Habit Formation in an Endogenous Growth Model with Pollution Abatement Activities *

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Abstract

This paper studies how the habit formation in consumption affects pollution abatement activities and the steady-state growth rate, in the context of an endogenous growth model in which agents derive disutility from the habit stock and pollution. The paper also examines how the effect of technological change in abatement on the optimal growth rate is enhanced or weakened by habit formation. We show that if agents persist in and care about their habits, sustained growth is only possible with rapid technological progress.

Keywords: habit formation, abatement cost and benefit, technological change.

JEL Classifications: D91, O40.

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1 Introduction

Most of the literature on sustained economic growth stresses the need for technological change towards less polluting production processes and increasing abatement activities in an expanding economy. Since R&D and abatement activities are costly, the society faces a trade-off between consumption, savings, and environmental investments. If individuals have consumption habit, their habit stock will play a role in society's decisions regarding this trade-off. An individual with consumption habit will dislike large and rapid reductions in consumption, and hence her response to environmental policies that induce technological change may also be affected by the strength of her habit formation.[1] This paper focuses on pollution abatement activities. We will show how resource allocation, and hence the long-term rate of economic growth, is affected by habit formation, and also how the effect of technological change in abatement on the optimal growth rate is enhanced or weakened by habit formation.[2]

Among a large number of theoretical studies that analyze the relationship between economic growth and the environment, Gradus and Smulders (1993) show that as individuals care more about the environment, increased abatement activities crowd out investment and lower the growth rate in the so-called "AK model," since there is no factor substitution such as knowledge stock or human capital accumulation.[3] Ligthart and van der Ploeg (1994) present evidence that a shift towards greener preferences raises the expenditure on public abatement, resulting in a lower rate of economic growth. In the theoretical literature on economic growth with habit formation, Ryder and Heal (1973) and Alvarez-Cuadrado *et al.* (2004) show that the neoclassical growth model incorporating habit formation follows a modified "golden rule," whereas Carroll *et al.* (1997, 2000) employ the AK model and show that the

[1] Klöckner *et al.* (2003) and Klöckner and Matthies (2004), psychologists, show that strong car choice habits block the process of normative decision-making. With a strong habit, personal norms to protect the environment have hardly any effect on behavior (Klöckner *et al.*, 2003).

[2] Extensive empirical evidence suggests the importance of habitual behavior of consumers in the dynamics of an economy (Brown, 1952; van de Stadt *et al.*, 1985; Naik and Moore, 1996).

[3] Bovenberg and Smulders (1995) develop a model in which growing environmental concern induces pollution-augmenting technological progress and higher growth rate. Smulders and Gradus (1996) study models in which the natural environment is a factor of production, and discuss whether a faster growth rate due an increase in the abatement propensity depends on the rate of discount and the rates of intratemporal and intertemporal substitution.

long-term rate of economic growth depends on the relative weight attributed to habit in utility.

An important question now emerges, namely: what is the impact of habit formation on the relationship between economic growth and the environment? On the one hand, environmental externalities reduce, in general, the steady-state growth rate in the absence of an engine of growth. On the other hand, there is a possibility that the presence of habit formation enhances the steady-state growth rate. Regarding the relationship between habit formation of consumption and the environment, Löfgren (2003) considers a model in which individuals experience habit formation in environmental quality, and illustrates the optimal tax path that is affected by habit formation. Valente (2006) introduces the disutility of pollution into a model of Alvarez-Cuadrado *et al.* (2004) without exogenous growth, and examines the steady-state consumption and capital stocks affected by the relative importance of habits.

