of this work.

2 The Model

In this section, we propose a growth model in which the process of human capital accumulation can explain the existence of nonlinearities in the economic growth path. To do this, we divide the model into two parts. In the first part of the model, we show how, through the acquisition of formal education, individuals can accumulate human capital in a nonlinear way. In the second part of the model, we put this result into a traditional Solow growth model in order to study the consequences of these nonlinearities on the aggregate level of output.

2.1 The Process of Human Capital Accumulation

Consider a closed economy in which markets are competitive and economic activity is performed over continuous time. Time is indexed by t, and individuals live for an infinite time horizon. Let L_t be the mass of population at time t, and assume that a *representative agent* i exists. Let u be the constant fraction of time that each individual devotes to work, while $e_{i,t}$ denotes the amount of time that agent i has already invested in education at time t.[4] Hereafter, for the sake of simplicity, we will omit the subscript i when there is no risk of confusion.

We temporary assume that individual i has already decided the amount of time to invest in education. That is, in this section, we do not make any particular assumption on the determinants of e_t; while, in the next section, we will specify a functional form for e_t.

Following the standard literature on human capital and economic growth, we consider education as the only input to produce human capital. Nevertheless, with respect to the standard

[2]This assumption is supported by several studies. For a rather detailed review of these studies see de Grip (2004).

[3]Other theoretical models suggest that human capital may have a nonlinear effect on economic growth. For example, by using an overlapping generation (OLG) model, Azariadis and Drazen (1990) explain the existence of multiple equilibria from threshold externalities in human capital accumulation. At the same time, Redding (1996) provides an endogenous growth model in which the presence of strategic complementarities between the accumulation of human capital and the investment in R&D sector may cause the existence of multiple equilibria in the economic growth path. However, these models are in contrast with micro evidence concerning the existence of non constant private returns to education in producing human capital.

[4]Obviously, the amount of time that individual i has already spent in liesure at time t will be $(1-u)t - e_{i,t}$. We abstract from the issue of optimal choice of allocation of time among work, education and leisure.

theory, we do not assume *a priori* the existence of constant returns to scale in producing human capital. Therefore, using a general human capital production function for individual i, we have that

$$h_t = h(e_t), \quad h'(e_t) > 0 \tag{1}$$

where h_t is the individual stock of human capital at time t, and $h(.)$ is an unknown function. Equation (1) states that the individual stock of human capital depends on the time already invested by individual in education at time t. Nevertheless, this equation does not explicitly describe the relationship between education and human capital. In order to obtain a more specific formulation for Equation (1), we must specify the mechanism through which individuals accumulate human capital.

As in MRW (1992), human capital variation will be determined by the difference between the human capital created in a given period and the human capital destroyed in the same period. Nevertheless, with respect to MRW, we will assume that the destruction (depreciation) of human capital will depend on the quantity of human capital subject to obsolescence. In particular, we will assume that the depreciation rate of human capital increases as the stock of human capital increases.

By analyzing the effect of technological change on schooling-specific obsolescence, that is the obsolescence of skills acquired at school, various papers provide theoretical and empirical arguments in favor of this hypothesis.[5] According to this literature, technological progress increases the depreciation rate of human capital. This is due to the fact that new technologies require the acquisition of new knowledge, which replaces part of the existing knowledge. Therefore, the higher the rate of technological innovation is, the higher the depreciation rate of human capital will be. In these studies, two causes may justify the assumption of an increasing depreciation rate of human capital:

1. The first cause is known as "vintage effect" and is well illustrated in Neumann and Weiss (1995). There, they give evidence that high skilled workers are more affected by depreciation of human capital than low skilled workers. Neumann and Weiss observe that, with respect to traditional sectors, high-tech sectors are subject to a higher rate of technological innovation. Since high-tech sectors use a highly specialized workforce - that is a workforce composed by engineers, computer scientists, biologists, etc. - at any given time, we may reasonably expect a higher rate of obsolescence for the most specialized workers, which are typically the most educated ones. By assuming that the degree of knowledge specialization increases with the grade of education, we may conclude that: "at the individual level, the knowledge obsolescence increases as specialization increases" [McPherson and Winston (1983)].

2. The second cause is known as "technical depreciation" and concerns the fact that the speed of technological change has accelerated over time. As stressed in de Grip (2004), in a context in which innovation accelerates over time, we must expect that even the obsolescence rate of knowledge accelerates continuously. This view is largely supported by the experience of the past several decades, where technological changes have produced an increasing demand of skills by firms. This happens independently of the level of education of a worker. Today an engineer, with respect to thirty years ago, experiences a higher rate of knowledge obsolescence, hence knowledge becomes obsolete more quickly in any production sector. For this reason, it is reasonable to assume that an increasing speed of innovation leads to an increasing depreciation rate of human capital.

[5] The main contributions to this literature are: Rosen (1976), Weiss and Lillard (1978), Johnson (1980), McPherson and Winston (1983), Neumann and Weiss (1995) and de Grip (2004).

Even if the above mentioned causes of human capital obsolescence can operate separately, we expect to observe both phenomena simultaneously. According to Rosen (1976), these two causes of human capital depreciation are indistinguishable. However, Neuman and Weiss (1995) argue that, since the *vintage effect* is not the same in all sectors while *technical depreciation* is, it is possible to identify both causes by comparing data from low and high technology sectors. Consistently with this point of view and with our future assumption of a constant exogenous rate of technological progress, we will consider only the *vintage effect*.

Now, we can write our human capital accumulation function:

$$\frac{dh_t}{de_t} = Bh_t - \phi(h_t)h_t, \quad \phi'(h) > 0, \tag{2}$$

where dh_t/de_t is the human capital variation with respect to e_t, $B > 0$ is a measure of gross productivity of the education sector, $\phi(.)$ is the depreciation coefficient of human capital and $\phi' > 0$ captures the vintage effect mentioned above. In Equation (2), as in Lucas (1988), the term Bh_t represents the gross creation of human capital due to the acquisition of formal education, while the term $\phi(h_t)h_t$ represents the depreciation of human capital due to obsolescence.

By considering Equations (1) and (2) and assuming a linear relationship between the stock of human capital and its depreciation rate, that is $\phi(h_t) = \sigma h_t$, we have:

$$\frac{dh_t}{de_t} = Bh(e_t) - \sigma h(e_t)^2 \tag{3}$$

where σ is a positive parameter that represents the unitary variation of the depreciation coefficient of human capital. We assume that the productivity of education sector is greater than the unitary variation of the depreciation coefficient due to the *vintage effect*, that is, $B > \sigma$. This hypothesis ensures that an additional investment in education will always increase the individual stock of human capital.

To study how our economic system produces human capital, we have to solve Equation (3).[6] By normalizing the productivity of an illiterate worker to one, *i.e.* $h(0) = 1$, we obtain the following solution:

$$h_t = \frac{B\exp[Be_t]}{B - \sigma + \sigma\exp[Be_t]} \tag{4}$$

Equation (4) represents our human capital production function, which depends on the individual choices about the schooling level. In particular, here, we have obtained a *logistic production function of human capital*. In other words, the process of human capital accumulation is nonlinear. This means that the individual stock of human capital is not directly proportional to the time invested in education by agent i.[7] By assuming $B = 1$ and taking the value of σ estimated by Arrazola et al. (2005) for Spain ($\sigma = 0.017$), in Figure 1 we provide a graphycal representation of Equation (4).[8]

[6] Equation (3) is an autonomous differential equation also known as Verhulst's equation [from Verhulst (1845)]. A formal derivation of this solution is provided in Appendix A.

