The Demand of Health and Economic Growth in an Aging Economy*

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Abstract

This paper constructs a endogenous growth model, introducing a health production function defined by Grossman (1972) into the overlapping generations model, incorporating an uncertain lifetime. The health status is a commodity produced at home by using agents' own health expenditure and transfers from their children. In the first part, we describe the economy in which there is no government and show that the rate of life expectancy has positive impact on growth rate. In the second part, we study the role and the effect of public funded health spending. By analyzing the model, we show that public funded health spending deteriorates (accelerates) the growth rate and accelerates the welfare level of initial old generation.

Key words: life expectancy, household production, economic growth, social welfare

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1 Introduction

It has been recognized that economic growth leads populations to live longer.[1] According to Weil (1997), population aging can be seen in both a reduction in the fraction of the population that is under 20, and an increase in the fraction over 64. Between 2005 and 2050, half of the increase in the world population will be accounted for by a rise in the population aged 65 years or over, whereas the number of children (persons under age 15) will decline slightly. Furthermore, in the more developed regions, the population aged 65 or over is expected to nearly double (from 245 million in 2005 to 406 million in 2050), whereas the member of persons aged 0-24 is likely to decline (from 372 million in 2005 to 131 million in 2050). The person aged 65 or over in the more developed countries represent 15.3% (2005) and is estimated 26.1% (2050) of all population; and that of the person aged 0-24 represent 30.7% (2005) and also estimated to 25.8% (2050). (See United Nations (2007)).

As health status tends to deteriorate with age, the trend of population aging increases health expenditure, though, it counts for about 9 % of GDP in OECD countries and the fact that the public sector is the main source of health funding in all OECD countries except the United States, Mexico, and Greece (See Table 1 for details), it is extremely necessary to study how health demand or public funded health spending affects the rate of economic growth and the welfare in an aging economy.

Population aging, along with the significance of the demographic changes had attracted a great deal of attention and observed to have a positive impact on economic growth. (for instance, Barro and Sala-i-Martin (1995) and Barro (1997)). Several explanations have been suggested to account for the strong positive relationships between life expectancy and eco-

[1] See Bloom and Canning (2000).

nomic growth. There are a number of mechanisms through which health[2] may have significant effects on economic growth. One mechanism is explained by the productivity improvement; more generally, healthier agents can take enough education in their child age and have higher labor productivity; healthier worker does not lose their working time for health, leads to higher productivity. Another mechanism is explained through the demographic variables. For example, improvements in health lead to a decline in child mortality which induces changes in the composition of the population and eventually causes an increase the proportion of working age young.[3] The mechanism which stated above respectively enhances the saving and economic growth.

In this paper, an alternative theory is provided to help explain the effect of population aging on economic growth by focusing on increasing health care costs. An increasing health care cost can be studied by time-related health care (see, for example, Lakdawalla and Philpson (2002) and Mizushima (2007)) and by good related health care (see, for example, Bednarek and Pecchenino (2002) and Tabata (2005)). Within these studies, we employ the good related health care and complement the study of Tabata (2005) by using some features of the work by Nakanishi and Nakayama (1993), with some microfoundations of the work in Grossman (1972), then construct a simple endogenous growth model.

This paper has two characteristic points. First, we consider Pecchenino and Pollard (1997) type life expectancy to examine the feature of population aging.[4] When we assume the population growth is constant through the time, an increase in the life expectancy increases the rate of old agents against the young agents, thus implies the population aging. Second,

[2]Population aging can be understood that agents to live better and longer.

[3]In Asia, the demographic changes may have contributed significantly to the "economic miracle" of the 1965-1990 (Bloom and Williamson (1998)).

[4]Follows by Pecchenino and Pollard (1997), in this paper, the life expectancy is decided independently by other variables.

we assume that a household's health status in old age is a commodity produced at home by using the health expenditure which is supplied by both young (children) and old generation (parents). In addition to this, we also assume that health expenditure is divided two types, one is indirect health expenditure; that is, exercise, food and preventive medicine; and the other is direct health expenditure; that is, hospital and nursing care.

Using the model which described above, in the first part, we examine the economy in which there is no government and show that the net growth rate in this economy increases when the rate of life expectancy increases. As the future life expectancy being subject to uncertainty, agents have an incentive to prepare future spending. Therefore a rise in life expectancy brings the precautionary saving motive (Leland (1968), Sandmo (1970), and Kimball (1990)) and enhances the economic growth.

In the second part, we introduce the government that is the main source of health funding. The government levies income tax on young generation and transfers these resources to old generation by reimbursing a part of the direct health expenditure. In this regime, we show that public funded health spending (PFH) changes the saving decisions of agents. It has both positive and negative impact on savings. As mentioned above, we assume that PFH is covered by payroll tax, an increase in PFH decreases disposable income. When we call this effect as tax burden effect, this effect has a negative impact on savings. The next effect is a health cost effect. Since PFH reduces the burden of direct health expense, the incentive to prepare the health expense in old age decreases. Therefore health cost effect also has a negative impact on saving. The last effect is a transfer effect. As mentioned in the first effect, PFH decreases disposable income, the health transfers from children towards parents also decreases. A decrease in the transfers from children brings the incentive to prepare the burden of health expense in old age, thus this effect has a positive impact on saving. By analyzing the model, we show that the effect of an increase in the rate of life expectancy on economic growth depends on the rate of reimbursement. When the rate of reimbursement is small (large), an increase in the rate of life expectancy increases (decreases) the economic growth.

By comparing the growth rate of the economy with and without PFH, we show that PFH decreases the growth rate and this trend tends to high when the rate of life expectancy increases. An increase in life expectancy can be interpreted as the rate of time preference, as incorporated in the models such as those of Yaari (1965) and Blanchard (1985). An increase in life expectancy increases time preference, leading to higher capital stock. In the regime without PFH, old age health expenditure does not reimbursed by government, the rate of time preference increases, results in higher capital stock than the regime with PFH.

When PFH funded by income tax in young age, the initial old generation can receive the reimbursement of health without burden any cost, PFH accelerates the welfare level of initial old generation. As to the welfare of following generation, PFH has both short run and long run effects on the welfare level. The short run effect is a health funded effect. This effect has both positive and negative impact on welfare. Since PFH increases the aggregate transfers from following generation, enhances the welfare level. On the other hand, PFH increases the tax burden, reduces the welfare level. The long run effect is a growth effect. As PFH decreases the growth rate, agents cannot receive the return from growth rate. This trend tends to large when it becomes the future generation. Then, in the future generation, growth effect dominates the positive or negative health funded effect and deteriorates the welfare level.

The remainder of this paper is organized as follows. We set up the model in section 2. Section 3 analyzes the equilibrium of the economy without public funded health spending. Section 4 analyzes the economy with public funded health spending. Section 5 examines the comparison of economic growth and welfare with and without public funded health spending. Section 6 contains some concluding remarks.

2 The Model

Consider an infinite-horizon economy composed of agents and perfectly competitive firms. A new generation, referred to as generation t, is born in each period $t = 1, 2, 3, \cdots$. Generation $t \geq 1$ is composed of a continuum of $N_t > 0$ units of agents who live for two periods, young and old age. The net rate of population growth is constant $n > 0 : N_t = (1 + n)N_{t-1}$.