Our approach is different. We introduce abatement activities into the model of Carroll *et al.* (1997, 2000), and this enables us to examine how the effect of technological change in abatement, in addition to the effect of optimal growth rate *per se*, is affected by habit formation.[4] One of the main findings of this paper is that the degree of persistence of habit stock has an important impact on abatement activities and economic growth, whereas it does not affect the steady-state growth rate in Carroll *et al.* (1997, 2000). In our model, if individuals adhere to their habit, the steady-state growth rate is higher, the optimal level of abatement activities is lower, and the optimal level of pollution is higher, which is consistent with findings by psychologists. An increase in the relative importance of habits raises the level of the optimal abatement activities so as to reduce the desired level of pollution. However, the effect of an higher importance of habit on the optimal growth rate is ambiguous, because there are two opposite effects: enhancing capital accumulation (positive effect) and raising the level of abatement activities (negative effect). Which effect dominates depends on the degree of persistence of habit and the level of technology

[4]Wendner (2000) and Ono (2002) introduce abatement activities of individuals into an overlapping generations model, but they do not discuss the technological change in abatement. Moreover, there is a generational conflict in the sense that young individuals presumably care about what the environmental quality will be when they are old, while the individuals may care less about what the environmental quality will be when they are dead. This is an additional factor affecting the dynamics of the economy.

of abatement present in the economy.

We also show that the effect of technological change in abatement on the optimal growth rate depends on the habit stock using simulations. When the level of technology is low, the more agents care about habit stock, the lower is the optimal growth rate. However, a strong persistence of habit stock enhances the effect of technological improvement on growth rate. Thus, if agents have persistent habit stock and care about their habit, more technological progress is required for sustained growth.

The remainder of the paper is organized as follows. In Section 2 we present the model. The social optimum is examined in Section 3, where we derive the conditions for optimality, then discuss the optimal balanced growth path, and finally investigate the optimal abatement activities. Section 4 examines the effects of technological change in abatement and habit stock on the steady-state growth rate. Some concluding remarks are presented in Section 5. Appendix A presents some stability properties of the steady state. All proofs are in Appendix B.

2 The model

Consider an economy with a continuum of identical individuals, called agents, where, without loss of generality, we normalize the number of individuals to one. An agent derives utility from the comparison of her current consumption to the habit stock, as well as from the absolute level of her current consumption; she derives disutility from pollution. The discounted sum of the intertemporal utility of a representative agent is given by

$$\int_0^\infty e^{-\theta t} u(c_t, h_t, P_t) dt, \tag{1}$$

where c_t denotes consumption in period t, h_t is the stock of reference consumption, and P_t is the level of aggregated pollution in the economy. For the first- and second-order partial derivatives of the utility function u, we assume that $u_c > 0$, $u_h < 0$, $u_P < 0$, $u_{cc} < 0$, $u_{PP} < 0$, $u_{cP} > 0$, and $u_{ch} > 0$.[5] To make the analysis tractable,

[5]The condition $u_{ch} > 0$ implies that the preference of an agent displays *adjacent complementarity* as characterized by Ryder and Heal (1973). An increase in the agent's current consumption increases her future consumption. In our analysis we focus on such situations where an agent habitually uses air-conditioning or a car, and where an increase in the production of such goods has an adverse effect on the environment.

we specify the utility function as

$$u(c_t, h_t, P_t) = \frac{(c_t h_t^{-\beta} P_t^{-\varepsilon})^{1-\gamma} - 1}{1-\gamma}, \tag{2}$$

where $\gamma > (1+\varepsilon)/\varepsilon$ is the inverse of the elasticity of intertemporal substitution, $\varepsilon > 0$ is the elasticity of utility with respect to pollution, and $\beta \in [0, 1]$ indexes the importance of the habit stock.[6]

Rewriting the term $ch^{-\beta}$ as $c^{1-\beta}(c/h)^{\beta}$, shows that β represents the geometric weight of relative consumption (to the habit stock). Therefore, the larger is β, the more importance the agent attaches to c/h, and the greater her disappointment will be when her current consumption is small relative to her past consumption.

The reference consumption level is determined as a weighted average of past consumption:

$$h_t = \rho \int_{-\infty}^{t} c_s e^{-\rho(t-s)} ds, \tag{3}$$

where $\rho > 0$ determines the relative weight given to the reference consumption level, or alternatively the discount rate for past consumption. The smaller is ρ, the higher is the degree of persistence of habitual behavior in consumption. For example, for $\rho = 0.1$, an agent places almost two thirds ($\exp(-s/10) = \exp(-0.5) = 0.61$) of the weight of current consumption on past consumption that she had $s = 5$ years ago. When the agent chooses current consumption, she attaches considerable importance to past consumption.