[7] Note that, in this model, the maximum level of human capital that an individual can accumulate ($H_{\max} = \frac{B}{\sigma}$) does not depend on any biological consideration on the capacity of human brain, but it depends positively on the productivity of education sector and negatively on the unitary variation of the depreciation coefficient of human capital.

[8] Although, as suggested by Van Leeuwen (2007), B is likely to be on occasion smaller than 1, in this context only a value of B too small may cause the absence of nonlinearities.

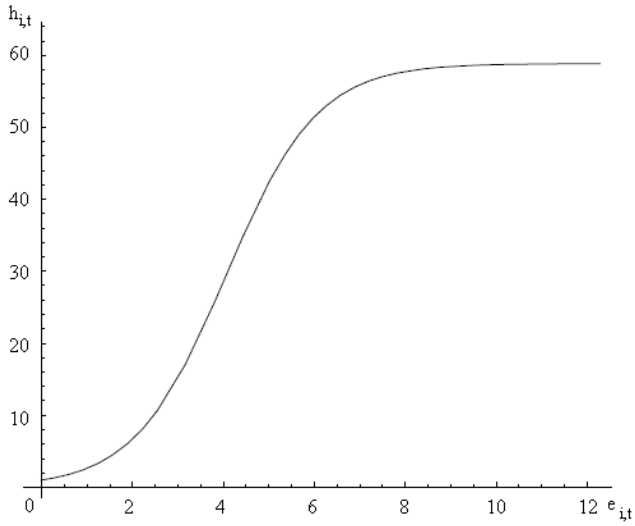

Figure 1

Figure 1 shows how returns to education in the production of human capital are increasing for low levels of education and decreasing for high levels of education. In particular, if human capital depreciation increases of about two percentage points for each additional unit of human capital, the production function of human capital starts to show decreasing returns to education after about 4.5 years of schooling.

We can also interpret Figure 1 in another way. In fact, Figure 1 states that the effort (in terms of time invested in education) necessary to acquire an additional unit of human capital initially decreases with the stock of human capital already accumulated by an individual and subsequently increases. This means that an additional year of schooling has different effects if it is attended by an individual with a low level of human capital or by an individual with a high level of human capital.[9]

By using a neoclassical framework, in the next section, we study the consequences of a logistic process of human capital accumulation on output growth.

2.2 A modified Solow growth model

2.2.1 Production function

Assume that output is produced by using a Cobb-Douglas production function with constant returns to scale,

$$Y_t = K_t^{\beta}(uH_tA_t)^{1-\beta}, \quad \beta \in [0,1] \tag{5}$$

where K_t is the total stock of physical capital, H_t is the total stock of human capital, and A_t is a technology parameter. In Equation (5), human capital and technological progress enter multiplicatively. That is, technological progress is *human capital-augmenting*. As we will see, this hypothesis will allow us to write the production function in terms of *units per effective-labor* by making the analysis much simpler.

[9] In this work, we do not consider the implications of our model in terms of welfare analysis, nevertheless this consideration may represent an interesting starting point for future studies.

Assuming that there are not external effects in the aggregation of human capital and considering that a representative agent exists, we can aggregate individual capacities as follows:

$$H_t = \int_0^{L_t} h_{i,t} di = L_t \overline{h}_t \tag{6}$$

where $\overline{h}_t$ is the average level of human capital at time t. Equation (6) states that, at the aggregate level, what matters is the average level of human capital. At the same time, according to (4), the average level of human capital will depend on the average level of education.

Therefore, by putting Equation (4) into Equation (6), we have

$$H_t = L_t \frac{B \exp[B\overline{e}_t]}{B - \sigma + \sigma \exp[B\overline{e}_t]} \tag{7}$$

where $\overline{e}_t$ represents the average level of education in our economic system at time t.

Finally, by considering Equation (7), we write Equation (5) in terms of quantities *per unit of effective-labor,*

$$y_t = {k_t}^{\beta} \left(\frac{B \exp[B\overline{e}_t]}{B - \sigma + \sigma \exp[B\overline{e}_t]} \right)^{1-\beta} \tag{8}$$

where k_t and y_t are respectively the amount of physical capital and output per unit of effective-labor, that is, $k_t \equiv \frac{K_t}{uL_t A_t}$ and $y_t \equiv \frac{Y_t}{uL_t A_t}$. In the production function described by (8), education is not a proxy variable for human capital as usual but enters as input in the production of human capital.

In the next section, we examine the dynamic behavior of the inputs into production. Since this is a fairly standard analysis, we will especially emphasize the main differences of our model with respect to the standard version of the Solow model.

2.2.2 Dynamics of the inputs into production

As in the exogenous growth theory, we assume that technology and labor grow at the exogenous rates g and n, respectively.[10] Moreover, since output is divided between consumption and investment, we can write the Solowian accumulation function of physical capital as follows:

$$\dot{k}_t = sy_t - (\delta + g + n)k_t \tag{9}$$

where s is the exogenous saving rate and δ is the exogenous depreciation rate of physical capital.

Now, we must substitute Equation (8) into Equation (9) in order to study the whole dynamics of our economic system

$$\dot{k}_t = s[{k_t}^{\beta} h(\overline{e}_t)^{1-\beta}] - (\delta + g + n)k_t. \tag{10}$$

Expression (10) is similar to the Solowian dynamic equation of capital per labor services, except for the presence of human capital. Given Equation (10), we can find the steady-state level of physical capital per capita (k^*) simply equalling $\dot{k}_t$ to zero and solving for k_t :

$$k^* : \quad s{k_t}^{\beta} h(\overline{e}_t)^{1-\beta} = (\delta + g + n)k_t \tag{11}$$

[10] Formally, we will have that $\dot{A}_t = gA_t$ and $\dot{L}_t = nL_t$.

Once we have found k^*, we can put it into Equation (8) in order to obtain the steady-state level of output given a certain average level of education.

2.2.3 The possibility of a poverty trap

Until now, in the equations of the model, physical and human capital have always been considered separately. Nevertheless, some macroeconomic studies suggest that output growth, and consequently physical capital accumulation, can influence the average level of education.

For example, Bils and Klenow (2000) find that the level of education registered in a country depends positively on its GDP growth. Bils and Klenow investigate the presence of a reverse causality between human capital and growth. In contrast with previous studies such as Barro (1991), Benhabib and Spiegel (1994) and Barro and Sala-i-Martin (1995), they conclude that the causality direction is from economic growth to schooling and not the opposite. In a more recent work, Glewwe and Jacoby (2004) find a positive relationship between the individuals' investment in education and their levels of wealth.