Firms

Firms are considered as perfectly competitive profit maximizers that produce output using a Romer (1986) type production function $Y_t = A(K_t)^\alpha(\bar{K}_t L_t)^{1-\alpha}$, where Y_t is the aggregate output, A is the parameter representing the technology level, K_t is the aggregate productive capital, L_t is the aggregate labor, and $\bar{K}_t$ is the aggregate capital stock in the economy so that there is an externality in production. The production function can be rewritten in an intensive form as $y_t = (k_t)^\alpha(l_t\bar{K}_t)^{1-\alpha}$, where $k_t \equiv K_t/N_t$ is a per capita capital stock in period t. We assume that capital depreciates completely in the process of production. Since firms are price takers, they take the wage w_t and real rental rate $1 + r_t$ as given and hire labor and capital up to the point where their marginal products equal to their factor prices in period t. Noting $k_t = \bar{K}_t$ and $l_t = 1$ in equilibrium, the wage and the real rental rate are given as follows:

$$w_t = (1-\alpha)Ak_t, \quad 1 + r_t = R_t = \alpha A. \tag{1}$$

Agents

The model of individual behavior is based on that developed by Pecchenino and Pollard (1997). The probability that an agent survives through the period of old age is $p \in (0, 1)$. The probability that an individual dies at the beginning of the period of old age, after having had a child is $1 - p$.

In young age, each agent is endowed with one unit of labor, which supplies inelastically to firm, and obtains wage income. A fraction p of young agents are of type a, whose parents are survive. Type d agents, whose parents die constitutes a fraction $1 - p$ of young agents. Type d agents in generation t consume a part of their income $c^t_{d,t}$ and save the remainder $s^t_{d,t}$ for consumption in old age. Type a agents differ from type d agents in that they care their parents and derive satisfaction from giving them a part of their income q^t_t. They also consume $c^t_{a,t}$ and save the remainder $s^t_{a,t}$. In what follows, we refer the type of young agents as index $i = a, d$. The budget constraint for a young agent in generation t is:

$$w_t = \phi(c^t_{a,t} + q^t_t + s^t_{a,t}) + (1-\phi)(c^t_{d,t} + s^t_{d,t}), \tag{2}$$

where ϕ is an index indicating a agent's type and take $\phi = 1$ or 0. ϕ, which is realized at the beginning of date t immediately after agents of generation t are born, is distributed independently and identically across agents and time with the probability distribution: $\phi = 1$ with probability $p \in (0, 1)$, $\phi = 0$ with probability $1 - p$.

In old age, type a agents, whose is survive is also constitute a fraction $p \in (0, 1)$ of old agents. If an agent dies, his or her annuitized wealth is transferred to the agents who live throughout old age (see Yaari (1965) and Blanchard (1985)). As the capital depreciate 100% in one period, agents take R_{t+1}/p units of returns.

We assume that old agents have the following household-produced health technology:

$$h_{t+1} = \delta I_{t+1}^t + (O_{t+1}^t)^\gamma (Q_{t+1})^{1-\gamma} \quad \delta > 1, \ \ \gamma \in (0,1), \tag{3}$$

where I_{t+1}^t is the indirect health expenditure; that is, exercise, food, and preventive medicine; O_{t+1}^t is the direct health expenditure; that is, hospital, medicine, and nursing care; and $Q_{t+1} = p(1+n)q_{t+1}^{t+1}$ is aggregate health transfers from his or her children; that is, the care towards their parents. The output from the above household-produced health technology equalized with the health status of old generation. When we assume that old agents leave bequest to his or her children, old agents have following budget constraint:

$$\frac{R_{t+1}}{p} s_t = c_{t+1}^t + I_{t+1}^t + O_{t+1}^t, \tag{4}$$

where s_t^t shows the aggregate saving; that is, $s_t^t = ps_{a,t}^t + (1-p)s_{d,t}^t$.

We assume that each agent in generation t has the expected utility function of the form:

$$Eu_{i,t} = \ln c_{i,t}^{\ t} + \phi\beta \ln q_t^t + EV(c_{t+1}^t, h_{t+1}; p) \quad i = a, d \tag{5}$$

where ϕ is the parameter which shows the type of agents; $EV(c_{t+1}^t, h_{t+1}; p)$ is the expected value in old age. We assume that the expected value $EV(c_{t+1}^t, h_{t+1}; p)$ takes the following log-linear form:

$$EV(c_{t+1}^t, h_{t+1}; p) = p[\ln \bar{\sigma} + \sigma \ln c_{t+1}^t + (1-\sigma)\ln h_{t+1}] + (1+p)0, \tag{6}$$

where $\sigma \in (0,1)$ is a weight attached to the utility from his or her consumption and $\bar{\sigma} \equiv 1/\sigma^\sigma(1-\sigma)^{1-\sigma}\delta^{1-\sigma}$. Each agent of generation t maximizes his or her utility (5) subjects (2), (3), (4), and (6). The timing is decided as follows:

1. Each agent maximizes his or her expected utility (5) subject to budget constraints (2) taking w_t and R_{t+1} as given.

2. If an agent survive in their old age, he or she maximizes his or her old period's value (6) subject to budget constraints (4).

3. An agent decides his or her household's health status by minimizing their cost $I^t_{t+1}+O^t_{t+1}$ subject to home production function (3), taking the transfer from his or her children Q_{t+1} as given.

3 Equilibrium

As a benchmark case, we first describe an economy in which there is no government. In Section 4, we introduce the public sector which bears the main source of health funding. In order to derive the equilibrium in benchmark case, we solve each agent's problem by following the three timing which we showed in Section 2 by backward.

At first, let us derive the indirect and direct health expenditure in his or her old age. An agent produces his or her health status by minimizing the cost, that is, minimizing $I^t_{t+1}+O^t_{t+1}$ subject to (25). The demand of each health expenditure is decided as $O^t_{t+1} = (\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}} Q_{t+1}$ and $I^t_{t+1} = \frac{1}{\delta}[h_{t+1} - (\frac{\gamma}{\delta})^{\frac{\gamma}{1-\gamma}}]Q_{t+1}$. Thus, we have the aggregate demand of health as follows:

$$I^t_{t+1} + O^t_{t+1} = \frac{1}{\delta} h_{t+1} - Q_{t+1}\left(\frac{\gamma}{\delta}\right)^{\frac{1}{1-\gamma}} \frac{1-\gamma}{\gamma}. \tag{7}$$

The second term in the right hand side of (7) shows the aggregate health transfers from his or her children. When the transfers from his or her children decrease (increase), the cost of health in old age increases (decreases).

Next, we examine the utility maximizing problem in old age. An agent who survives in his or her old age, decides his or her old period's consumption and health level by solving the following problem:

$$\max EV_{t+1} = p\Big(\ln\bar{\sigma} + \sigma\ln c_{t+1}^t + (1-\sigma)\ln h_{t+1}\Big),$$

s.t.

$$\frac{R_{t+1}}{p}(p s_{a,t}^{\ t} + (1-p) s_{d,t}^{\ t}) \quad = \quad c_{t+1}^t + I_{t+1}^t + O_{t+1}^t, \tag{7}$$

The first order condition for this problem yields the solution for the consumption and health status in his or her old age as follows:

$$c_{t+1}^t \quad = \quad \sigma\Big[\frac{R_{t+1}}{p}(p s_{a,t}^{\ t} + (1-p) s_{d,t}^{\ t}) + Q_{t+1}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big], \tag{8}$$

$$h_{t+1} \quad = \quad (1-\sigma)\delta\Big[\frac{R_{t+1}}{p}(p s_{a,t}^{\ t} + (1-p) s_{d,t}^{\ t}) + Q_{t+1}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big]. \tag{9}$$

Since the first term in bracket on the right hand side of (8) and (9) shows the return from saving and the second term shows the health transfers from his or her children, the inside of the bracket shows the aggregate income in his or her old period. Therefore, in old age, the expected income is allocated to the consumption and health status according to the weight parameter. We then have the value in his or her old age by substituting (8) and (9) into (6):

$$EV(c_{t+1}^t, h_{t+1}; p) = \frac{R_{t+1}}{p}(p s_{a,t}^{\ t} + (1-p) s_{d,t}^{\ t}) + Q_{t+1}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma} \tag{10}$$