Differentiating the habit stock (3) with respect to time yields the evolution of the habit stock:

$$\dot{h}_t = \rho(c_t - h_t), \tag{4}$$

showing that not only the habit stock but also its evolution is influenced by current consumption decisions. Thus, ρ represents the speed at which the habit stock is adjusted. When ρ is large, then the habit stock adjusts quickly to actual consumption, so that c/h changes little. If utility would depend only on relative consumption c/h (and when ρ is large), then consumption growth would hardly affect the agent's utility.

The production technology is given by

$$Y_t = AK_t, \tag{5}$$

[6] These conditions imply that $\gamma > 1$ which is in accordance with empirical evidence and avoids the possibility of the nonexistence of an equilibrium (see Appendix B). Note that the condition $\gamma > (1+\varepsilon)/\varepsilon$ implies that $u_{PP} < 0$.

where Y_t denotes aggregate output and K_t the aggregate capital stock in period t. For convenience we assume that capital stock does not depreciate. The constant A is assumed to be sufficiently large to enable positive growth of output, as analyzed later.

Following Bretschger and Smulders (2007), we assume that pollution P_t in period t is generated by the use of the capital stock in production, but reduced by abatement activities M_t in the same period, and that a learning-by-abatement effect exists:

$$P_t = \frac{K_t}{\ell_P} \min\{1, (M_t/K_t)^{-(\phi-1)}\}, \qquad \ell_P = \max\{\ell_0, M_t\},$$

where ℓ_P, ℓ_0, and ϕ are parameters. Learning requires large enough abatement levels. We assume that $\ell_0 < K_t < M_t$, which leads to the specification in Gradus and Smulders (1993):

$$P_t = \left(\frac{K_t}{M_t}\right)^{\phi}, \tag{6}$$

where ϕ implies an exogenous elasticity of P_t with respect to the capital-abatement ratio K_t/M_t.

3 Social optimum

In this section we consider the social planner's problem and characterize the economy. We derive the optimal growth rate and show the existence of optimal abatement activities.

3.1 Conditions for optimality

The social planner takes the disutility of pollution into account when maximizing the welfare of the representative agent. The planner's problem is to maximize (1) subject to the flow resource constraint of the economy $\dot{K}_t = AK_t - C_t - M_t$, in the context of production technology (5), evolution of the habit stock (4), and pollution production technology (6). The current value Hamiltonian is given by

$$\mathcal{H} = u(C_t, H_t, P_t) + \lambda_t(AK_t - C_t - M_t) + \mu_t\rho(C_t - H_t),$$

where λ_t is the shadow price associated with the capital stock, μ_t is the shadow price associated with the habit stock (expected to be negative), and $c_t = C_t$ and $h_t = H_t$.

The optimality conditions are

$$u_c + \mu_t \rho = \lambda_t, \tag{7}$$

$$-\phi u_P \frac{P_t}{M_t} = \lambda_t, \tag{8}$$

$$\dot{\lambda}_t = (\theta - A)\lambda_t - u_P \phi \frac{P_t}{K_t}, \tag{9}$$

$$\dot{\mu}_t = (\theta + \rho)\mu_t - u_h, \tag{10}$$

together with the transversality conditions:

$$\lim_{t\to\infty} \lambda_t K_t e^{-\theta} = \lim_{t\to\infty} \mu_t H_t e^{-\theta} = 0. \tag{11}$$

Condition (7) implies that the sum of the marginal utility of consumption and the shadow value of the habit stock formed by additional consumption (measured in terms of final goods) is equal to the shadow price of the capital stock. The social planner chooses the level of abatement activities according to the rule in (8). The absolute value of the marginal utility derived by an additional unit of abatement activities is equal to the shadow price of capital. Conditions (9) and (10) are the Euler equations associated with the capital stock and the habit stock, respectively. Combining (7) with (8), the optimality allocation rule between consumption and abatement activity can be written as