Therefore, in line with these studies, we assume that, at the aggregate level, there exists a positive relationship between the physical capital per capita of a country and the average level of education of its population.[11] This assumption can be formalized as follows:

$$\overline{e}_t = k_t^b, \quad b > 0. \tag{12}$$

Equation (12) states that in a rich country individuals invest more time in education than those in a poor country. This means that the average stock of human capital depends positively on the stock of physical capital per unit of effective-labor and on the constant elasticity coefficient of e with respect to k, that is, b.

By combining Expressions (4), (8) and (12) we can obtain a more specific formulation for y_t in which output depends only on physical capital. In this way, we can obtain the dynamics of y_t from the dynamics of k_t,

$$y_t = k_t^{\beta} \left(\frac{B \exp[\underline{B} b k_t]}{B - \sigma + \sigma \exp[\underline{B} b k_t]} \right)^{1-\beta} \tag{13}$$

where $\underline{B} \equiv B^{\frac{1}{b}}$. Now, the steady-state solutions of our augmented Solow model can be found as follows:

$$k^* : \quad s{k_t}^{\beta} \left(\frac{B \exp[\underline{B} b k_t]}{B - \sigma + \sigma \exp[\underline{B} b k_t]} \right)^{1-\beta} = (\delta + g + n) k_t \tag{14}$$

or

$$k^* : \; F(k_t) \; = (\delta + g + n) \tag{14'}$$

where $F(k_t) = s{k_t}^{\beta-1} \left(\frac{B \exp[\underline{B} b k_t]}{B - \sigma + \sigma \exp[\underline{B} b k_t]} \right)^{1-\beta}$.

While in the Solow growth model the function $F(k_t)$ is monotonically decreasing, here $F(k_t)$ may be a non-monotonic function.[12] This means that, as in Figure 2, we could observe the

[11] The introduction of these complementarities in the accumulation of physical and human capital will allow us to explain the level of output only in terms of physical capital per unit of effective-labor, as in the Solow model. In this way, we will obtain a growth model in which nonlinear dynamics in the economic growth path may appear.

[12] In Appendix B, we study the conditions under which $F(k_t)$ is non-monotonic.

existence of three different stationary equilibria, two of which are stable and one is unstable.[13] In particular, the low equilibrium (k_L) and the high equilibrium (k_H) are stable, while the middle equilibrium (k_M) is unstable. The low equilibrium is also known in literature as *poverty trap*, that is, an equilibrium characterized by low levels of capital (human and physical) and income per capita.

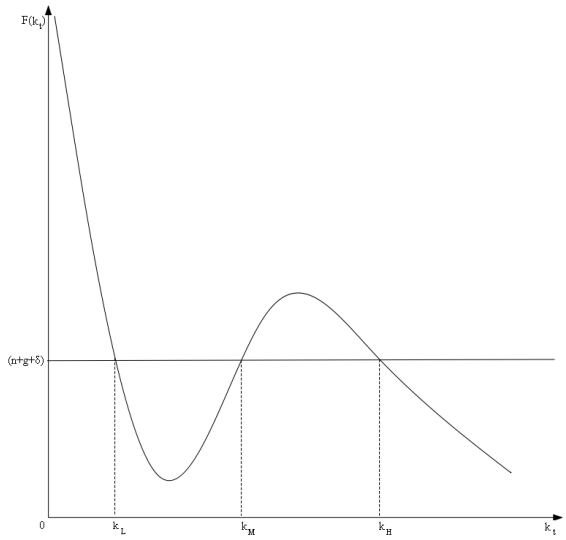

Figure 2

Let us examine the dynamics of physical capital implied by Figure 2. Assuming an initial level of physical capital k_0 such that $0 < k_0 < k_L$ (or $k_M < k_0 < k_H$), Figure 2 states that the accumulation of physical capital per unit of effective-labor will exceed the depreciation of physical capital due to osolescence, technical progress and population growth, therefore we will observe an increase in the level of k_t. This increase in k_t will rise the level of output either directly as input for the production of y_t or indirectly causing an increase in the level of education and consequently of human capital. In this situation, k continues to rise until it reaches the value k_L (or k_H), at this level physical capital will remain constant.

On the contrary, if $k_L < k_0 < k_M$ (or $k_H < k_0 < +\infty$), the depreciation of physical capital due to osolescence, technical progress and population growth will exceed the accumulation of physical capital per unit of effective-labor, therefore we will observe a decrease in the level of k_t until it will reach the value k_L (or k_H). We can conclude that the convergence of k_t towards the value k_L or the value k_H depends on the initial value of physical capital per unit of effective-labor, k_0.

In the next section, we provide a numerical example in which we show that, under plausible values of parameters, the model presented in this section can explain the existence of multiple equilibria.

2.2.4 Investing in education to escape from the trap

Here, we study the consequences of parameters changes on our theoretical model. To do this, we use a numerical analysis in which the values of exogenous parameters are those observed for

[13] As well discussed by Galor (1996), the non-monotonic behavior of $F(k_t)$ is a necessary, but not sufficient, condition for the existence of multiple equilibria. For example, if s is sufficiently high (or $n + g + \delta$ is sufficiently low) the function $F(k_t)$ will match the function $(n + g + \delta)$ only one time.

the European Union (EU-15) in 2001.[14] There, the depreciation coefficient of physical capital was not available for all countries; thus, we have taken the depreciation coefficient observed by MRW (1992). Table 1 contains the parameter values used to draw Figure 3.

Table 1: Parameters used in the numerical example	
B	1
s	0.127
b^*	0.504
β	0.250
σ^{**}	0.017
δ^{***}	0.030
g	0.028
n	0.014
*Levine et al. (2000)	
** Arrazola (2005)	
*** MRW (1992)	

In Table 1, the value of σ refers to the one estimated by Arrazola et al. (2005) for Spain, while the parameter b has been estimated by using the data set provided by Levine et al. (2000) and described in the next section.[15]

Figure 3 shows that, under plausible values of the exogenous parameters, the function $F(k_t)$ is non-monotonic. Nevertheless in Figure 3, since we have used parameters from developed countries, $F(k_t)$ equals the constant value $n + g + \delta$ in only one point. This means that for countries of EU-15 we have a unique steady-state equilibrium (k^*).

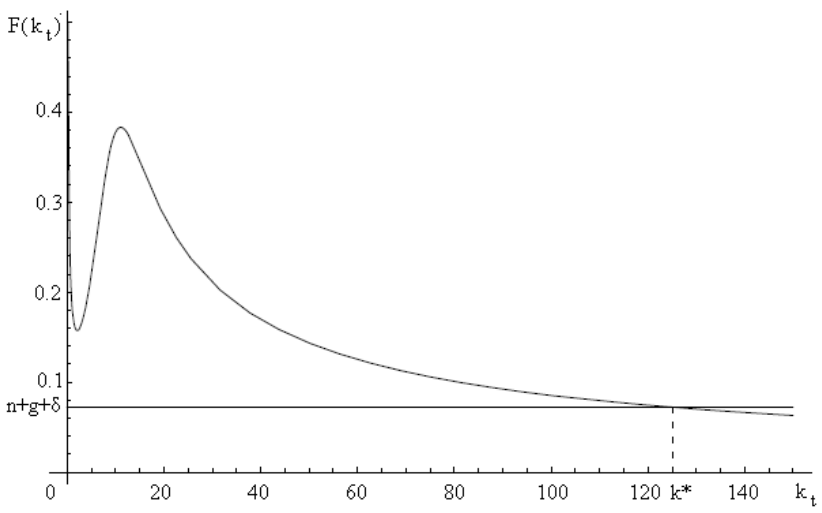

Figure 3

[14]Source Eurostat (2004).