Finally, we derive the saving function, which is, maximizing (5) subjects to (2) and (10). The first order condition for this problem yields the solution for the young period's consumption, transfer to parents, and saving as follows:

$$\begin{aligned} c_{a,t}^{\ t} &= \frac{1}{p(2+\beta+p)}\Big[w_t + p\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big] \\ q_t^t &= \frac{\beta}{p(2+\beta+p)}\Big[w_t + p\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big] \\ c_{d,t}^{\ t} &= \frac{1}{(1-p)(2+\beta+p)}\Big[w_t + p\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big] \end{aligned} \tag{11}$$

$$s_{a,t}^{t} = \frac{1}{p(2+\beta+p)}\Big\{[p(1+p)-(1-p)(1+\beta)]w_t - p(1+\beta)\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big\}$$
$$s_{d,t}^{t} = \frac{1}{(1-p)(2+\beta+p)}\Big\{[(1-p)(1+\beta)-p^2]w_t - p(1+\beta)\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big\}$$

Since aggregate saving is the weighted sum of each agent[5], we have:

$$s_t = \frac{p}{2+\beta+p}\Big[w_t - (2+\beta)\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big]. \tag{12}$$

The market clearing condition of capital is $K_{t+1} = s_t N_t$, which expresses the equality of the total savings by young agents in generation $t, s_t N_t$, to the stock of aggregate physical capital in period $t+1, K_{t+1}$. Dividing both sides by N_t leads the following:

$$(1+n)k_{t+1} = s_t. \tag{13}$$

In period 1, there are young agents in generation 1 and the initial old agents in generation 0. The initial old agents of generation 0 is endowed with k_1 units of capital. Each initial old agents rents his or her capital to the insurance firms and earns an income R_1/pk_1, which is then spent for consumption and health expenditure. The measure of initial old agents is $pN_0 > 0$. The utility of an agent in generation 0 is $p(\ln c_1^0 + \ln h_1)$.

Definition 1 *An economic equilibrium is a sequence of allocations and prices which satisfy the following conditions at each date.*

- *Agents and firms optimize, taking the wage rate and the rate of interest; that is, (1) and (12) hold.*
- *Markets for goods, capital, and labor clear; that is, (13) and $l_t = 1$ is hold.*
- *The transfer q_{t+1}^{t+1} from generation $t+1$, which is taken as given by each agent of generation $t \geq 1$ in his or her maximization problem is realized.*

[5]Aggregate saving in period t is derived as $s_t = ps_{a,t}^t + (1-p)s_{d,t}^t$.

In equilibrium, we guess the aggregate transfer from generation $t+1, Q_{t+1}$ from (12):

$$\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}=\frac{1}{2+\beta}w_t-\frac{2+\beta+p}{p(2+\beta)}s_t. \tag{14}$$

To express the equilibrium in this economy in a compact manner, we note, first, that the third condition in Definition together with (11) and (14) imply:

$$Q_{t+1}=(1+n)\frac{\beta}{(2+\beta)}\Big[w_{t+1}-s_{t+1}\Big]. \tag{15}$$

We then have the saving function as follows:

$$s_t=\frac{p}{2+\beta+p}\Big[w_t-\frac{(1+n)\beta}{R_{t+1}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}(w_{t+1}-s_{t+1})\Big]. \tag{16}$$

Substituting (1) and (13) into (16) to obtain:

$$\begin{aligned}&(1+n)^2\frac{p\beta}{\alpha A}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}k_{t+2}\\&-(1+n)\Big[\frac{p\beta(1-\alpha)}{\alpha}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}+2+\beta+p\Big]k_{t+1}+p(1-\alpha)Ak_t=0.\end{aligned} \tag{17}$$

The general solution to the second-order linear difference equation (17) is given by :

$$k_t = Z_1b_1^t+Z_2b_2^t, \tag{18}$$

$$b_1 = A\Big(\frac{\Delta-\sqrt{\Theta}}{(1+n)2\Omega}\Big)>0, \tag{19}$$

$$b_2 = A\Big(\frac{\Delta+\sqrt{\Theta}}{(1+n)2\Omega}\Big)>0,$$

where Z_1 and Z_2 is arbitrary constants. In addition, $\Delta\equiv\frac{p\beta(1-\alpha)}{\alpha}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}+2+\beta+p$ $\Theta\equiv[\frac{p\beta(1-\alpha)}{\alpha}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}]^2+2(2+\beta-p)\frac{\beta(1-\alpha)}{\alpha}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}+(2+\beta+p)^2$, and $\Omega\equiv\frac{p\beta}{\alpha}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}$.

Lemma 1 *The equilibrium condition must hold*

$$g_{t+1}\equiv\frac{k_{t+1}}{k_t}\leq\frac{(1-\alpha)A}{1+n}.$$

The equilibrium path which satisfies this condition is b_1. Then the general solution is rewritten as $k_t=Z_1b_1^t$.

Proof. We have the characteristic equation of (17) as follows:

$$
\begin{aligned}
&(1+n)^2 \frac{p\beta}{\alpha A}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}} \frac{1-\gamma}{\gamma} b^2 \\
&-(1+n)\Big[\frac{p\beta(1-\alpha)}{\alpha}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}} \frac{1-\gamma}{\gamma} + 2 + \beta + p\Big] b + p(1-\alpha)A = 0.
\end{aligned}
$$

Define the left hand side of characteristic equation as

$$
\begin{aligned}
f(b) \equiv\ &(1+n)^2 \frac{p\beta}{\alpha A}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}} \frac{1-\gamma}{\gamma} b^2 \\
&-(1+n)\Big[\frac{p\beta(1-\alpha)}{\alpha}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}} \frac{1-\gamma}{\gamma} + 2 + \beta + p\Big] b + p(1-\alpha)A. \qquad (20)
\end{aligned}
$$

The budget constraint (2) is rewritten as $s_t \leq w_t$. Substituting equilibrium values (1) and (13) into the condition, we just derived before, we have $k_{t+1} \leq (1-\alpha)Ak_t/(1+n)$. Arranging this condition, to obtain

$$
g_{t+1} \equiv \frac{k_{t+1}}{k_t} \leq \frac{(1-\alpha)A}{1+n}. \qquad (21)
$$

Substituting (21) into (20), we have $f[(1-\alpha)A/(1+n)] = -(1-\alpha)A(2+\beta) < 0$ and $b_1 < (1-\alpha)A/(1+n) < b_2$.

As any equilibrium path must satisfy the resource constraint (2), the general solution of (18) must be such that Z_2 is zero, that is, the equilibrium path is given by $k_t = Z_1 b_1^t$. ■

Proposition 1 *Suppose that*

$$
A > \frac{(\Delta + \sqrt{\Theta})(1+n)}{2p(1-\alpha)},
$$

then there exists a unique equilibrium such that $k_t = k_1 b_1^{t-1}$, *where* b_1 *is given as (19) and* $b_1 > 1$ *for each* $t \geq 1$.

Proof. The necessary and sufficient condition that the growth rate is positive is $b_1 > 1$. From (19), the condition is derived as

$$
A > \frac{(\Delta + \sqrt{\Theta})(1+n)}{2p(1-\alpha)}.
$$

The equilibrium condition must satisfy its initial condition. The initial condition is $k_1 = Z_1 b_1$, arranging this, we have $Z_1 = k_1/b_1$. Thus the equilibrium path is given by $k_t = k_1(b_1)^{t-1}$ for each $t \geq 1$. ■

Proposition 1 makes it clear that when the productivity parameter A is sufficiently large, the economy grows at the positive constant rate, b_1. We can see that the output or GDP of this economy grow at the same constant rate b_1. In addition, wage w_t, young-period consumption c_t, young-period transfer q_t, and saving s_t also grow at the same constant rate b_1. The growth rate depends on the parameters A, p, γ, and β. The following proposition shows the results of comparative statics results.

Proposition 2 *At the economy without public funded health spending, an increase in technology parameter A and life expectancy p increases economic growth. In contrast, an increase in population growth n and the altruism towards their parents β decreases economic growth.*

Proof. (i)Since $b_1 = g = A[(\Delta - \sqrt{\Theta})/((1+n)\Omega)] > 1$, it is obvious that $\partial b_1/\partial A > 0$.

