Child Mortality Decline, Inequality and Economic Growth

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Abstract

The aim of this paper is to analyze the effect of child mortality and fertility reductions on economic growth. We develop a two period overlapping generations model where altruistic agents differ in their human capital endowment. Parents care about the number of their surviving children and the future level of human capital of each of them. Children probability of surviving to the adult age is an increasing concave function of parent's human capital. This framework allows us to generate the demographic transition and has the effect of creating multiple development regimes such that the growth rate of the economy depends on initial human capital endowments. For a low level of income, the economy converges to a malthusian steady state. Here, the relationship between population growth and income is positive: small increases in income lead to reductions in child mortality and increases in the number of children. In addition, the optimal spending in children's education is zero. For a high level of income, the economy is on a high development path. In particular, we show the existence of a quality-quantity trade off: as income rises, child mortality decreases and parents choose to have a lower number of children and to devote more resources to children's education spending. This leads to a decreasing growth rate of population and a higher growth rate of human capital.

Keywords: Child Mortality, Fertility, Inequality, Human Capital, Growth.
JEL Classification: I20, J13, O40, C14.

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1 Introduction

Through the last two centuries, economic development has gradually contributed to the increase in the human life span. In 1840 life expectancy at birth was 40 years in England, 44 years in Denmark and 45 years in Sweden (Livi-Bacci, 2001). According to recent life tables, in 2007 life expectancy at birth in the United Kingdom, Denmark and Sweden is 79, 78 and 81 years respectively. In most developed countries, life expectancy at birth is around 80 years (CIA, TheWorld Factbook 2007). Developing countries have also shown a rapid increase in life expectancy which, however, stop to increasing since 1980. Indeed, in several poor countries the HIV/AIDS epidemic reverse the positive trend in life expectancy (Becker et al., 2005; Cutler et al., 2006). Figure 1 shows as life expectancy has been rising from 1960 to 2004.

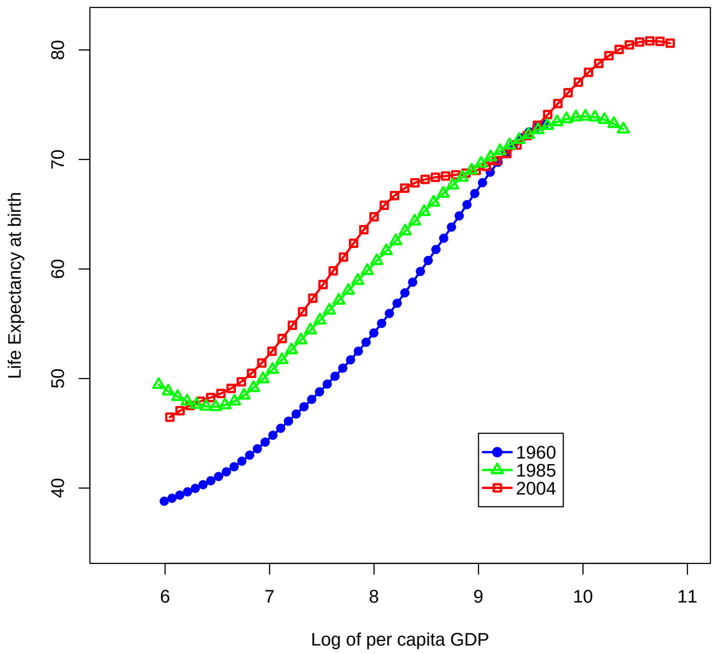

Figure 1: Life Expectancy versus Log of GDP per capita (1960, 1985, 2004). Nonparametric kernel smoother. Per capita GDP data are from Penn World Table 6.2. Life expectancy at birth data are from World Development Indicators CD-ROM, World Bank (2006).

A large body of the literature suggests that one of the most important factors for this raise in life expectancy is the increase in the level of education (Grossman, 1982; Shultz, 1999). Higher agent's education, indeed, implies a higher willingness to invest in health care either because education makes people better decision makers or because more educated people have better informations about health. Education can improve health through a better choice of health inputs: it reduces smoking, improves eating habits and increases exercise (Adams, 2002). In addition, home environment and parents' schooling are correlated with lower child mortality (Grossman, 1982; Shultz, 1999). This a central point since the increase in life expectancy at birth mostly comes from the reductions in child mortality. At the same time as mortality rates decline there is a sharp decline in the fertility rates. Figure 2 gives an insight of this relationship between child mortality rates and fertility rates. In addition, Figure 2 shows that fertility, especially in 1960, is increasing for low levels of income and decreasing for high levels of income.