$$u_c + \rho\mu_t = -\phi u_P \frac{P_t}{M_t}. \tag{12}$$

From (8) and (9), we obtain $\dot{\lambda}_t/\lambda_t = \theta - A + m_t$, where $m_t := M_t/K_t$. Differentiating (7), the Keynes-Ramsey rule is given by

$$\frac{\dot{C}_t}{C_t} = \Theta_t \left(\frac{(u_c - \mu_t \rho)}{u_c}(A - \theta - m_t) - \frac{\rho\big(\mu(\theta+\rho) - u_h\big)}{u_c} - \frac{u_{cP}}{u_c}\dot{P}_t - \frac{u_{ch}}{u_c}\dot{H}_t \right), \tag{13}$$

where $\Theta_t := -u_c/C_t u_{cc}$ is the elasticity of intertemporal substitution between current and future consumption. The first two terms in brackets represent the marginal return to postponed consumption, corresponding to the values of the capital and habit stocks. The third and fourth terms are disutility of pollution and habit stocks due to increased consumption without increases in abatement activities in the future. All items are measured as final goods.

From the resource constraint and (4), the growth rates of the aggregate capital stock and the aggregate habit stock are given by

$$\frac{\dot{K}_t}{K_t} = A - \frac{C_t}{K_t} - m_t, \qquad \frac{\dot{H}_t}{H_t} = \rho\left(\frac{C_t}{H_t} - 1\right). \tag{14}$$

3.2 Optimal balanced growth path

Following Chen (2007), we transform the economic system $\{\lambda, \mu, K, H\}$ into the system $\{\chi, q, \omega\}$, where $\chi := C/K$, $q := \mu/\lambda$, and $\omega := H/K$. This transformation will allow us to study the optimal balanced growth path of the economy.

We define a balanced growth path as a path along which all variables grow at the same rate, that is, $\dot{Y}/Y = \dot{C}/C = \dot{H}/H = \dot{K}/K = \dot{M}/M$. Then, χ, ω, q, m, and P are all constant along the optimal balanced growth path.

Eliminating $\lambda_t = \mu_t/q_t$ in (7) and substituting $C_t/H_t = \chi_t/\omega_t$, the shadow price of the habit stock (measured in terms of final goods) can be written as $\mu_t = (q_t u_c)/(1-\rho q_t)$. This represents the agent's willingness to pay for the reduction of one unit of habit stock. Substituting this in (10) using (2), the growth rate of the shadow price of the habit stock becomes $\dot{\mu}_t/\mu_t = \theta + \rho + (\beta(1-\rho q_t)\chi_t)/q_t\omega_t$. Also, from (12), the level of abatement activities can be written as $m_t = \phi\varepsilon\chi_t(1-\rho q_t)$.

Using (14) with the definitions of m_t, χ_t, q_t, and ω_t, and noting that $\dot{P}_t/P_t = -\phi\dot{m}_t/m_t$ (see equation 6), the economy can now be described as a system of three differential equations in the variables χ, q, and ω, as follows:

$$\dot{\chi} = \chi\left(\frac{(1-\gamma)A - \theta - \beta(\gamma-1)\rho}{\gamma + \phi\varepsilon(\gamma-1)} + \chi + \frac{\rho\beta\chi}{\omega} - \rho q\phi\varepsilon\chi + \frac{\rho q}{1-\rho q}\Phi\right), \tag{15}$$

$$\dot{q} = q\left(A - \phi\varepsilon\chi(1-\rho q) + \rho + \frac{\beta(1-\rho q)}{q}\frac{\chi}{\omega}\right), \tag{16}$$

$$\dot{\omega} = \omega\left(\rho\Big(\frac{\chi}{\omega} - 1\Big) - A + \chi + \phi\varepsilon\chi(1-\rho q)\right), \tag{17}$$

where $\Phi := (1+\phi\varepsilon(\gamma-1))(A+\rho)/(\gamma+\phi\varepsilon(\gamma-1))$.