(ii)As the growth rate in this economy is given by $g \equiv k_{t+1}/k_t = b_1$(constant), we use the implicit function theorem. By differentiating both sides of (17) with respect to p, we obtain the following:

$$\frac{dg}{dp} = -\frac{(\frac{(1-\alpha)A}{1+n} - g)(1 - \frac{\beta(1+n)}{\alpha A}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}g)}{(2(1+n)\Omega g - \Delta)}.$$

From the resource constraint (See lemma 1), the first braces in the numerator is positive. Substituting (19) into the second braces in the numerator yields $\frac{1}{2p}(2p - \Delta + \sqrt{\Theta})$. To examine the sign, we assume that $\sqrt{\Theta} < \Delta - 2p$. Arranging this, we have the relation: $(2+\beta+p)^2 < (2+\beta-p)^2$. It is contradict with the assumption, thus we have $\sqrt{\Theta} > \Delta - 2p > 0$. From these conditions, the sign of numerator is positive. Substituting (19) into the denominator follows $-(1+n)\sqrt{\Theta} < 0$. Therefore we have $dg/dp > 0$.

(iii) Since $g = b_1 = A[(\Delta - \sqrt{\Theta})/((1+n)\Omega)] > 1$, it follows $\partial g/\partial n < 0$.

(iv) By differentiating sides of (17) with respect to β, we obtain the following:

$$\frac{dg}{d\beta} = -\frac{g\Big\{(1+n)\frac{p}{\alpha A}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}g - \Big[\frac{p(1-\alpha)(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}+\alpha}{\alpha}\Big]\Big\}}{(2(1+n)\Omega g - \Delta)}.$$

From the resource constraints, the nominator is negative (See Lemma 1) and the denominator is positive (see case (i)), then, we have $dg/d\beta < 0$. ■

The net growth rate in this economy increases when the life expectancy p increases. The extra saving caused by future income being subject to uncertainty is known as precautionary saving. Precautionary saving is associated with convexity of the marginal utility function or a positive third derivative of the utility function (see for example, Leland (1968), Sandmo (1970), and Kimball (1990)). Households with a log utility function have a positive motive for precautionary saving, thus a rise in life expectancy encourages private saving and economic growth.

The logic leading to the effect of population growth n and altruism β on economic growth is explained by consumption–smoothing motive. An increase in population growth and the rate of altruism increases old period's transfers from his or her children, therefore, the consumption-smoothing motive increases young period's consumption and decreases saving and growth rate.

4 Public Policy

In this section, we introduce a government that funds the health spending of old generation. We assume that a government funds the health spending by reimbursing a part of direct health expenditure.

At each time, government levies payroll tax τ on young agents (generation t), then transfers these resources to old generation (generation $t+1$) as the subsidies of health spending. For analytical simplicity, we assume that government strategically decides reimbursement rate ϵ on the direct health spending for a given rate of life expectancy p. Thus, we have the budget constraint of government as follows:

$$\tau_t w_t = p\epsilon O_t^{t-1}. \tag{22}$$

The left hand side of (22) represents the aggregate income of government and that of right hand side shows the aggregate spending of public funded health. The tax rate has decided to satisfy the budget constraint (22). When the rate of life expectancy p or the reimbursement rate ϵ increases, the tax rate also increases. Taking R_{t+1}, w_t, p, τ_t and ϵ as given, each young agent maximizes his or her utility (5) subjects to (3) and following constraints:

$$\begin{aligned} (1-\tau_t)w_t &= \phi(c_{a,t}^{\ t} + q_t^t + s_{a,t}^{\ t}) + (1-\phi)(c_{d,t}^{\ t} + s_{d,t}^{\ t}), & (23) \\ \frac{R_{t+1}}{p} s_t &= c_{t+1}^t + I_{t+1}^t + (1-\epsilon)O_{t+1}^t. & (24) \end{aligned}$$

Solving this problem using the similar method to that used in Section 3, we have the each demand of health expenditure as follows:

$$O_{t+1}^t = \Big(\frac{\gamma}{(1-\epsilon)\delta}\Big)^{\frac{1}{1-\gamma}} Q_{t+1}, \quad I_{t+1}^t = \frac{1}{\delta}\Big[h_{t+1} - \Big(\frac{\gamma}{(1-\epsilon)\delta}\Big)^{\frac{\gamma}{1-\gamma}} Q_{t+1}\Big]. \tag{25}$$

As public health funding (PHF) decreases the cost of direct health expenditure O_{t+1}^t, the demand of direct health increases. On the other hand, the demand of indirect health expenditure I_{t+1}^t decreases. Since direct health goods and indirect health goods is perfect substitute (see the household health production function (3)), the demand of health goods that a cost is lower rises.

From (25), we have the aggregate health demand as follows:

$$I_{t+1}^t + (1-\epsilon)O_{t+1}^t = \frac{1}{\delta}h_{t+1} - Q_{t+1}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\Big(\frac{1-\gamma}{\gamma}\Big). \tag{26}$$

By comparing (7) and (26), we found that PFH also decreases the aggregate health demand of old agents. By using (26), we have the consumption and health demand in his or her old age as follows:[6]

$$
\begin{aligned}
c_{t+1}^{t} &= \sigma\Big[\frac{R_{t+1}}{p}(ps_{a,t}^{t}+(1-p)s_{d,t}^{t})+Q_{t+1}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big],\\
h_{t+1} &= (1-\sigma)\delta\Big[\frac{R_{t+1}}{p}(ps_{a,t}^{t}+(1-p)s_{d,t}^{t})+Q_{t+1}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big].
\end{aligned}
$$

Finally, the utility maximization problem in his or her young period is solved in the same manner as Section 3.[7] The first order condition for this problem yields the solution for the young period's consumption, transfer towards their parents, and saving as follows:

$$
\begin{aligned}
c_{a,t}^{t} &= \frac{1}{p(2+\beta+p)}\Big[(1-\tau_t)w_t+p\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big]\\
q_{t}^{t} &= \frac{\beta}{p(2+\beta+p)}\Big[(1-\tau_t)w_t+p\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big]\\
c_{d,t}^{t} &= \frac{1}{(1-p)(2+\beta+p)}\Big[(1-\tau_t)w_t+p\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big]
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
s_{a,t}^{t} &= \frac{1}{p(2+\beta+p)}\Big\{[p(1+p)-(1-p)(1+\beta)](1-\tau_t)w_t-p(1+\beta)\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big\}\\
s_{d,t}^{t} &= \frac{1}{(1-p)(2+\beta+p)}\Big\{[(1-p)(1+\beta)-p^2](1-\tau_t)w_t-p(1+\beta)\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big\}
\end{aligned}
$$

Then, we have the aggregate saving in period t as follows:[8]

$$
s_t=\frac{p}{2+\beta+p}\Big[(1-\tau_t)w_t-(2+\beta)\frac{Q_{t+1}}{R_{t+1}}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\Big]. \tag{28}
$$

---

[6]The procedure is in the same way that of Section 3; that is;

$$
\begin{aligned}
&\max EV_{t+1}=p\Big(\ln\bar{\sigma}+\sigma\ln c_{t+1}^{t}+(1-\sigma)\ln h_{t+1}\Big),\\
&\text{s.t.}
\end{aligned}
$$

$$
\frac{R_{t+1}}{p}(ps_{a,t}^{t}+(1-p)s_{d,t}^{t}) = c_{t+1}^{t}+I_{t+1}^{t}+(1-\epsilon)O_{t+1}^{t}, \tag{26}
$$

[7]Each young agent maximizes the expected utility (5) in anticipation of the expected value in their old age.

[8]Aggregate saving in period t is derived as $s_t=ps_{a,t}^{t}+(1-p)s_{d,t}^{t}$.