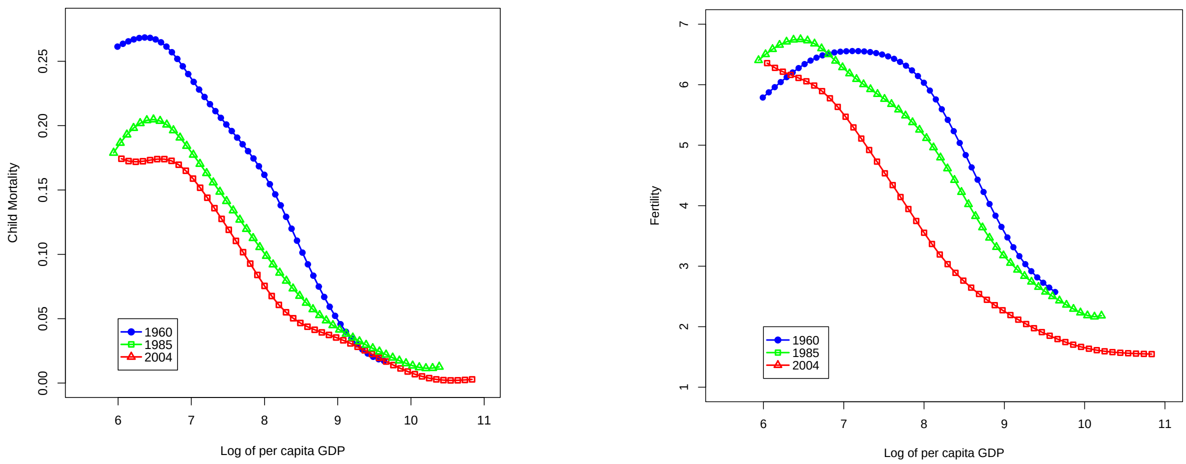

Figure 2: Child Mortality and Fertility Rate (1960, 1985, 2004). Nonparametric kernel smoother. Per capita GDP data are from Penn World Table 6.2. Fertility rate and Child Mortality data are from World Development Indicators CD-ROM, World Bank (2006).

There are many theories which explain this path in mortality and fertility rates. In particular, we refer to the seminal paper of Becker et al. (1990) which analyzes the relationship between economic growth and fertility choice. This approach shows that initial levels of human capital and technology, determine whether a country grows over time or stagnates at low income levels. Societies with low levels of human capital choose large families and invest little in each member since the return to human capital is scarce. On the other hand, in societies with high levels of human capital, the rate of return on human capital is high relative to the rate of return on children and choose to have small families and devote more resources to the investment in education. Our work is also related to the papers of Galor and Weil (1999), Galor (2005) and Kalemli-Ozcan (2002). The papers of Galor and Weil (1999) and Galor (2005) describe the process of economic and demographic development of Western Europe as passing through three distinct regimes. The first is called the Malthusian Regime. Here the relationship between income per capita and population growth is positive: small increases in income lead to population growth. In the second regime, called the post-Malthusian regime, both per capita income and population present a positive growth rate and their relationship remains positive as in the Malthusian regime. The final stage of development is the Modern Growth Regime. In this latter, both income per capita and the level of technology present a positive growth rate whereas population growth declines. Galor and Weil (1999) focus on the technology, the evolution of population and the output growth as the key elements which can explain the transition process through to these three stages. They argue that the technological progress raises the rate of return to human capital inducing parents to invest in children education. In particular, technological progress has two effects on population growth. On the one hand, improved technology increases households' budget constraints, allowing them to spend more resources on raising children. On the other hand, it induces a reallocation of these increased resources toward children education, that is children "quality". In the Post-Malthusian Regime, the former effect dominates, and so population grows. However, since the return to child quality continues to rise, the shift away from child quantity becomes more significant causing a reduction in the population growth rate and an increase in the output growth rate.

Based on this approach, Kalemli-Ozcan (2002) focuses on the effects of reductions in child mortality on fertility, education and economic growth. In particular, child mortality depends on parent's income. At low levels of income per capita, population increases with income causing a reduction of income per capita. Thus, the economy is in a stable Malthusian steady state where fertility is high and human capital investment is low. At high levels of income per capita population growth falls as income per capita increases. This leads to a unstable growth steady state with low fertility and high human capital investment.

Finally we refer to Lagerlof (2003)'s paper which models demographic and economic long-run development in a setting where mortality depends on agent's human capital and subjects to epidemic shocks. The transition from the Malthusian trap to the sustained growth is generated from a series of mild epidemic shocks. When the economy experiences a phase of relatively mild epidemic shocks, mortality rates fall leading to a positive population growth rate. However, birth rates remain unchanged and parents do not invest in children's education. When the education time becomes positive the economy transits into the modern growth regime. In this regime, the economy experience a quality-quantity substitution in children, i.e. birth rates fall since education time make children more expensive. Once the growth rate of human capital is high the impact of further epidemics becomes negligible and the economy remains in the modern growth regime. Our paper departs from this literature by stressing the effect of different initial human capital endowments on fertility and education choices. We develop an overlapping generations model where altruistic parents care about the number of their surviving children and future level of human capital of each of them. We assume that parents spend a proportion of their income to raise each born child and invest in education of surviving children. The child's probability of surviving to the adult age is increasing concave function of parent's human capital (see Figure 3)[1].