[15]In order to obtain b, we have regressed the logarithm of the average years of education, e_t, on the logarithm of the current stock of physical capital per capita, k_t. That is, by using the OLS method, we have estimated the following log-linear transformation of Equation (12): $\ln(e_t) = b \ln(k_t)$.

Figure 4 shows what happens to F when the elasticity coefficient of e with respect to k, that is b, changes. In particular, if the value of b is 0.10, the function F will match the constant value $n + g + \delta$ in three different points (k_L, k_M, k_H). These points are three different steady-state equilibria, two of which are stable (k_L, k_H). In this situation, as discussed in the previous section, countries with a stock of physical capital per capita lower than k_M will converge to the equilibrium characterized by a low level of physical capital per capita, k_L: that is, these countries fall in a *poverty trap*. At the same time, countries with a stock of physical capital higher than k_M will converge to the highest equilibrium, k_H. Figure 4 also shows how a sufficiently large increase in b ($b = 0.50$) will shift the function F up enough to eliminate the lower steady-state. This means that a highly educated society, that is a society in which individuals spend a large part of their time to study, will converge to a high level of physical capital per capita. In fact, above a certain level of b, the lower and the middle equilibria disappear, and we will observe only one (high) equilibria. This result supports the idea that, policies devoted to increase the average years of schooling in less developed countries can help these countries to escape from a situation of poverty trap.[16]

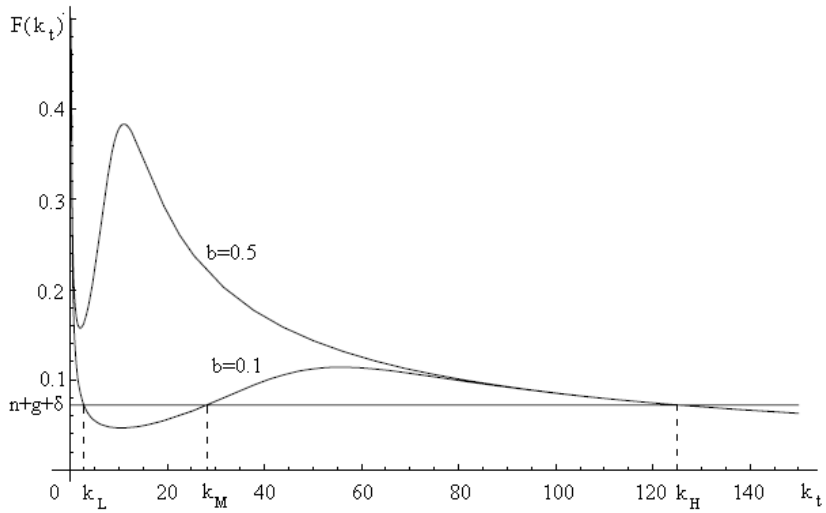

Figure 4

Figure 5 shows that if β rises our graph becomes smoother. This is due to the fact that we are keeping the hypothesis of constant returns to scale in production. That is, if β increases, the elasticity coefficient of human capital $(1 - \beta)$ decreases and the nonlinear effect of education becomes relatively less important. Note that when β increases multiple equilibria may appear, disappear or remain. In particular, in Figure 5 an increase in β implies only a higher steady-state level of k.

[16] In Figure 4 an increase in b has the same effect that an increase in the saving rate, s, has in Galor (1996). In both cases $F(k)$ shifts up and only one equilibrium remains.

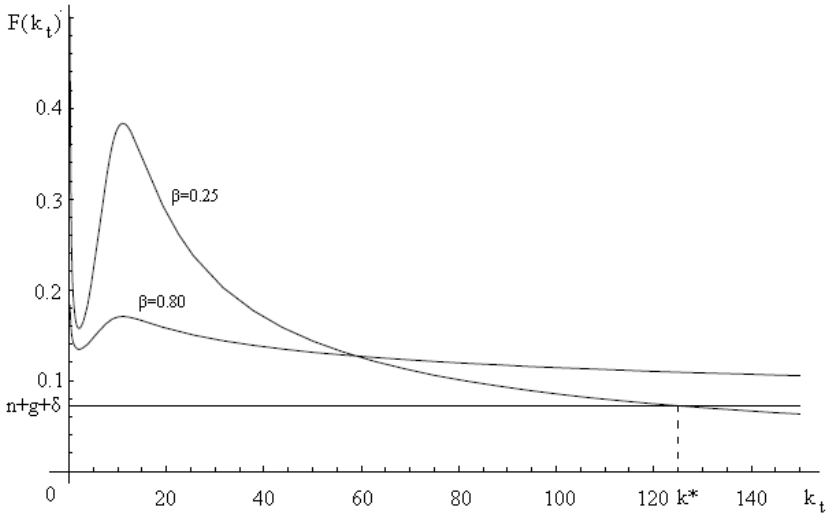

Figure 5

Finally, let us discuss the effect of a change in the value of σ. Figure 6 shows that multiple equilibria tend to disappear when the unitary depreciation of human capital due to vintage effect rises. In fact, an increase of σ slows down the process of human capital accumulation which causes the existence of nonlinear dynamics in our model. Therefore, if σ increases enough, we will observe only one equilibrium as in the traditional Solow model. However, in this situation, given the small contribution of human capital to the production of output, the unique steady-state equilibrium will be lower than the highest equilibrium observed in a situation with multiple equilibria. Symmetrically, when σ decreases, the accumulation of human capital becomes easier and nonlinearities more pronounced. This means that, given the assumption of an increasing depreciation coefficient of human capital (due to vintage effect), the dynamics of output growth tends to be nonlinear, and these nonlinearities are more important when σ is smaller.

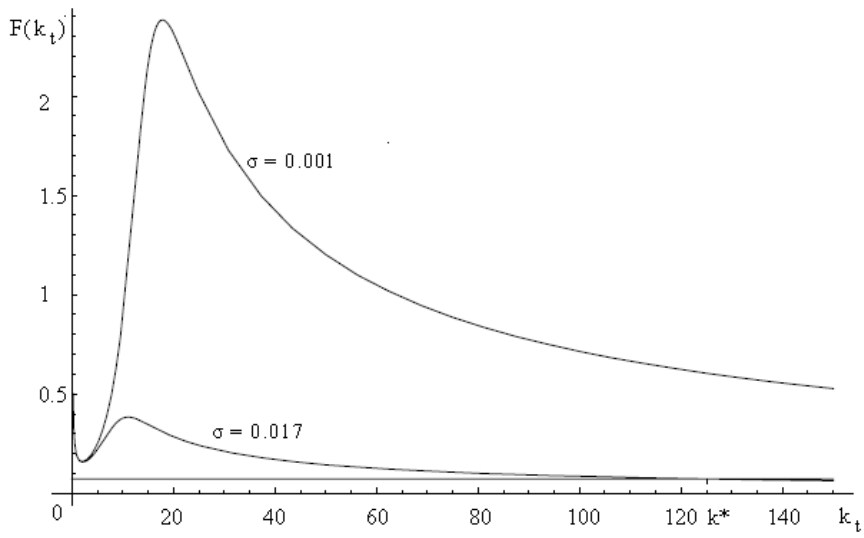

Figure 6

In the next section, we evaluate the impact of education on TFP growth. We accomplish this by using parametric and semiparametric methods that allow the effect of education on economic growth to be nonlinear.