In equilibrium, we guess that the aggregate transfer from their children as follows:

$$Q_{t+1} = \frac{(1+n)\beta}{(2+\beta)}\Big[(1-\tau_{t+1})w_{t+1} - s_{t+1}\Big]. \tag{29}$$

Substituting (29) into (28), we have the following saving function:

$$s_t = \frac{p}{2+\beta+p}\Big[(1-\tau_t)w_t - \frac{(1+n)\beta}{R_{t+1}}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}[(1-\tau_{t+1})w_{t+1} - s_{t+1}]\Big]. \tag{30}$$

By comparing the saving function with and without PFH (See (16) and (30)), we found that PFH gives three effects on saving. First effect is shown in the first term in the bracket on the right hand side of (30). An income tax decreases disposable income, this effect has negative impact on saving. We call this effect as direct tax effect. The second effect is a health cost effect. Since PFH reduces the cost bearing of health, the incentive to prepare the expense of health in his or her old period decreases. Therefore health cost effect also has a negative impact on saving. The last effect is a transfer effect. As mentioned in first effect, PFH decreases disposable income, the transfer from their children on health also decreases. A decrease in the transfer from their children brings the incentive to prepare the expense on health in his or her old period, thus this effect has a positive impact on saving.

Since we have not considered the debt, the budget of government must be balanced each time; that is, (22) holds each time. Substituting (25) and (29) into (22), we have the relation as follows:

$$\tau_t w_t = (1+n)X\Big[(1-\tau_t)w_t - s_t\Big],$$

where $X \equiv p\epsilon(\frac{\gamma}{(1-\epsilon)\delta})^{\frac{1}{1-\gamma}}\frac{\beta}{2+\beta}$. From this relation, we have the equilibrium tax rate as:

$$\tau_t = \frac{(1+n)X(w_t - s_t)}{[1+(1+n)X]w_t}. \tag{31}$$

Using (28) and (31), we can rewrite the saving function as follows:

$$s_t = \frac{p}{2+p+\beta+(2+\beta)(1+n)X}\Big[w_t - \frac{(1+n)\beta}{\alpha A}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\frac{1-\gamma}{\gamma}(w_{t+1}-s_{t+1})\Big]. \tag{32}$$

We then substitute equilibrium conditions (1) and (31) into (32) to obtain:

$$\begin{aligned}&(1+n)^2\frac{p\beta}{\alpha A}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\frac{1-\gamma}{\gamma}k_{t+2}\\ &-(1+n)\Big\{\frac{p\beta(1-\alpha)}{\alpha}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\frac{1-\gamma}{\gamma}+[2+p+\beta+(2+\beta)(1+n)X]\Big\}k_{t+1}\\ &+p(1-\alpha)Ak_t = 0\end{aligned} \tag{33}$$

The general solution of this second-order linear difference equation is given by:

$$\begin{aligned}\tilde{k}_t &= \tilde{Z}_1\tilde{b}_1^t + \tilde{Z}_2\tilde{b}_2^t\\ \tilde{b}_1 &= A\Big(\frac{\tilde{\Delta}-\sqrt{\tilde{\Theta}}}{(1+n)2\tilde{\Omega}}\Big)\\ \tilde{b}_2 &= A\Big(\frac{\tilde{\Delta}+\sqrt{\tilde{\Theta}}}{(1+n)2\tilde{\Omega}}\Big),\end{aligned} \tag{34}$$

where $\tilde{Z}_1$ and $\tilde{Z}_2$ is arbitrary constants. In addition, $\tilde{\Delta} \equiv \frac{p\beta(1-\alpha)}{\alpha}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}(\frac{1}{1-\varepsilon})^{\frac{\gamma}{1-\gamma}}\frac{1-\gamma}{\gamma}+2+\beta+p+(1+n)(2+\beta)X$, $\tilde{\Theta} \equiv [\frac{p\beta(1-\alpha)}{\alpha}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}(\frac{1}{1-\varepsilon})^{\frac{\gamma}{1-\gamma}}\frac{1-\gamma}{\gamma}]^2+(2+\beta)\{2\frac{p\beta(1-\alpha)}{\alpha}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}(\frac{1}{1-\varepsilon})^{\frac{\gamma}{1-\gamma}}\frac{1-\gamma}{\gamma}[1+(1+n)X]+(2+\beta+p)[1+2(1+n)X]+(2+\beta)[(1+n)X]^2\}$, and $\tilde{\Omega} \equiv \frac{p\beta}{\alpha}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}(\frac{1}{1-\varepsilon})^{\frac{\gamma}{1-\gamma}}\frac{1-\gamma}{\gamma}$.

The largest root b_2 does not satisfy the resource constraint[9], the general equation is rewritten as $\tilde{k}_t = \tilde{Z}_1\tilde{b}_1^t$.

Proposition 3 *Suppose that*

$$A > \frac{(\tilde{\Delta}+\sqrt{\tilde{\Theta}})(1+n)}{2p(1+n)},$$

then, there exists a unique equilibrium such that $\tilde{k}_t = \tilde{k}_1\tilde{b}_1^{t-1}$, *where* $\tilde{b}_1$ *is given as (34) and* $\tilde{b}_1 > 1$ *for each* $t \geq 1$.

[9]Derive the condition in the same way as Lemma 1, we have $f[(1-\alpha)A/(1+n)] = -(1-\alpha)A[2+\beta+(2+\beta)(1+n)X] < 0$.

Proof. The necessary and sufficient condition that the growth rate is positive is $\tilde{b}_1 > 1$. From (34), the condition is derived as

$$A > \frac{(\tilde{\Delta} + \sqrt{\tilde{\Theta}})(1+n)}{2p(1+n)}.$$

The equilibrium condition must satisfy its initial condition. The initial condition is $\tilde{k}_1 = \tilde{Z}_1 \tilde{b}_1$, arranging this, we have $\tilde{Z}_1 = \tilde{k}_1/\tilde{b}_1$. Thus the equilibrium path is given by $\tilde{k}_t = \tilde{k}_1(\tilde{b}_1)^{t-1}$ for each $t \geq 1$. ■

Suppose that the productive parameter A is sufficiently large, the economy grows at a constant rate of $g_{t+1} \equiv k_{t+1}/k_t = \tilde{b_1}$ at each date $t \geq 1$. It is easy to see that the output or GDP of this economy grow at the same constant rate $\tilde{b_1}$. The following proposition shows the comparative static effects on the growth rate.

Proposition 4 *At the economy with public funded health spending, an increase in the rate of reimbursement of health spending ϵ decreases the economic growth. On the other hand, the effect of an increase in the rate of life expectancy p depends on the rate of reimbursement ϵ. When ϵ is small (large), an increase in the rate of life expectancy p increases (decreases) the economic growth.*

Proof. (i) As the growth rate in this economy is given by $g \equiv k_{t+1}/k_t = \tilde{b}_1$ (constant), we use the implicit function theorem. By differentiating both sides of (33) with respect to ϵ, we obtain the following:

$$\frac{dg}{d\epsilon} = -\frac{\frac{p\beta}{\alpha A}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}(1-\epsilon)^{-\frac{1}{1-\gamma}}g[(1+n)g-(1-\alpha)A] - (2+\beta)(1+n)\frac{p\beta}{2+\beta}(\frac{\gamma}{\delta(1-\epsilon)})^{\frac{\gamma}{1-\gamma}}g}{(1+n)[2(1+n)\tilde{\Omega}g - \tilde{\Delta}]}$$

From the resource constraint (See lemma 1), the first term in the numerator is negative. Substituting (34) into the denominator follows $-(1+n)\sqrt{\tilde{\Theta}} < 0$. Therefore we have $dg/d\epsilon < 0$.