Now consider the optimal growth rate and the optimal values of χ, q, ω, and m. Let us denote g^o as the optimal growth rate, and in general use the superscript "o" for optimal values: χ^o, q^o, ω^o, and m^o. From the fact that $\dot{H}/H = \dot{K}/K = g^o = A - \chi^o - m^o$ and $\chi^o/\omega^o = (g^o+\rho)/\rho$, we obtain the optimal values of χ_t and ω_t as

$$\chi^o(m^o) = A - m^o - g^o, \tag{18}$$

$$\omega^o(m^o) = \frac{\rho\,(A - m^o - g^o)}{g^o+\rho}. \tag{19}$$

Substituting (18) and (19) into $\dot{q} = 0$ in (16) yields the optimal value of q_t:

$$q^o(m^o) = -\frac{\beta(g^o+\rho)}{\rho\,(A - m^o + \rho - \beta(g^o+\rho))}. \tag{20}$$

These valuables are expressed as a function of m^o. Substituting (18)–(20) into $\dot{\chi} = 0$ in (15) and setting $\dot{m}/m = 0$, we obtain the optimal growth rate:

$$g^o(m^o) = \frac{A - \theta - m^o}{\gamma(1-\beta)+\beta}, \tag{21}$$

which is the same as the steady-state growth rate in the model of Carroll *et al.* (1997, 2000) in the special case where the abatement is set to zero and capital depreciation is introduced.[7] In the absence of abatement activities, the effect of an increase in the importance of habit stock β on the steady-state growth rate is positive when $\gamma > 1$. In the present model, however, the steady-state growth rate is affected by the parameter associated with the degree of persistence of habit stock ρ as well as by β, through abatement activities m^o (see the optimality allocation rule (12)). Therefore, whether the optimal growth rate increases or decreases as a result of changes in β and ρ depends on how much the optimal abatement activities are affected.

The transversality conditions associated with the capital stock and the habit stock are given by (11). These conditions imply that $\dot{\lambda}_t/\lambda_t + \dot{K}_t/K_t - \theta < 0$ and $\dot{\mu}_t/\mu_t + \dot{H}_t/H_t - \theta < 0$. Since we obtain that $\dot{\lambda}_t/\lambda_t + \dot{K}_t/K_t - \theta = \dot{\mu}_t/\mu_t + \dot{H}_t/H_t - \theta = -(A - m^o) + g^o < 0$, the transversality conditions associated with both of the stocks are satisfied.[8] In Appendix A we show that the steady state is saddle-path stable under the assumption that ϕ is sufficiently small.[9]

3.3 Optimal abatement activities

We now explore the optimal abatement activity. Dividing both sides of (12) by u_c and using the steady-state value of q^o in (20), we obtain the optimality allocation rule between consumption and abatement activity on the steady-state growth path and the optimal level of abatement activities satisfying this rule.

Proposition 1 *(i) There exists a unique positive level of abatement activities $m^o := m^o(\rho, \beta)$ satisfying the allocation rule between consumption and abatement activity on the steady-state growth path:*

$$1 - \frac{\beta\left(g^o(m^o) + \rho\right)}{A + \rho - m^o} = \phi\varepsilon\frac{\chi^o(m^o)}{m^o}. \tag{22}$$

[7] The negative impact of m^o on the steady-state growth rate is the same as what would result if we had introduced capital depreciation in the model in Carroll *et al.* (1997, 2000). A major difference between m^o and capital depreciation is that the former is determined endogenously, while the latter is determined exogenously.

[8] When $\gamma < 1$, the transversality conditions are satisfied if and only if $\theta > (1-\gamma)(1-\beta)(A - m^o)$.

[9] Ryder and Heal (1973) use general functions of utility and find a multiplicity of optimal stationary solutions with satiation in utility. In contrast, we specify the utility function and assume that $0 \leq \beta \leq 1$ so that we have saddle-point stability; see Carroll *et al.* (1997, 2000) and Alvarez-Cuadrado *et al.* (2004). However, when ϕ is large, the positive growth is not feasible or indeterminacy may occur.

(ii) Increases in the relative weight on recent consumption ρ and in the relative importance of habit stock β lead to a larger optimal abatement.