3 Empirical Analysis

In this section, we want to test the effects of education on economic growth. By relaxing the assumption of a linear relationship between education and growth, we show that important nonlinearities emerge in this relationship. In particular, as predicted by our model, we will observe an inverted-U shaped relationship between education and TFP growth.

3.1 Data and Variables

Data come from Levine, Loayza, and Beck (2000). Their sample consists of 78 countries with data for the period 1960-1995. In order to eliminate the business cycle effects, data have been collected in groups of five-year averages.[17]

The dependent variable is the TFP growth rate. TFP growth was estimated from a Cobb-Douglas production function with constant returns to scale by setting the capital share at 1/3 and the labor share at 2/3.[18] Among the explanatory variables, particular attention is paid to the effect of education (EDU) on TFP growth. Here, education is measured by the average years of schooling of the population aged 15 years and older. Many variables influence the process of human capital accumulation: quantity and quality of schooling, work experience, general and specific skills. Unfortunately, with respect to the years of schooling, the other dimensions are not so easy to quantify.

In this paper, we also estimate the effects of some control variables on TFP growth: the investment share (INV), measured as fixed investment share of GDP; the financial depth (LLY), measured as the ratio of liquid liabilities to GDP; the trade openness ratio (TRADE), measured as the ratio of exports plus imports to GDP; the inflation rate (PI); the population growth rate (POP); and two dummy variables (D70, D80), to control for time period effects with reference to decades '70 and '80. Except for the average years of schooling, all the explanatory variables are expressed in terms of growth rates.

3.2 Econometric Models

As in Hall and Jones (1999) and in line with our theoretical framework, we develop the econometric analysis by starting from an aggregate production function equal to previous Equation (5):

$$Y_t = K_t^{\alpha}(A_t H_t)^{1-\alpha} \tag{15}$$

where A_t represents the exogenous component of technological progress, $H_t \equiv h_t L_t$ and labor is assumed to be homogeneous within a country.[19]

Moreover, as suggested by Hall and Jones (1999) and Bils and Klenow (2000), we consider the following human capital production function:

$$h_t = \exp[f(\overline{e}_t)]. \tag{16}$$

[17] In Appendix C we have reported the country list.

[18] These values are rather standard in the growth accounting literature [see, e.g, Benhabib and Spiegel (2002)].

[19] Since data on hours per worker are not available for most countries, we use the number of workers to measure the labor input.

According to Bils and Klenow, Equation (16) represents the appropriate way to incorporate the average years of schooling into an aggregate production function. Here, the function $f(\overline{e}_t)$ captures the efficiency of a unit of labor with $\overline{e}_t$ years of schooling, while the derivative $f'(\overline{e}_t)$ captures the returns to schooling in producing human capital, that is, an additional year of schooling raises the efficiency of labor by $f'(\overline{e}_t)$. Therefore, $f'(\overline{e}_t)$ also represents the returns to schooling estimated in a Mincerian wage regression.[20]

In this section we want to estimate the shape of $f(\overline{e}_t)$. As suggested by Josephson (2006), the best way to do this is taking as dependent variable the TFP growth rate instead of real GDP growth rate.[21]

Let us write Equation (15) in terms of output per worker

$$y_t = k_t^{\alpha}(A_t h_t)^{1-\alpha} \tag{17}$$

where $y_t \equiv Y_t/L_t$ and $k_t \equiv K_t/L_t$. Now, we can decompose output per worker into inputs and productivity simply by taking the logarithm of (17)

$$\ln y_t = \alpha \ln k_t + (1-\alpha)\ln A_t + (1-\alpha)\ln h_t \tag{18}$$

that is,

$$\ln TFP_t = (1-\alpha)\ln A_t + (1-\alpha) f(\overline{e}_t) \tag{19}$$

where $\ln TFP_t = \ln y_t - \alpha \ln k_t$. As specified in the previous section, we have imposed $\alpha = 1/3$ in order to obtain the values of TFP.

Finally, we can write Equation (19) in terms of growth rates

$$g_{TFP_t} = (1-\alpha) g_{A_t} + (1-\alpha) g_{H_t} \tag{20}$$

where $g_{TFP_t} = \ln TFP_{t+1} - \ln TFP_t$, $g_{A_t} = \ln A_{t+1} - \ln A_t$ and $g_{H_t} = f(\overline{e}_{t+1}) - f(\overline{e}_t)$. In line with the *growth accounting* literature, the term g_{A_t} can be calculated from (20) as residual.

We will use different specifications for the last equation in order to test different hypotheses concerning the returns to education. The first specification refers to the hypothesis that the percentage gains in human capital from each year of schooling is constant. This hypothesis can be formalized as follows:

$$g_{TFP_t} = \beta_0 + \beta_1 \overline{e}_t + \gamma X_t^T + \varepsilon_t \tag{21}$$

where β_0 is the intercept and represents the gain (or loss) of TFP when a country's labor force is completely illiterate, β_1 is the coefficient of $\overline{e}_t$, γ is a vector of coefficients, X_t is a vector of control variables expressed in terms of growth rates and ε_t is a white noise error.

To verify the hypothesis of a nonlinear contribution of education to TFP growth, as suggested by our theoretical model, we must estimate alternative human capital accumulation functions. With respect to previous specification, now, we consider a model in which education enters in a quadratic form,

$$g_{TFP_t} = \beta_0 + \beta_1 \overline{e}_t + \beta_2 \overline{e}_t^2 + \gamma X_t^T + \varepsilon_t \tag{22}$$

Note that to observe increasing returns for low levels of education and decreasing returns for high levels of education, the coefficient β_2 should be negative. To make the regression model

[20] Note that if $f(\overline{e}_t) = 0$ for all $\overline{e}_t$ this is a standard production function with undifferentiated labor.

[21] Josephson (2006) shows that since the hypothesis of *homogeneity* between TFP and human capital cannot be rejected, "any changes in TFP growth is proportional to changes in human capital in a one-to-one relation."

more sensitive to changes in educational attainment, especially for high levels of education, we also estimate a cubic specification,

$$g_{TFP_t} = \beta_0 + \beta_1 \overline{e}_t + \beta_2 \overline{e}_t^2 + \beta_3 \overline{e}_t^3 + \gamma X_t^T + \varepsilon_t \tag{23}$$

The introduction of a cubic term, in the case of a positive value for β_3, will make the last equation closer to a logistic specification, where the decrease of returns to schooling slows down for high levels of education. To avoid the problem of heteroscedasticity for cross-section data, models (21)-(23) have been estimated by using a traditional GLS method.[22]

Finally, we want to compare our parametric estimates with the results of a semiparametric analysis in which the contribution of education to the TFP growth can be estimated without restrictions. To do this, we estimate a PLR (Partial Linear Regression) model in which the effect of education on TFP growth is *a priori* unknown. This model can be specified as follows:

$$g_{TFP_t} = \beta_0 + \psi(\overline{e}_t) + \gamma X_t^T + \varepsilon_t \tag{24}$$

here, $\psi(\overline{e}_t)$ is an unknown function, which has been estimated by using a normal kernel with bandwidth obtained by cross-validation. By looking at the shape of $\psi(\overline{e}_t)$, we expect to observe a positive relationship between education and TFP growth for relatively low levels of education and a negative relationship for relatively high levels of education.