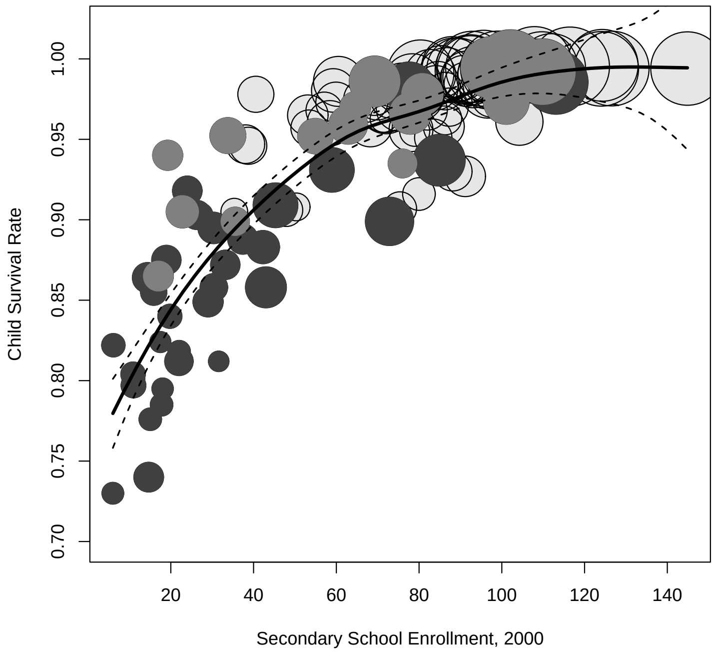

Figure 3: Child Survival Rate (under-5) versus Secondary School Enrollment. Nonparametric kernel smoother, year 2000, n=127. Source: World Development Indicators CD-ROM, World Bank (2006). Circles are proportional to the country's per capita income, the black ones indicate Sub-Saharan countries and the gray ones indicate east Asian and Pacific countries (W.D.I, 2006).

This framework allows us to generate the demographic transition and has the effect of creating multiple development regimes such that the growth rate of the economy depends on initial human capital endowments.

The structure of the paper is as follows. The model is set out in Section 1. Section 2 shows the optimal fertility and education choices. Section 4 contains the analysis of human capital accumulation. Finally, some concluding remarks are made in section 5.

[1]The confidence interval indicates the degree of variability in the estimate.

2 The model

Consider an overlapping-generations economy that operates in a perfectly competitive world. Activity extends over a infinite discrete time. In every period, the economy produces a single material good, the price of which is normalized to 1.

2.1 Production Technology

Production function is linear in the stock of human capital[2]:

$$Y_t = H_t. \tag{1}$$

Hence, firms employ the whole labor force to produce as long as the wage per unit of human capital is lower or equal to one. The equilibrium in the labor market thus implies that the wage per unit of human capital is constant through time and equal to one, i.e., $w_t = 1$, for all t.

2.2 Agents' preferences

Consider an overlapping generations economy where members of generation t live for two periods: childhood and adulthood. All decisions are made in the adult period of life. Individuals have an endowed level of human capital h_t, determined from previous generations decisions.

Parents have n_t children of which a fraction $1 - \pi$ dies before reaching adulthood. In particular, we suppose that children's probability of reaching adulthood depends on parents human capital endowment, i.e. $\pi_t = \pi(h_t)$.

Individual's preferences are defined over a consumption above a subsistence level $\tilde{c} > 0$, the number of surviving children $\pi(h_t)n_t$, i.e. children's quantity, and the human capital of children h_{t+1}, i.e. children's quality (see Galor, 2005). The utility function of an agent of generation t, U^t, is given by:

$$U^t = (1-\gamma)\log(c_t) + \gamma \log(\pi(h_t)n_t h_{t+1}), \tag{2}$$

where the parameter $\gamma > 0$ is the altruism factor.

Agents allocate their income, i.e. $w_t h_t$, between consumption c_t, child rearing and education spending per child e_t. In particular, raising each born child takes a fraction $\phi \in (0, 1)$ of an adult's income. This implies that having many children is more costly for parents who have high income. The investment in education e_t is devoted only to each surviving children. Thus the agent's budget constraint is given by:

[2]For simplicity we abstract from physical capital.

$$c_t = w_t h_t (1 - \phi n) - \pi(h_t) e_t n_t, \tag{3}$$

where w_t from equation (1) is equal to 1.

The human capital of children h_{t+1} depends on parents' human capital h_t and education spending e_t, that is:

$$h_{t+1} = (\theta + e_t)^{\alpha} h_t^{1-\alpha}, \tag{4}$$

where $\theta > 0$ and $\alpha \in (0, 1)$. The presence of θ implies that children's human capital is positive even if parents do not invest in education (De la Croix and Doepke, 2004).

2.3 Endogenous Child Mortality

Many contributions focus on the positive relationship between parent's human capital and child's health status. Shultz (1993), for example, shows that higher level of parents human capital are correlated with lower child mortality, even after holding per capita income constant (see Figure 3). In particular, women's education is the most significant determinant of child mortality. A year of additional schooling for the mother is often associated, in a low-income country, with 5-10 percent reduction in her child's probability of dying in the first five years of life (Shultz, 1993).