Let us define $MAC(m^o) := 1 - (\beta(g^o(m^o) + \rho)) / (A + \rho - m^o)$ for the left-hand side and $MAB(m^o) := \phi\varepsilon\chi^o(m^o)/m^o$ for the right-hand side of the rule (22). The

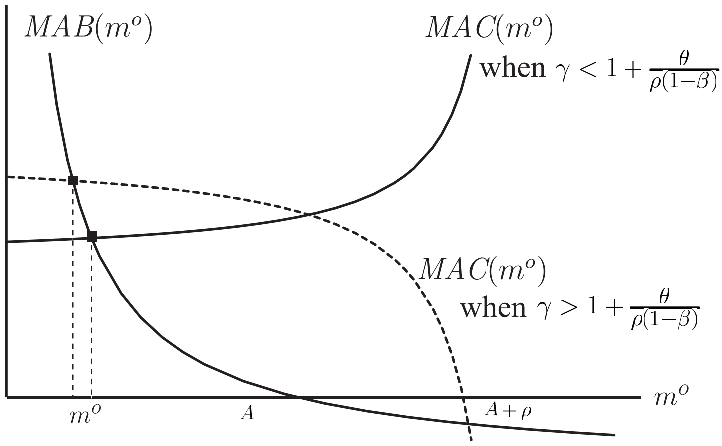

Fig. 1: The existence of optimal abatement m^o

intersection of two curves $MAC(m^o)$ and $MAB(m^o)$ is the optimal level of abatement activities m^o, as shown in Fig. 1. When $\gamma < 1 + \theta/\rho(1-\beta)$, the $MAC(m^o)$ curve is increasing in m^o; when $\gamma > 1 + \theta/\rho(1-\beta)$, the $MAC(m^o)$ curve is decreasing in m^o. In either case the $MAC(m^o)$ curve intersects with the $MAB(m^o)$ curve once at the interval $m^o < A + \rho$.

The intuition behind Proposition 1(ii) is as follows. When ρ is large, agents are more likely to postpone their current consumption because they derive disutility from the faster accumulation of habit stock, which encourages capital accumulation. A larger capital stock causes more pollution so that the social planner increases abatement activities. Therefore, the larger is ρ, the larger will be m^o. Conversely, when ρ is small, agents attach more value to their past consumption. Since they invest less in capital stock, the level of output as well as the level of pollution decreases, which results in a low level of abatement activities. This implies that the more agents adhere to their habit stock, the smaller will be m^o.

Similarly, when β is large, agents invest more in capital stock in order to avoid accumulating habit stock. Thus, larger capital stock requires greater abatement activities to reduce pollution, which is a positive effect of β on the optimal abatement activities. However, unlike the case where ρ increases, a larger value of β decreases the relative value of abatement activities to the value of consumption and the habit stock, which is a negative effect of β on m^o. Although the positive effect dominates the negative effect as shown in Appendix B, the increment in optimal abatement activities when β increases is smaller than when ρ increases.

Consider next the optimality allocation rule (22). The left-hand side ($MAC(m^o)$ curve) represents the marginal benefit of additional consumption (the sum of the direct marginal utility and the marginal disutility through additional habit stock) and the right-hand side ($MAB(m^o)$ curve) represents the marginal cost of giving up one unit of consumption in order to allocate goods to abatement activities, measured in terms of the marginal utility of consumption, u_c. In other words, $MAC(m^o)$ can be interpreted as the marginal cost of giving up a reduction in pollution and $MAB(m^o)$ can be interpreted as the marginal benefit of abatement activities.

Increases in ρ and β reduce the marginal cost of abatement activities, because a unit of consumption gives agents larger disutility of an additional habit stock. In addition, an increase in β reduces the marginal benefit of abatement activities since agents place more importance to the habit stock than to polluting emissions. The