3.3 Estimation Results

Table 2 contains our estimates for equations (21)-(24). The coefficients of control variables have the expected signs in all regressions, even if not all of these coefficients are statistically significant. In particular, investments and trade openness play a positive and statistically significant (at 1%) role in the process of TFP growth, while the coefficient of dummy variable for period 1970-1980 is negative and statistically significant (at 1%).[23]

[22] GLS and PLR estimates are obtained by using R programs provided by Abbey (1988) and Wood (2006), respectively.

[23] Usually, economists ascribe the period of *stagflation* registered during the 70s to the oil shocks observed in the same period.

Table 2: Cross-Country Regressions on TFP growth rate (1960-95)				
	GLS	GLS	GLS	PLR
	(eq.21)	(eq.22)	(eq.23)	(eq.24)
Control Variables				
$Intercept$	-0.0117 (0.719)	-0.0167 (1.025)	-0.0253 (1.481)	0.0027 (0.141)
$D70$	-0.0122 (3.532)**	-0.0113 (3.269)**	-0.0109 (3.156)**	-0.0111 (3.214)**
$D80$	-0.0053 (1.558)	-0.0046 (1.361)	-0.0047 (1.397)	-0.0047 (1.394)
POP	0.1571 (0.849)	0.2074 (1.122)	0.1649 (0.886)	0.1671 (0.899)
INV	0.0492 (2.860)**	0.0596 (3.404)**	0.0656 (3.678)**	0.0623 (3.536)**
PI	0.0009 (0.193)	-0.0016 (0.332)	-0.0020 (0.406)	-0.0015 (0.308)
$TRADE$	0.0109 (2.554)**	0.0106 (2.514)**	0.0115 (2.701)**	0.0112 (2.638)**
LLY	0.0003 (0.041)	0.0003 (0.050)	0.0001 (0.019)	0.0000 (0.006)
Main Variables				
EDU	0.0020 (2.013)**	0.0074 (3.290)**	0.0141 (3.089)**	NP
EDU^2		-0.0005 (2.670)**	-0.0020 (2.158)**	
EDU^3			0.0001 (1.687)*	
Obs	469	469	469	469
$Adj-R^2$	0.060	0.0727	0.0765	0.0765
$Log-Lik$	986.9842	990.6067	992.0618	992.2814
GCV				0.0009
$d.f.$	11	12	13	13.388
F-test	4.3685**	2.3578	1.1025	
t-statistics are given in parentheses: **(1%),*(5%).				
NP: variable enters in the nonparametric part of regression.				

The first column of Table 2 shows estimates of the model represented by (21), where we have assumed a linear contribution for education. As we can see, at this level of the analysis, the effect of education on TFP growth is positive and statistically significant at 1%. In particular, the estimate of β_1 is 0.002, this value indicates that an additional year of schooling raises the level of TFP by 0.2 percent. This value is far from the value provided by Psacharopoulos (1994) for private return to schooling, which was 10.1 percent for the world as a whole. Following Lange and Topel (2006), we can say that: "the possibility that social returns are smaller than private ones typically views education as merely redistributive-able individuals use schooling to signal their ability, which may raise wages even if it has not impact on productivity."

Column (2) of Table 2 reports the estimated parameters for the quadratic model represented by (22). The quadratic model explains data much better than the linear one. Here, the positive coefficient on the linear term (0.0074) and the negative coefficient on the squared term (-0.0005) suggest that an inverted U-shaped relation between education and productivity growth exists. Moreover, the coefficients on both the linear and squared terms are statistically significant at 1%. In other words, consistently with our theoretical model, these findings suggest that education

enters nonlinearly in Equation (19). In particular, returns to schooling are increasing for low levels of education and decresing for high levels of education. However, a quadratic specification implies that after a certain level of education (14.9 years of schooling) returns to schooling become negative, that is, human capital starts to decrease. For this reason, we have included a cubic term to allow for a more sensitive analysis of nonlinearities.

Column (3) of Table 2 reports the estimation results for Equation (23). This model fits data in a better way with respect to the linear and quadratic models. According to our findings, the magnitudes of the linear and quadratic terms are higher than in the quadratic specification and still significant at 1%. The coefficient of cubic term is positive (0.0001) and statistically significant at 5%. This means that, in line with the logistic model proposed in Section 2, the decrease of returns to schooling slows down for high levels of education.

Although the cubic model yields a statistically significant third order coefficient, this appears to be driven by the highest levels of education where, given the small number of observations, the confidence interval is rather large. On the other hand, this specification indicates that the relationship between education and TFP growth is more complex than previously thought. Therefore, we have also estimated the PLR model described by (24) in order to better characterize Equation (19).

In Column (4) of Table 2, we have reported estimates of the parametric part of Equation (24). As we can see, this specification provides the best fit with respect to previous specifications. Figure 7 presents the estimated function $\psi(\overline{e})$, which represents the nonparametric part of the model. This function has been obtained by using penalized regression splines with smoothing parameters selected by a GCV (Generalized Cross-Validation) criterion.[24] Here, the degree of smoothness is estimated as part of the model fitting, and, as suggested by Wood (2000), a Bayesian approach is employed to estimate the confidence interval.

The last row of Table 2 presents the results of the F-test for the null hypotheses of a linear, quadratic and cubic model respectively against the PLR alternative. The results suggest that the hypothesis of a linear model must be rejected at the 1% level, whereas we cannot reject the hypothesis of a quadratic or cubic model.[25]

[24] GCV criterion is a technique used to guide the selection of optimal parameters in smoothing splines and related regularization problems. The optimal combination of parameters is, typically, the one that produces a minimum GCV score. The GCV score function is defined as: $GCV(\alpha, \rho) = (\sum_{i=1}^{n} \frac{e_i^2}{n})/(1 - \frac{m}{n})^2$ where e is the vector of error terms, m is the number of parameters, n is the number of data points, $\alpha = \frac{K}{n}$ (K is the number of sub-samples) and ρ is the order of polynomial used to fit the data into the sub-samples.

[25] Even if the test statistics calculated from these models may not have exact or even asymptotic F distributions, following Hastie and Tibshirani (1990), we can say that the F-test provides a good and useful approximation.