Following empirical evidence (Figure 3) the survival probability of children is assumed to satisfy the following properties:

$$\partial \pi_t / \partial h_t > 0, \tag{5}$$

$$\partial^2 \pi_t / \partial h_t^2 < 0, \tag{6}$$

$$\lim_{h_t \to 0} = \underline{\pi} \geq 0, \tag{7}$$

$$\lim_{h_t \to \infty} \pi(h_t) = \bar{\pi} \leq 1. \tag{8}$$

Hence, we specify the children probability of surviving as follows (see Blackburn and Cipriani, 2002):

$$\pi_t = \frac{\underline{\pi} + \bar{\pi} \delta (h_t)^{\eta}}{1 + \delta (h_t)^{\eta}}, \tag{9}$$

where the parameters $0 < \eta \leq 1$ and $\delta > 0$ jointly determine both the turning point in $\partial \pi_t / \partial h_t$ and the speed at which $\pi(h_t)$ traverses the interval $(\underline{\pi}, \bar{\pi})$. For a given value of η, an increase (decrease) in δ reduces the turning point, while for a given value of such a point, an increase (decrease) in η raises the speed of transition (the limiting case of which is when $\pi(h_t)$ changes value from $\underline{\pi}$ to $\bar{\pi}$ instantaneously, which corresponds to the case of a step function (Blackburn and Cipriani, 2002). For simplicity we assume that $\eta = 1$ and $\delta = 1$:

$$\pi_t = \frac{\underline{\pi} + \bar{\pi} h_t}{1 + h_t}. \tag{10}$$

3 Fertility and Education choices

Members of generation t choose the number of children, the education spending for each of them and their own consumption. Substituting equations (3) and (4) into equation (2), agents maximization problem is given by:

$$U^t = (1-\gamma)\log(h_t\,(1-\phi n_t) - \pi(h_t)e_t n_t) + \gamma \log(\pi(h_t) n_t\,(\theta + e_t)^{\alpha}\, h_t^{1-\alpha}), \tag{11}$$

subject to:

$$h_t\,(1-\phi n_t) - \pi(h_t)e_t n_t \geq \tilde{c}, \tag{12}$$

$$(n_t, e_t) \geq 0. \tag{13}$$

For agents that have enough income so as to assure a consumption above $\tilde{c}$, the optimal education level and the optimal number of children are given by:

$$n_t = \frac{\gamma h_t\,(1-\alpha)}{\phi h_t - \pi(h_t)\theta}, \tag{14}$$

and:

$$e_t = \frac{h_t \phi \alpha - \pi(h_t)\theta}{\pi(h_t)\,(1-\alpha)}. \tag{15}$$

In particular, the optimal consumption is above $\tilde{c}$ when human capital is above $\tilde{h} = \tilde{c}/(1-\gamma)$ (for the technical aspects see appendix A). Hence, when $h_t > \tilde{h}$ a fraction $1-\gamma$ of h_t is devoted to the consumption and a fraction γ of h_t is devoted to raising children and the education spending for each child (see figure 4). In the other hand, when $h_t \leq \tilde{h}$, agents devote their income to secure a consumption equals to $\tilde{c}$, and the remaining part, that is $h_t - \tilde{c}$, is devoted to raising children and the education spending for each child (see figure 4), that is:

$$[\phi h_t + e_t \pi(h_t)]\, n_t = \begin{cases} \gamma h_t & \text{if } h_t > \tilde{h} \\ h_t - \tilde{c} & \text{if } h_t \leq \tilde{h}, \end{cases} \tag{16}$$

from which we can see $h_t \geq \tilde{c}$.

Given equation (15) we can see that there is an interior solution for the optimal education choice if agents have enough human capital such that $h_t > \bar{h}$ (the human capital level $\bar{h}$ is given in appendix A). Hence, given the human capital level $\tilde{h}$, we can distinguish two cases depending if $\tilde{h} < \bar{h}$ and $\tilde{h} > \bar{h}$. However, we suppose that

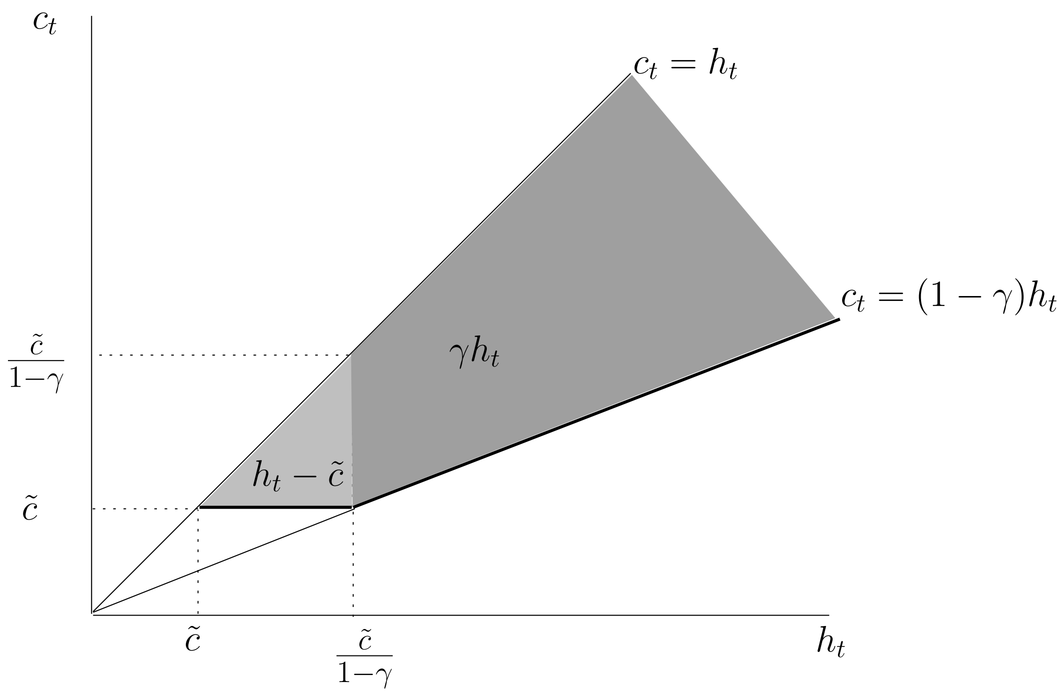

Figure 4: Consumption path.