(ii) By differentiating both sides of (33) with respect to p, we have the following:

$$\frac{dg}{dp} = -\frac{[\frac{(1-\alpha)A}{1+n} - g][1 - \frac{(1+n)\beta}{\alpha A}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}(\frac{1}{1-\epsilon})^{\frac{\gamma}{1-\gamma}}\frac{1-\gamma}{\gamma}g] - (1+n)\beta\epsilon(\frac{\gamma}{\delta(1-\epsilon)})^{\frac{1}{1-\gamma}}g}{2(1+n)\tilde{\Omega}g - \tilde{\Delta}}.$$

Define the numerator as $F(\epsilon) \equiv [\frac{(1-\alpha)A}{1+n} - g][1 - \frac{(1+n)\beta}{\alpha A}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}(\frac{1}{1-\epsilon})^{\frac{\gamma}{1-\gamma}}\frac{1-\gamma}{\gamma}g] - (1+n)\beta\epsilon(\frac{\gamma}{\delta(1-\epsilon)})^{\frac{1}{1-\gamma}}g$. Then we have $\lim_{\epsilon\to 0} F(\epsilon) > 0$ (the results is same with proposition 2) and $\lim_{\epsilon\to 1} F(\epsilon) \to -\infty$. Thus there exists $\tilde{\epsilon} \in (0,1)$ such that $F(\epsilon) = 0$, then $F(\epsilon) > 0$ for $0 < \epsilon \leq \tilde{\epsilon}$ and $F(\epsilon) < 0$ for $\tilde{\epsilon} \leq \epsilon < 1$. Since denominator is negative, the aggregate sign follows with the sign of numerator; that is, $\frac{dg}{dp} > 0$ for $\epsilon(0, \tilde{\epsilon}]$, $\frac{dg}{dp} < 0$ for $\epsilon \in [\tilde{\epsilon}, 1)$. ■

An increase the rate of reimbursement ϵ means the higher tax burden and higher rate of public funded health, then the negative tax burden and wealth cost effect on saving dominates the positive transfers effect, results in lower growth rate.

5 The effect of Public Funded Health Spending

The aim in this section is to consider what role of public funded health spending plays in accelerating or decelerating economic growth and social welfare. As the rate of reimbursement determines the size of public funded health spending (PFH), these questions amount to asking how the rate of reimbursement on health and the rate of life expectancy will cause the economy to grow faster or slower.

5.1 Public Funded Health Spending and Economic Growth

Let superscripts n and p denote, respectively, "the economy with no public funded health spending or the economy with $\epsilon = 0$", "the economy with public funded health spending or the economy with $\epsilon \in (0,1)$". To determine how public funded health spending benefit (or hurts) the growth rate, we compare the growth rate (19) and (34): It is complicated to compare the growth rate by analytically, thus we numerically examine the growth rate. The following proposition formalizes this observation and the results is give in Figure 1.

Proposition 5 *The public funded health spending decreases the growth rate and this trend tends to high when the rate of life expectancy increases.*

An increase in life expectancy can be interpreted as the rate of time preference, as incorporated in the models such as those of Yaari (1965) and Blanchard (1985), yields the higher capital stock. In the regime without PFH, old age health expenditure does not reimbursed by government, the rate of time preference increases, results in higher capital stock than the regime with PFH.

5.2 Public Funded Health Spending and Social Welfare

In subsection 5.1, we have shown that, in the economy with PFH, economic growth decelerates. In this subsection, we will analyze the effect of PFH on social welfare. Since initial old generation is special generation who can take health reimbursement without burden any cost, in examine the effect of PFH on welfare, it is extremely necessary to analyze the welfare of initial old generation and other generation.

The welfare of initial old generation is defined as: $W^0 = pEV_1(c_1, h_1; p)$. By noting $EV_1(c_1, h_1; p) = \ln\{\frac{\alpha A}{p}k_1 + \frac{\beta(1+n)}{2+\beta}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}[(1-\alpha)Ak_1 - (1+n)k_2]\}$ in equilibrium without PFH, and $EV_1(c_1, h_1; p) = \ln\{\frac{\alpha A}{p}k_1 + \frac{\beta(1+n)}{2+\beta}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}(\frac{1}{1-\epsilon})^{\frac{\gamma}{1-\gamma}}\frac{1}{1+(1+n)p\epsilon(\frac{\gamma}{(1-\epsilon)\delta})^{\frac{1}{1-\gamma}}\frac{\beta}{2+\beta}}[(1-\alpha)Ak_1 - (1+n)k_2]\}$ in equilibrium with PFH. To compare the welfare level, we substitute the welfare with PFH from that of without PFH, we have the following:

$$\begin{aligned} W^{0,n} - W^{0,p} &= \frac{\beta(1+n)}{2+\beta}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma} \\ &\Big\{(1-\alpha)A\Big[1-\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\frac{1}{1+(1+n)p\epsilon(\frac{\gamma}{(1-\epsilon)\delta})^{\frac{1}{1-\gamma}}\frac{\beta}{2+\beta}}\Big] \\ &-(1+n)g^n\Big[1-\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\frac{1}{1+(1+n)p\epsilon(\frac{\gamma}{(1-\epsilon)\delta})^{\frac{1}{1-\gamma}}\frac{\beta}{2+\beta}}\frac{g^p}{g^n}\Big]\Big\} \end{aligned} \quad (35)$$

Noting that $(1-\alpha)A-(1+n)g^n$ (see Lemma 1) and $g^n > g^p$ (see Proposition 5), a necessary and sufficient condition of the right-hand side of (35) to be positive is given by:

$$1 < (1-\epsilon)^{\frac{\gamma}{1-\gamma}}\Big[1+(1+n)p\epsilon\Big(\frac{\gamma}{(1-\epsilon)\delta}\Big)^{\frac{1}{1-\gamma}}\frac{\beta}{2+\beta}\Big].$$

Provided above condition holds, each member of generation 0 will be better off with PFH. As is evident from (35), the welfare gain from PFH is greater for the member of generation 0 if the rate of reimbursement is higher; and it is greatest when ϵ tends to 1.

Consider now the member of generation $t \geq 1$. The welfare of each member is measured by his or her expect utility given by (5). To examine the welfare, let us define the Benthamite social welfare function; that is, the welfare level of period t is measured by the sum of the utility of generation $t-1$ and generation t who live in period t. This sum is formulated as follows:

$$W^t = p(\ln c^t_{a,t} + \beta \ln q^t_t) + (1-p)\ln c^t_{d,t} + pEV(c^t_{t+1}, h_{t+1}; p). \quad (36)$$

The welfare of each agent is measured by $\ln c^{n,t}_{a,t} = \ln\{\frac{1}{p(2+\beta+p)}\} + \ln\{(1-\alpha)Ak_t + \frac{p}{\alpha A}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\frac{(1+n)\beta}{2+\beta}[(1-\alpha)Ak_{t+1} - (1+n)k_{t+2}]\}$, $\ln q^{n,t}_t = \ln\{\frac{\beta}{p(2+\beta+p)}\} + \ln\{(1-\alpha)Ak_t + \frac{p}{\alpha A}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\frac{(1+n)\beta}{2+\beta}[(1-\alpha)Ak_{t+1} - (1+n)k_{t+2}]\}$, $\ln c^{n,t}_{d,t} = \ln\{\frac{1}{(1-p)(2+\beta+p)}\} + \ln\{(1-\alpha)Ak_t + \frac{p}{\alpha A}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\frac{(1+n)\beta}{2+\beta}[(1-\alpha)Ak_{t+1} - (1+n)k_{t+2}]\}$, and $EV_{t+1}(c^{n,t}_{t+1}, h^n_{t+1}; p) = \ln\{\frac{\alpha A}{p}(1+n)k_{t+1} + \frac{(1+n)\beta}{2+\beta}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}[(1-$