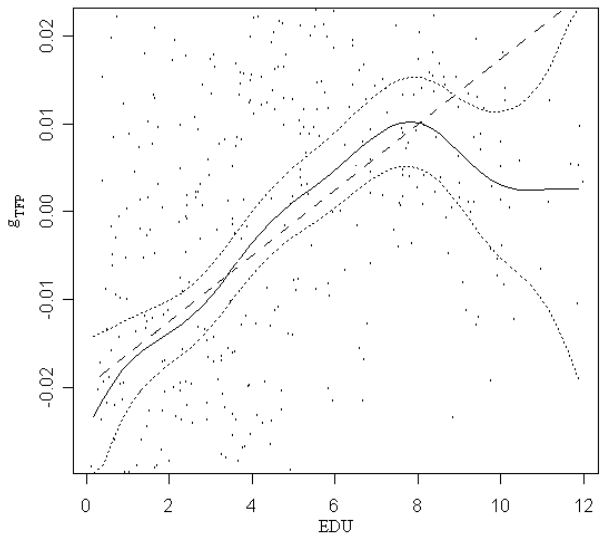

Figure 7

Having established that model (24) is the most preferred, we can analyze the behavior of function $\psi(\overline{e})$ more in depth. In Figure 7, function $\psi(\overline{e})$ shows how education and TFP growth are positively related for relatively low levels of education and negatively related for relatively high levels of education. That is, as suggested by our theoretical model, an increase in the level of schooling seems to be more profitable for countries with low levels of education than for countries with high levels of education. This result confirms the previous findings provided by Krueger and Lindahl (2001) when they find that before 7.5 years of schooling returns to education are increasing and after are decreasing. In fact, by looking at Figure 7, we can see that even $\psi(\overline{e})$ shows a peak at about 7.5 years of schooling.

By integrating the quadratic or the cubic specification and putting the result into the human capital production function, consistently with our theoretical model and also with Trostel's results, we obtain the expected logistic shape for (16).

4 Conclusions

In this paper we have studied the effect of education on economic growth. In particular, we have shown theoretically and empirically that education can have a nonlinear effect on the process of economic growth. By assuming that higher educated workers are more affected by depreciation of human capital than lower educated workers, in Section 2 we have provided a theoretical model, which explains why the returns to scale in producing human capital through education initially increase and later decrease.

Subsequently, we have used this result to modify a Solow growth model augmented by the presence of human capital. There, we have shown that a positive relationship between physical capital and education leads to a situation in which multiple equilibria may arise in the GDP growth path. Through a numerical example, we have also shown that, under plausible values of the exogenous parameters, our model may generate the expected nonlinearities and consequently the possibility of multiple equilibria in the economic growth path. Moreover, we have studied the effects of parameter variations on these equilibria. In particular, we have seen how investing in education is one way for poor countries to exit from an equilibrium characterized by low levels of physical capital per capita.

In the last part of the paper, we have investigated the empirical relationship between education and growth. As Liu and Stengos (1999) and Kalaitzidakis et al. (2001), we have seen that a PLR model, in which education enters into the nonparametric part of the model, fits data better than traditional regressions. In fact, by comparing traditional parametric regressions with a semiparametric regression, we found evidence in favor of a nonlinear relationship between education and growth. In particular, we have seen that for low levels of education, an increasing relationship emerges between years of schooling and TFP growth rate, while for high levels of education, the same relationship is decreasing.

In conclusion, even if education may not be the only cause for a nonlinear economic growth path, future growth regressions should account for these nonlinearities. At the same time, as suggested by this paper, theoretical studies should take into account the idea that the process of human capital accumulation may be a source of these nonlinearities.

5 References

Abbey S., (1988), "Robust measures and the estimator limit," *Geostandards Newsletter*, n. 12, 241-248.

Arrazola M.,de Hevia J., Risueño M. and Sanz J. F., (2005), "A proposal to estimate human capital depreciation: Some evidence for Spain," Hacienda Pública Española / Revista de Economía Pública, n. 172: 9-22.

Azariadis C. and Drazen A., (1990), "Threshold Externalities in Economic Development," *The Quarterly Journal of Economics*, n. 105(2), 501-26.

Barro R. J., (1991), "Economic Growth in a Cross Section of Countries," *Quarterly Journal of Economics*, n.106, 407-443.

Barro R. J., and Sala-i-Martin X., (1995), *Economic Growth.* NY: McGraw-Hill.

Benhabib J. and Spiegel M.M., (1994), "The Role of Human Capital in Economic Development: Evidence from Aggregate Cross-Country Data," *Journal of Monetary Economics*, n. 34, 143-173.

Benhabib J. and Spiegel M., (2002), "Human Capital and Technology Diffusion," Mimeo: Federal Reserve Bank of San Francisco.

Bils M. and Klenow P.J., (2000), "Does Schooling Cause Growth?," *American Economic Review*, n. 90(5), 1160-83.

de Grip A., (2004), "Evaluating Human Capital Obsolescence," Research Centre for Education and the Labour Market (R.O.A.), Maastricht University.

Eurostat. Oct. 18, 2004. <http://epp.eurostat.cec.eu.int>.

Galor O., (1996), "Convergence? Inferences from Theoretical Models," *Economic Journal*, n. 106, 1056-69.

Glewwe P. and Jacoby H. G., (2004), "Economic growth and the demand for education: is there a wealth effect?" *Journal of Development Economics*, n. 74(1), 33-51.

Grossman G. M. and Helpman E., (1991). *Innovation and Growth in the Global Economy.* MIT Press, Cambridge, MA.

Hall R. and Jones C. I., (1999), "Why Do Some Countries Produce So Much More Output per Worker than Others?" *Quarterly Journal of Economics*, n. 114(1), 83—116.

Hustle T. J. and Tibshirani R. J., (1990). *Generalized Additive Models.* New York: Chapman and Hall.

Isaksson A., (2002), "The Importance of Human Capital for the Trade-Growth Link," *Working Paper* , n. 2, UNIDO.

Johnson W. R., (1980), "Vintage Effects in the Earnings of White American Men," *Review of Economics and Statistics*, n. 62, 399-407.

Josephson C., (2006), "What technical change really is in a disaggregate production function," *The Business Review*, n. 5(2).

Kalaitzidakis P., Mamuneas T., Savvides A. and Stengos T., (2001), "Measures of human capital and nonlinearities in economic growth," *Journal of Economic Growth*, n. 6, 229-254.

King R. G. and Rebelo S., (1990), "Public policy and economic growth: developing neoclassical implications," *Journal of Political Economy*, n. 98, S126–50.

Krueger A. B. and Lindahl M., (2001), "Education for Growth: Why and for Whom?," *Journal of Economic Literature*, n. 39(4), 1101-36.

Lange F. and Topel R., (2006). *The Social Value of Education and Human Capital.* Handbook of Education Economics, Vol. 1. Eds. Eric Hanushek and Finis Welch.

Levine R., Loayza N. and Beck T., (2000), "Financial Intermediation and Growth: Causality and Causes," *Journal of Monetary Economics,* n. 46(1), 31-77.

Liu Z. and Stengos T., (1999), "Non-Linearities in Cross-Country Growth Regressions: A Semiparametric Approach," *Journal of Applied Econometrics*, n. 14, 527-538.

Lucas R. E., (1988), "On the mechanics of economic development," J*ournal of Monetary Economics*, n. 22, 3-42.

Lucas R. E., (1990), "Why Doesn't Capital Flow from Rich to Poor Countries?" *American Economic Review*, Papers and Proceedings, n. 80(2), 92-96.