$\tilde{h} < \bar{h}$ on the consideration that only when income is sufficiently high so as to assure a consumption above the subsistence level, parents begin to invest in children education (the case $\tilde{h} > \bar{h}$ is analyzed in appendix A). Thus, given $\tilde{h} < \bar{h}$, there are the following regimes:

$$\begin{aligned} &\tilde{c} \leq h_t \leq \tilde{h}, \\ &\tilde{h} < h_t \leq \bar{h}, \\ &h_t > \bar{h}. \end{aligned} \tag{17}$$

The optimal number of children and the level of education chosen by members of generation t, in the three regimes, are given by (see appendix A):

$$n_t = \begin{cases} \frac{1}{\phi}\left(1 - \frac{\tilde{c}}{h_t}\right) & \text{if } \tilde{c} \leq h_t \leq \tilde{h}, \\ \frac{\gamma}{\phi} & \text{if } \tilde{h} < h_t \leq \bar{h}, \\ \frac{\gamma h_t (1-\alpha)}{\phi h_t - \pi(h_t)\theta} & \text{if } h_t > \bar{h}. \end{cases} \tag{18}$$

and:

$$e_t = \begin{cases} 0 & \text{if } h_t \leq \bar{h}, \\ \frac{h_t\phi\alpha - \pi(h_t)\theta}{\pi(h_t)(1-\alpha)} & \text{if } h_t > \bar{h}. \end{cases} \tag{19}$$

Therefore, when human capital is $\tilde{c} \leq h_t \leq \tilde{h}$, the optimal choice for education is zero while the optimal number of children increases in h_t, i.e. $\partial n_t/\partial h_t > 0$, $\partial^2 n_t/\partial h_t^2 < 0$ (see appendix A.1). For less educated parents, indeed, the opportunity cost of raising children is low while providing education is expensive relative to their income.

When human capital is $\tilde{h} < h_t \leq \bar{h}$, optimal education choice is zero while the optimal number of children ceases to increase in parents' human capital and becomes a constant.

When $h_t > \bar{h}$, the optimal number of children decreases in h_t, that is $\partial n_t/\partial h_t < 0$ (see appendix A.1). Indeed, as income raises the cost of having more children increases and parents choose to have a lower number of children and to give more education to each of them. Thus since for agents with a high human capital level, the rate of return on human capital is higher that the return on children, they choose to have a low number of children and to devote more resources to the education of each child. The lowest possible fertility rate is given by:

$$\lim_{h_t \to \infty} n_t = \frac{\gamma(1-\alpha)}{\phi}.$$

Fertility as a function of human capital is plotted in figure 5

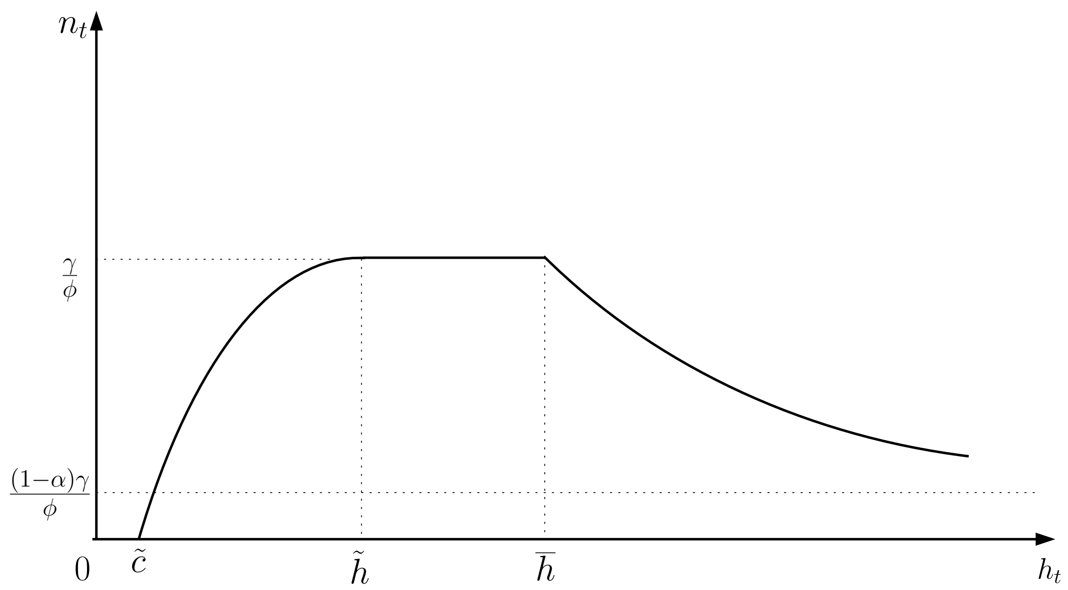

Figure 5: Fertility as a function of human capital.

Equation (19) shows that the optimal education spending chosen by skilled parents is increasing concave with respect to parents' human capital, that is $\partial e_t/\partial h_t > 0$, $\partial^2 e_t/\partial h_t^2 > 0$ (see appendix A.2).