$\alpha)Ak_{t+1}-(1+n)k_{t+2}]\}$ in equilibrium without PFH, substituting these values into (36) yields:

$$\begin{aligned} W^{t,n} &= -p(1+\beta)\ln p + p\beta\ln\beta - (1-p)\ln\{1-p\} - (1+p\beta)\ln\{2+p+\beta\} \\ &+ (1+p+p\beta)\ln\{k_1(g^n)^t\} \\ &- (1+p\beta)\ln\Big\{\frac{(1-\alpha)A}{g^n} + \frac{p}{\alpha A}\frac{(1+n)\beta}{(2+\beta)}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}[(1-\alpha)A-(1+n)g^n]\Big\} \\ &+ p\ln\Big\{\frac{\alpha A}{p}(1+n) + \frac{(1+n)\beta}{2+\beta}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}[(1-\alpha)A-(1+n)g^n]\Big\} \end{aligned} \tag{37}$$

Similarly, the equilibrium level of welfare of each agent of generation $t \geq 1$ with PFH can be obtained by noting $\ln c_{a,t}^{p,t} = \ln\{\frac{1}{p(2+\beta+p)}\} + \ln\{(1-\frac{(1+n)X[(1-\alpha)Ak_t-(1+n)k_{t+1}]}{[1+(1+n)X](1-\alpha)Ak_t})(1-\alpha)Ak_t + \frac{p}{\alpha A}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}(\frac{1}{1-\epsilon})^{\frac{\gamma}{1-\gamma}}\frac{1-\gamma}{\gamma}\frac{(1+n)\beta}{2+\beta}[(1-\frac{(1+n)X[(1-\alpha)Ak_{t+1}-(1+n)k_{t+2}]}{[1+(1+n)X](1-\alpha)Ak_{t+1}})(1-\alpha)k_{t+1}-(1+n)k_{t+2}]\}$, $\ln q_t^t = \ln\{\frac{\beta}{p(2+\beta+p)}\} + \ln\{(1-\frac{(1+n)X[(1-\alpha)Ak_t-(1+n)k_{t+1}]}{[1+(1+n)X](1-\alpha)Ak_t})(1-\alpha)Ak_t + \frac{p}{\alpha A}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}(\frac{1}{1-\epsilon})^{\frac{\gamma}{1-\gamma}}\frac{1-\gamma}{\gamma}\frac{(1+n)\beta}{2+\beta}[(1-\frac{(1+n)X[(1-\alpha)Ak_{t+1}-(1+n)k_{t+2}]}{[1+(1+n)X](1-\alpha)Ak_{t+1}})(1-\alpha)k_{t+1}-(1+n)k_{t+2}]\}$, $\ln c_{d,t}^t = \ln\{\frac{1}{(1-p)(2+\beta+p)}\} + \ln\{(1-\frac{(1+n)X[(1-\alpha)Ak_t-(1+n)k_{t+1}]}{[1+(1+n)X](1-\alpha)Ak_t})(1-\alpha)Ak_t+\frac{p}{\alpha A}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}(\frac{1}{1-\epsilon})^{\frac{\gamma}{1-\gamma}}\frac{1-\gamma}{\gamma}\frac{(1+n)\beta}{2+\beta}[(1-\frac{(1+n)X[(1-\alpha)Ak_{t+1}-(1+n)k_{t+2}]}{[1+(1+n)X](1-\alpha)Ak_{t+1}})(1-\alpha)k_{t+1}-(1+n)k_{t+2}]\}$, and $EV_{t+1}(c_{t+1}^{p,t}, h_{t+1}^p; p) = \ln\{\frac{\alpha A}{p}(1+n)k_{t+1}+\frac{(1+n)\beta}{2+\beta}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}(\frac{1}{1-\epsilon})^{\frac{\gamma}{1-\gamma}}\frac{1-\gamma}{\gamma}[(1-\frac{(1+n)X[(1-\alpha)Ak_{t+1}-(1+n)k_{t+2}]}{[1+(1+n)X](1-\alpha)Ak_{t+1}})(1-\alpha)k_{t+1}-(1+n)k_{t+2}]\}$:

$$\begin{aligned} W^{t,p} &= -p(1+\beta)\ln p + p\beta\ln\beta - (1-p)\ln\{1-p\} + (1+p\beta)\ln\{2+p+\beta\} \\ &+ (1+p+p\beta)\ln\{k_1(g^p)^t\} - (1+p+p\beta)\ln\{1+(1+n)X\} \\ &- (1+p\beta)\ln\Big\{\frac{(1-\alpha)A}{g^p} + (1+n)X(1+n) \\ &+ \frac{p}{\alpha A}\frac{(1+n)\beta}{(2+\beta)}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}[(1-\alpha)A-(1+n)g^p]\Big\} \\ &+ p\ln\Big\{\frac{\alpha A}{p}(1+n) + \frac{(1+n)\beta}{2+\beta}\Big(\frac{1}{1-\epsilon}\Big)^{\frac{\gamma}{1-\gamma}}\Big(\frac{\gamma}{\delta}\Big)^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}[(1-\alpha)A-(1+n)g^p]\Big\} \end{aligned} \tag{38}$$

The welfare gain (or loss) from public funded health spending can then be obtained by subtracting (38) from (37):

$$
\begin{aligned}
&W^n - W^p = t[1+p(1+\beta)]\ln\left\{\frac{g^n}{g^p}\right\} + (1+p\beta) \\
&\ln\left\{\frac{\frac{(1-\alpha)A}{g^n} + \frac{p}{\alpha A}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\frac{(1+n)\beta}{2+\beta}[(1-\alpha)A-(1+n)g^n]}{\frac{(1-\alpha)A}{g^p} + (1+n)^2 p\epsilon(\frac{\gamma}{(1-\epsilon)\delta})^{\frac{1}{1-\gamma}}\frac{\beta}{2+\beta} + \frac{p}{\alpha A}(\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}(\frac{1}{1-\epsilon})^{\frac{\gamma}{1-\gamma}}\frac{(1+n)\beta}{2+\beta}[(1-\alpha)A-(1+n)g^p]}\right\} \\
&+p\ln\left\{\frac{\frac{\alpha A}{p}(1+n) + (\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}\frac{(1+n)\beta}{2+\beta}[(1-\alpha)A-(1+n)g^n]}{\frac{\alpha A}{p}(1+n)[1+(1+n)p\epsilon(\frac{\gamma}{(1-\epsilon)\delta})^{\frac{1}{1-\gamma}}\frac{\beta}{2+\beta}] + (\frac{\gamma}{\delta})^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma}(\frac{1}{1-\epsilon})^{\frac{\gamma}{1-\gamma}}\frac{(1+n)\beta}{2+\beta}[(1-\alpha)A-(1+n)g^p]}\right\} \\
&+(1+p+p\beta)\Big[1+(1+n)p\epsilon\Big(\frac{\gamma}{(1-\epsilon)\delta}\Big)^{\frac{1}{1-\gamma}}\frac{\beta}{2+\beta}\Big] \qquad (39)
\end{aligned}
$$

The direct PFH effect of PFH of each member of generation $t \geq 1$, which is shows up in the last term on the right-hand side of (39), is positive for each generation. When we call this effect as health funded effect, an increase the rate of reimbursement ϵ increases the health funded effect. The first term in the right hand side of (39) shows the direct growth effect on welfare. We call this effect as growth effect. The second and third term respectively shows the young and old age welfare. The health funded effect and growth effect have both positive and negative impact effects on these welfare. At the both young and old age welfare, PFH increases the aggregate transfers from young generation, and then the welfare level of agents increases. On the other hand, PFH reduces disposable income, then the welfare level of agents' decreases.

The later the generation of the household belongs to, the greater is the effect from growth effect dominates other effect, thus PFH decelerates the welfare level. The numerically examples of generation $t = 50$ is shown in Figure 2 and Figure 3. Figure 2 shows the case where the elasticity of direct health expenditure on health is low and that of the case where the elasticity is high is shown in Figure 3. Since PFH reimburses a part of direct health expenditure, the higher rate of the elasticity of direct health increases the welfare level.

From these results, we find that PFH accelerates the welfare of initial old generation, though that of future is deteriorates. Therefore, when government funds the health spending, it is crucially necessary to propose the additional policy that enhances the welfare of future generation.