Mankiw N. G., Romer D. and Weil D. N., (1992), "A contribution to the empirics of economic growth," *Quarterly Journal of Economics*, n. 107, 407-37.

Mincer J., (1974). *Schooling, Experience, and Earnings.* Columbia University Press, New York.

Mc Pherson M.S. and Winston G. C., (1983), "The economics of Academic Tenure: A. Relational Perspective," *Journal of Economic Behavior and Organization*, n. 4, 163-184.

Nelson R. R. and Phelps S. E., (1966), "Investment in Humans, Technological Diffusion, and Economic Growth," *American Economic Review*, n. 56, 69-75.

Neuman S. and Weiss A., (1995), "On the Effects of Schooling Vintage on Experience-Earnings Profiles: Theory and Evidence," *European Economic Review,* n. 39, 943-955.

Psacharopoulos G., (1994), "Returns to Investment in Education: A Global Update," *World Development*, XXII, 1325-1343.

Redding S., (1996), "Low-skill, low-quality trap: strategic complementarities between human capital and R&D," *Economic Journal*, n. 106, 458-70.

Romer P. M, (1986), "Increasing Returns and Long-run Growth," *Journal of Political Economy*, n. 94(5), 1002-37.

Romer P. M., (1990), "Endogenous Technical Change," *Journal of Political Economy*, n. 98, S71-S102.

Rosen S., (1976), "A Theory of Life Earnings," *Journal of Political Economy*, n. 84, 45-68.

Solow R. M., (1956), "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics*, n. 70(1), 65-94.

Stiglitz J. E. and Boadway R. W., (1994), *Economics and The Canadian Economy*, New York: W.W. Norton & Company.

Trostel A. P., (2004), "Returns to scale in producing human capital from schooling," *Oxford Economic Papers*, n. 3.

van Leeuwen B., (2007). *Human Capital and conomic Growth in India, Indonesia, and Japan: A quantitative analysis, 1890-2000.* Utrecht University.

Verhulst P. F., (1845), "Recherches Mathematiques sur La Loi D'Accroissement de la Population," *Nouveaux Memoires de l'Academie Royale des Sciences et Belles-Lettres de Bruxelles*, n.18 (1), 1-45.

Weiss Y. and Lillard L. A., (1978), "Experience, Vintage, and Time Effects in the Growth of Earnings: American Scientists, 1960-1970," *Journal of Political Economy*, n. 86, 427-447.

Wood S. N., (2006). *Generalized Additive Models: An Introduction with R.* CRC/Chapman & Hall.

A The Production Function of Human Capital

Note that we can write Equation (3) as follows:

$$\frac{dh}{Bh - \sigma h^2} = de. \tag{A.1}$$

Thus, by integrating both sides, we have

$$\int_{h(0)}^{h(e_t)} \frac{dh}{Bh - \sigma h^2} = \int_0^{e_t} de \tag{A.2}$$

that is,

$$\int_{h(0)}^{h(e_t)} \frac{dh}{h - \eta h^2} = Be_t \tag{A.3}$$

where $\eta \equiv \frac{\sigma}{B}$.
Now, we can solve Equation (A.3) in the following way:

$$\int_{h(0)}^{h(e_t)} \left(\frac{1}{h} + \frac{\eta h}{1 - \eta h} \right) dh = Be_t \tag{A.4}$$

and hence

$$\log h - \log(1 - \eta h)|_{h(0)}^{h(e_t)} = Be_t. \tag{A.5}$$

After some algebra we can easily write

$$\frac{h(e_t)}{1 - \eta h(e_t)} = \frac{h(0)}{1 - \eta h(0)} \exp[Be_t] \tag{A.6}$$

By imposing $h(0) = 1$, we can conclude that

$$h(e_t) = \frac{\exp[Be_t]}{1 - \eta + \eta \exp[Be_t]} \tag{A.7}$$

Since $\eta \equiv \frac{\sigma}{B}$, Equation (A.7) corresponds to the logistic production function of human capital described by (4).□

B The Nonmonotonic Behavior of $F(k)$

If $F(k_t)$ is monotone, it will match the constant value $n + g + \delta$ only once. Therefore, the nonmonotonic form of $F(k_t)$ is a necessary condition for the existence of multiple equilibria. To show that $F(k_t)$ can be nonmonotonic for some value of its parameters, we must study the sign of its derivative with respect to k_t.

We can write $F(k_t)$ as follows:

$$F(k_t) \equiv sf(k_t)g(k_t) \tag{A.1}$$

where $f(k_t) \equiv k_t^{\beta-1}$ and $g(k_t) \equiv (\frac{B\exp[\underline{B}bk_t]}{B-\sigma+\sigma\exp[\underline{B}bk_t]})^{1-\beta}$.

Thus, the derivative of $F(k_t)$ with respect to k_t is:

$$F'(k_t) = s[f'(k_t)g(k_t) + f(k_t)g'(k_t)] \tag{A.2}$$

Since s is a positive constant, we can say that $F'(k_t) \gtreqless 0$ if and only if:

$$\frac{g'(k_t)}{g(k_t)} \gtreqless -\frac{f'(k_t)}{f(k_t)} \tag{A.3}$$

or

$$\frac{b\underline{B}(B-\sigma)}{B-\sigma+\sigma\exp[\underline{B}bk_t]} \gtreqless \frac{1}{k_t} \tag{A.4}$$

If $B > \sigma$, we have:

$$\lim_{k_t\to 0^+} \frac{g'(k_t)}{g(k_t)} < \lim_{k_t\to 0^+} -\frac{f'(k_t)}{f(k_t)} \tag{A.5}$$

Thus, $F'(k_t)$ will be negative as $k_t \to 0^+$.

Now, we must state the conditions under which $F'(k_t)$ is positive, i.e.:

$$\frac{b\underline{B}(B-\sigma)}{B-\sigma+\sigma\exp[\underline{B}bk_t]} > \frac{1}{k_t} \tag{A.6}$$

Given (A.5) and (A.6), we can conclude that, for some configuration of parameters, $F(.)$ is a nonmonotone function.

C The Country-List

Table C.1: The Country-List

Algeria	Korea, Republic of
Argentina	Lesotho
Australia	Malawi
Austria	Malaysia
Belgium	Malta
Bolivia	Mauritius
Brazil	Mexico
Cameroon	Nepal
Canada	Netherlands
Central African Republic	New Zealand
Chile	Nicaragua
Colombia	Niger
Congo	Norway
Costa Rica	Pakistan
Cyprus	Panama
Denmark	Papua New Guinea
Dominican Republic	Paraguay
Ecuador	Peru
Egypt, Arab Rep.	Philippines
El Salvador	Portugal
Finland	Rwanda
France	Senegal
Gambia, The	Sierra Leone
Germany	South Africa
Ghana	Spain
Greece	Sri Lanka
Guatemala	Sudan
Guyana	Sweden
Haiti	Switzerland
Honduras	Syria
India	Thailand
Indonesia	Togo
Iran, Islamic Republic of	Trinidad and Tobago
Ireland	United Kingdom
Israel	United States
Italy	Uruguay
Jamaica	Venezuela
Japan	Zaire
Kenya	Zimbawe