Parents face a trade-off between the optimal number of children and the amount of resources to invest on the education of each child. For parents with a low level of human capital, the opportunity cost of raising children is low, while providing education is expensive relative to their income. Unskilled parents, therefore, prefer to have many children but invest little in the education of each child. As long as income is sufficiently high the optimal number of children decreases in income and the investment in education increases in income. For parents with a sufficiently level of human capital, indeed, the opportunity cost of child rearing is high and the rate of return in education is high. Hence, they prefer to invest in the education or "quality" of a small number of children.

4 Dynamic of Human Capital

Given the optimal education choice from equation (19) above, we can now characterize the dynamic of human capital accumulation as follows:

$$h_{t+1} = \begin{cases} (\theta)^{\alpha} h_t^{1-\alpha} & \text{if } h_t \leqslant \bar{h} \\ \\ \left(\theta + \frac{h_t \phi \alpha - \pi(h_t)\theta}{\pi(h_t)(1-\alpha)}\right)^{\alpha} h_t^{1-\alpha} & \text{if } h_t > \bar{h}. \end{cases} \tag{20}$$

The economy shows multiple development regimes if:

$$h_{t+1\,h_t=\bar{h}} < \bar{h},$$

which is satisfied when the following condition holds (see appendix B):

$$\phi < \underline{\pi}.$$

Therefore, an economy which starts with a human capital level below $\bar{h}$, converges to a stable equilibrium $h_L = \theta$ which is a Malthusian steady state (see appendix B). An economy with an initial human capital level above $\bar{h}$ grows in the long run if the following condition holds (for technical details see appendix B):

$$\alpha > \frac{\overline{\pi}}{\phi + \overline{\pi}}. \tag{21}$$

We collect these results in Proposition 1 below, the technical aspects of which are proved in Appendix AB.

Proposition 1 *An economy with an initial human capital level below $\bar{h}$ converges to a Malthusian steady state h_L. An economy with an initial human capital level above $\overline{h}$ grows in the long run.*

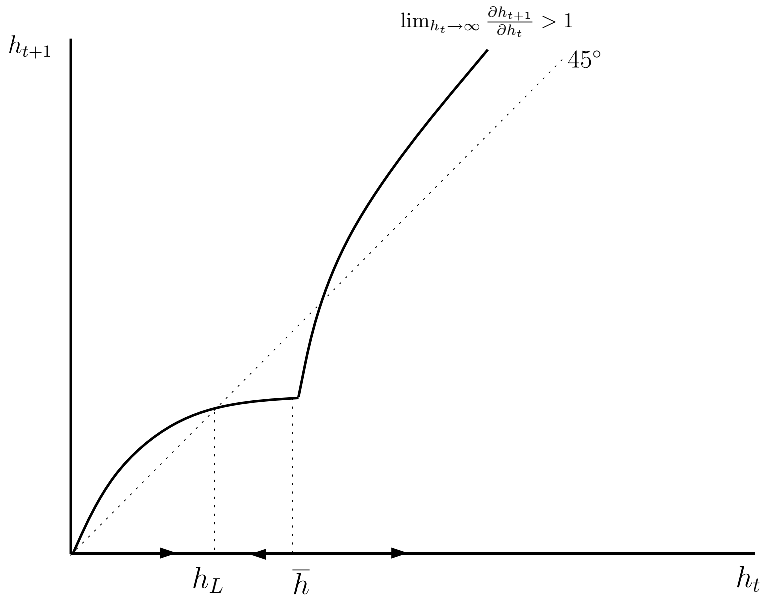

Figure 6: Multiple development regimes.

Figure 6 depicts the dynamic of human capital. The initial stock of human capital determines the allocation of total resources between parents' consumption, education spending and the number of children. Given this initial conditions, the economy develops along one of the two paths, either to the left or to the right of $\bar{h}$. Agents endowed with a low level of human capital do not invest in children education and devote their income to the consumption and the number of children. This leads the economy to a Malthusian equilibrium where education is zero, fertility is high and the survival probability of children is low. When human capital is above $\bar{h}$, skilled agents choose to invest in education of their children since the rate of return in the investment in human capital is higher than the rate of return on the number of children. A lower number of children with increased levels of human capital investment lead to endogenous growth.

5 Concluding remarks

In this paper we analyze the effect of income inequality on fertility, child mortality and education choices. This framework allows us to generate the demographic transition and has the effect of creating multiple development regimes such that the growth rate of the economy depends on initial conditions. The initial human capital endowments is the key factor in explaining the persistence in income inequality across households. For low level of income, the optimal spending in children education is zero, fertility increases in human capital and child mortality is high. Therefore, when income is below the subsistence level the economy converges to a Malthusian steady state where parents do not invest in children education and choose to invest in the quantity of their children. When income is above the subsistence level, the economy is on a high development path: as income rises, child mortality decreases, parents choose to have a lower number of children and to devote more resources to children's education spending.