6 Conclusion

In this paper, we focus on the increased amount of health expenditure in an aging economy. In most developed countries, the public sector is the main source of health funding; we examine the effect of public funded health spending on economic growth and social welfare. For this purpose, in the first part of this paper, we construct the benchmark model of health demand and then introduce the government is the authority of public funded health spending. To examine these issues, we employ a two-period overlapping generations model that incorporated uncertainties about lifespan. In addition to this, we assume that the health status is a commodity produced at home by using agents' own health expenditure and transfers from their children.

By analyzing the model, we show that the public funded health spending decreases the economic growth and accelerates (decelerates) the welfare level of initial old (future) generation. Therefore, when government funds the health spending, it is crucially necessary to propose the additional policy that enhances the welfare of future generation.

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Country	Health Expenditure as a Share of GDP % (2005)	Public Funded Health Spending % (2005)
Australia	9.5 (2004)	67.5 (2004)
Austria	10.2	75.7
Belgium	10.3	72.3
Canada	9.8	70.3
Denmark	9.1	84.1
France	11.1	79.8
German	10.7	76.9
Greece	10.1	42.8
Ireland	7.5	78.0
Italy	8.9	76.6
Japan	8.0 (2004)	81.7 (2004)
Korea	6.0	53.0
Luxembourg	8.3	90.6
Mexico	6.4	45.5
The Netherlands	9.2 (2004)	62.5 (2002)
Spain	8.2	71.4
Sweden	9.1	84.6
Switzerland	11.6	59.7
Turkey	7.6	71.4
U.K.	8.3	87.1
U.S.	15.3	45.1
OECD	9.0	72.5

Table 1: The Data of Health: Source OECD (2007)

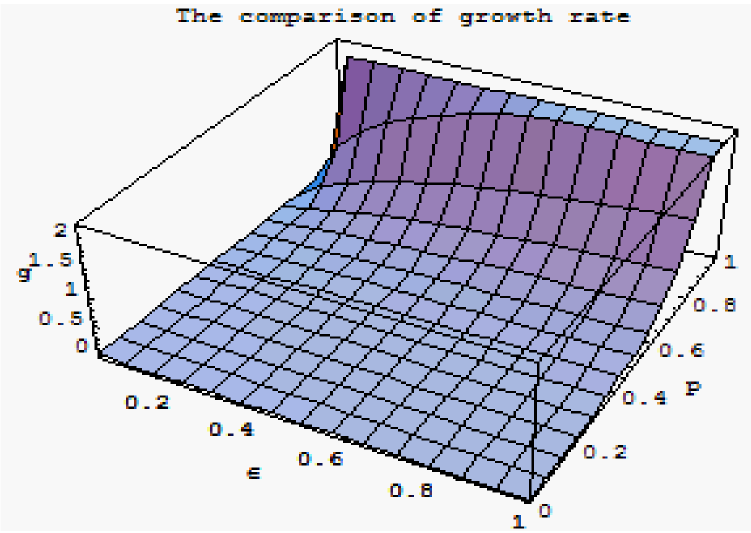

Figure 1: The difference of economic growth $(g^n - g^p)$; $\alpha = 0.33, \gamma = 0.3, \delta = 1.3, n = 0.26, \beta = 0.7, A = 1.5$

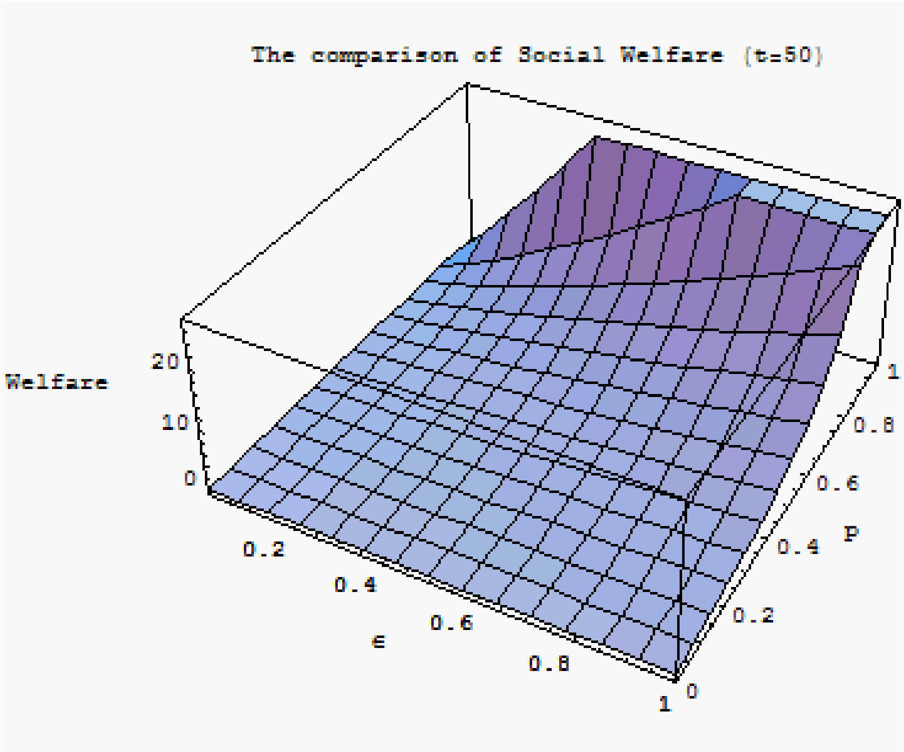

Figure 2: The welfare of generation $t = 50$ $\alpha = 0.33, \gamma = 0.3, \delta = 1.3, \beta = 0.7$

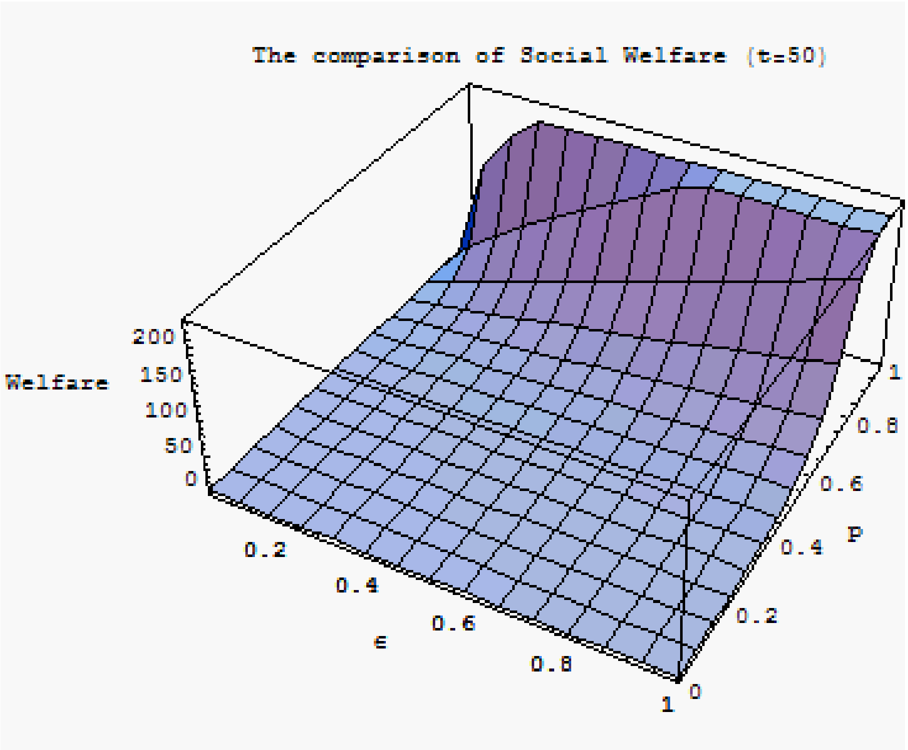

Figure 3: The welfare of generation $t = 50$ $\alpha = 0.33, \gamma = 0.7, \delta = 1.3, \beta = 0.7$