Appendix

A Optimal Conditions

Given agents maximization problem by equations (11), (12) and (13) the first order conditions yield equations (14) and (15) for the optimal number of children and the optimal education spending respectively. Substituting equations (14) and (15) into the budget constraint we obtain the optimal consumption as follows:

$$c_t = h_t \left(1 - \gamma\right),$$

from which, consumption is above the subsistence level, i.e. $\tilde{c}$, if:

$$h_t \geq \frac{\tilde{c}}{(1-\gamma)}, \tag{22}$$

where we define the human capital level $\tilde{h} = \tilde{c}/\left(1-\gamma\right)$ such that $c_t = \tilde{c}$.

When $c_t > \tilde{c}$ a fraction $1-\gamma$ of h_t is devoted to the consumption and a fraction γ of h_t is devoted to raising children and the education spending for each child, that is:

$$c_t = h_t \left(1 - \gamma\right),$$

$$\left[\phi h_t + e_t \pi(h_t)\right] n_t = \gamma h_t.$$

When $c_t = \tilde{c}$, the difference between income and the subsistence consumption is devoted to raising children and the education spending for each child, that is:

$$c_t = \tilde{c},$$

$$\left[\phi h_t + e_t \pi(h_t)\right] n_t = h_t - \tilde{c}. \tag{23}$$

Given equation (15) there is a corner solution for education if:

$$h_t \leq \frac{\pi(h_t)\theta}{\phi\alpha},$$

where using equation (10) we obtain the following solutions for h_t:

$$h_1 = \frac{-(\phi\alpha - \overline{\pi}\theta) + \sqrt{(\phi\alpha - \overline{\pi}\theta)^2 + 4\phi\alpha\underline{\pi}\theta}}{2\phi\alpha} > 0, \tag{24}$$

$$h_2 = \frac{-(\phi\alpha - \overline{\pi}\theta) - \sqrt{(\phi\alpha - \overline{\pi}\theta)^2 + 4\phi\alpha\underline{\pi}\theta}}{2\phi\alpha} < 0, \tag{25}$$

where $h_1 > 0$ and $h_2 < 0$.

We define the human capital $h_1 = \bar{h}$ such that when $h_t < \bar{h}$ the optimal choice for education is zero. Given the human capital level $\tilde{h}$, we distinguish two cases depending if $\tilde{h} < \bar{h}$ or $\tilde{h} > \bar{h}$. When $\tilde{h} < \bar{h}$ we have the three regimes given by (17). In the first regimes, i.e. $\tilde{c} \leq h_t \leq \tilde{h}$, the optimal choice for education is zero, that is:

$$e_t = 0$$

Substituting this solution into equation (23) the optimal number of children is given by:

$$n_t = \frac{1}{\phi}\left(1 - \frac{\tilde{c}}{h_t}\right). \tag{26}$$

When $h_t = \tilde{h}$, it follows that:

$$n_t = \frac{\gamma}{\phi}.$$

When $\tilde{h} < h_t \leq \bar{h}$, consumption is above the subsistence level $c_t > \tilde{c}$ and the optimal spending in education is zero. Hence, agents maximize the following utility function:

$$U^t = (1-\gamma)\log(h_t(1-\phi n_t)) + \gamma\log(\pi(h_t)n_t h^{1-\alpha}\theta^\alpha),$$

which yields the following optimal decision rule for the number of children:

$$n_t = \frac{\gamma}{\phi}.$$

When $h_t > \bar{h}$ the optimal number of children and the optimal choice for education are given by equations (14) and (15) respectively.

In the second case, i.e. $\tilde{h} < \bar{h}$, we have three regimes given by:

$$\tilde{c} \leq h_t \leq \bar{h},$$

$$\bar{h} < h_t \leq \tilde{h}, \tag{27}$$
$$h_t > \tilde{h}.$$

The first order conditions give the following solutions for the optimal number of children:

$$n_t = \begin{cases} \frac{1}{\phi}\left(1 - \frac{\tilde{c}}{h_t}\right) & \text{if } \tilde{c} \leq h_t \leq \bar{h}, \\ \frac{(h_t - \tilde{c})(1-\alpha)}{h_t\phi - \pi(h_t)\theta} & \text{if } \bar{h} < h_t \leq \tilde{h}, \\ \frac{\gamma h_t(1-\alpha)}{\phi h_t - \pi(h_t)\theta} & \text{if } h_t > \tilde{h}. \end{cases} \tag{28}$$

Thus fertility behaves as in the first case (see equation (18)) expect for $\bar{h} < h_t \leq \tilde{h}$. In this regime, indeed, agents maximize the following utility function:

$$U^t = (1-\gamma)\log(\tilde{c}) + \gamma \log(\pi(h_t)\left(\frac{h_t - \tilde{c}}{h_t\phi - e_t\pi(h_t)}\right) h^{1-\alpha}\left(e+\theta\right)^{\alpha}).$$

The first order conditions yield equation (15) for education, and the following solution for the optimal number of children:

$$n_t = \frac{\left(h_t - \tilde{c}\right)\left(1-\alpha\right)}{h_t\phi - \pi(h_t)\theta}.$$

A.1 Optimal Fertility

Given the optimal fertility in equation (18), when human capital is low, that is $\tilde{c} \leq h_t \leq \tilde{h}$, the optimal number of children increases in human capital and has a concave shape with respect to h_t, that is:

$$\frac{\partial n_t}{\partial h_t} = \frac{\tilde{c}}{\phi h_t^2} > 0, \tag{29}$$

and:

$$\frac{\partial^2 n_t}{\partial h_t} = -\frac{2\tilde{c}}{\phi u h_t^3} < 0. \tag{30}$$

When $h_t > \bar{h}$ the optimal number of children decreases in h_t, that is:

$$\frac{\partial n_t}{\partial h_t} = -\frac{\gamma\left(1-\alpha\right)\pi\left(h_t\right)\theta}{\left(\phi h_t - \pi\left(h_t\right)\theta\right)^2} < 0, \tag{31}$$

and:

$$\frac{\partial^2 n_t}{\partial h_t^2} = \frac{2\gamma\left(1-\alpha\right)\pi\left(h_t\right)\theta\phi}{\left(\phi h_t - \pi\left(h_t\right)\theta\right)^3} > 0. \tag{32}$$

If we suppose that $\tilde{h} > \bar{h}$ optimal fertility choice is given by equation (28). The optimal number of children behaves as in the case $\tilde{h} < \bar{h}$ except for a human capital level $\bar{h} < h_t \leq \tilde{h}$. Indeed, in this regime fertility increases in human capital, that is:

$$\frac{\partial n_t}{\partial h_t} = \frac{(1-\alpha)\left[h_t^2\left(\phi\tilde{c} - \overline{\pi}\theta\right) + 2h_t\left(\phi\tilde{c} - \theta\underline{\pi}\right) + \phi\tilde{c} - \theta\underline{\pi} - \theta\tilde{c}\left(\overline{\pi} - \underline{\pi}\right)\right]}{\left[\theta\left(\underline{\pi} + \overline{\pi}h_t\right) + \left(1 + h_t\right)\phi h_t\right]^2} > 0,$$

where we suppose that:

$$\phi\tilde{c} - \overline{\pi}\theta > 0,$$

$$\tilde{c}\left[\phi - \theta\left(\overline{\pi} - \underline{\pi}\right)\right] - \theta\underline{\pi} < 0.$$

We obtain the two solutions:

$$h_1 = \frac{\left(\phi\tilde{c} - \theta\underline{\pi}\right) + \sqrt{\theta\tilde{c}\left(\phi\tilde{c} - \theta\underline{\pi}\right)\left(\overline{\pi} - \underline{\pi}\right)}}{\left(\phi\tilde{c} - \overline{\pi}\theta\right)} > 0,$$

$$h_2 = \frac{\left(\phi\tilde{c} - \theta\underline{\pi}\right) - \sqrt{\theta\tilde{c}\left(\phi\tilde{c} - \theta\underline{\pi}\right)\left(\overline{\pi} - \underline{\pi}\right)}}{\left(\phi\tilde{c} - \overline{\pi}\theta\right)} < 0.$$

Thus when $h_t > h_1$ it follows that $\partial n_t / \partial h_t > 0$.

Finally when $h_t > \tilde{h}$, the number of children decreases in h_t as we can see in equations (31) and (32).

A.2 Optimal Education

Given the optimal education choice in equation (19), when $h_t > \bar{h}$ the spending in education of each child increases in h_t and has a concave shape with respect to h_t, that is:

$$\frac{\partial e_t}{\partial h_t} = \frac{\phi\alpha\left[\pi(h_t) - h_t\pi'(h_t)\right]}{\left[\pi(h_t)\right]^2(1-\alpha)} > 0,$$

since $\pi(h_t) - h_t\pi'(h_t) > 0$.

The second derivative is given as follows:

$$\frac{\partial^2 e_t}{\partial h_t^2} = -\phi\alpha\left(1-\alpha\right)\pi(h_t)\left\{\frac{h_t\pi''(h_t)\pi(h_t)\left(1-\alpha\right) + 2\left[\pi(h_t) - h_t\pi'(h_t)\right]\pi(h_t)\left(1-\alpha\right)\pi'(h_t)}{\left[\pi(h_t)\left(1-\alpha\right)\right]^4}\right\},$$

which is negative since $\pi(h_t) - h_t\pi'(h_t) > 0$.

B Human Capital

Given human capital accumulation in equation (20) we have that the economy shows multiple development paths if:

$$h_{t+1\,h_t=\bar{h}} < \bar{h},$$

that is:

$$\bar{h}(\phi\alpha - \overline{\pi}) + \phi\alpha - \underline{\pi} < 0, \tag{33}$$

where since $\overline{\pi}$ tend to 1, it follows that $\phi < \overline{\pi}$, that is:

$$\alpha < \frac{\overline{\pi}}{\phi},$$

since $\alpha < 1$. Hence, equation (33) is satisfied if the following condition holds:

$$\phi < \underline{\pi},$$

which implies that $\phi\alpha - \underline{\pi} < 0$. Thus, when $h_t \leqslant \bar{h}$, the economy shows the stable steady state h_L, that is:

$$h_L = \theta,$$

where:

$$\frac{\partial h_{t+1}}{\partial h_t}_{h_t=\theta} = (1-\alpha) < 1.$$

When $h_t > \bar{h}$, the economy grows in the long run at a constant rate if:

$$\lim_{h_t \to \infty} \frac{\partial h_{t+1}}{\partial h_t} > 1,$$

that is:

$$\frac{\alpha\phi}{\overline{\pi}\,(1-\alpha)} > 1,$$

which is satisfied if:

$$\alpha > \frac{\overline{\pi}}{(\phi + \overline{\pi})}.$$

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