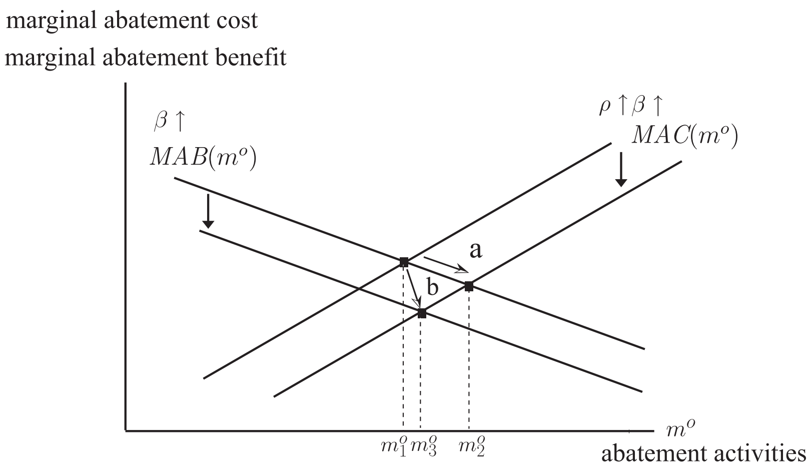

Fig. 2: The effect of changes in ρ and β on marginal abatement cost and marginal abatement benefit in the case where $\gamma > 1 + \theta/\rho(1-\beta)$

effects of changes in ρ and β, both on the marginal abatement benefit and on the marginal abatement cost, are shown in the simplified diagram in Fig. 2.

When ρ increases, the curve $MAB(m^o)$ shifts downwards, while the curve $MAC(m^o)$ remains unchanged. Consequently, the intersection of both curves shifts as shown by arrow a, resulting in a shift from m_1^o to m_2^o. In contrast, when β increases, both curves shift downwards and the intersection of both curves shifts as shown by arrow b. The optimal level of abatement activities shifts from m_1^o to m_3^o.[10] Thus, when agents express more concern about consumption relative to habit stock or recent consumption, the optimal level of abatement activities is higher.

Let us examine the optimal level of pollution, denoted by P^o. Equation (6) can be rewritten as $P = m^{-\phi}$, which shows that $\partial P^o/\partial m^o < 0$. An immediate consequence of Proposition 1 is our next result.

Proposition 2 *Increases in the parameters associated with the habit stock ρ and β reduce the optimal level of pollution in the long run: $\partial P^o/\partial \rho < 0$ and $\partial P^o/\partial \beta < 0$.*

When agents care more about recent consumption, the level of optimal abatement activities is higher and the level of optimal pollution is lower. Conversely, when agents adhere more to their consumption habits, a lower level of abatement activities is desired, and this results in a higher level of optimal pollution. In the latter case, the technological improvement in abatement is of immediate importance. To this issue we now turn.

4 Technological change in abatement and economic growth

In this section we examine the relationship between technological change in abatement and habit stock in consumption, and its effect on economic growth. We show that the effect of habit stock (technological change) on the steady-state growth is affected by technological change (habit stock).

Let us first examine the effect of changes in ρ and β on $g^o(m^o)$. Turning to the optimal growth rate in (21), it is obvious that an increase in m^o reduces $g^o(m^o)$: $\partial g^o(m^o)/\partial m^o < 0$, whereas the effect of an increase in β on the optimal growth rate is ambiguous. In fact, we can prove the following result.

Proposition 3 *An increase in the relative importance of recent consumption reduces the optimal growth rate: $\partial g^o\left(m^o(\rho;\beta)\right)/\partial \rho < 0$. When ρ and ϕ are sufficiently small, an increase in the relative importance of habit stock raises the optimal growth rate: $\partial g^o\left(m^o(\beta;\rho)\right)/\partial \beta > 0$.*

Since an increase in ρ reduces the marginal abatement cost and does not change the marginal abatement benefit, it is optimal to allocate more goods to abatement

[10] We can show the same property of the effect of a change in ρ and β on the marginal abatement cost and on the marginal abatement benefit in the case where $\gamma < 1 + \theta/\rho(1-\beta)$.

activities than to investments in capital stock; this reduces the steady-state growth rate. Without abatement activities, ρ does not affect the steady-state growth rate as shown in Carroll *et al.* (1997, 2000).

Observing the optimal growth rate in (21), there are two opposing effects of β on the growth rate: the negative effect of the increased abatement activities and the positive effect of the increased capital stock. Which effect dominates depends on the combinations of the values of β, ϕ, and ρ, as shown in Fig. 3. When ρ and ϕ

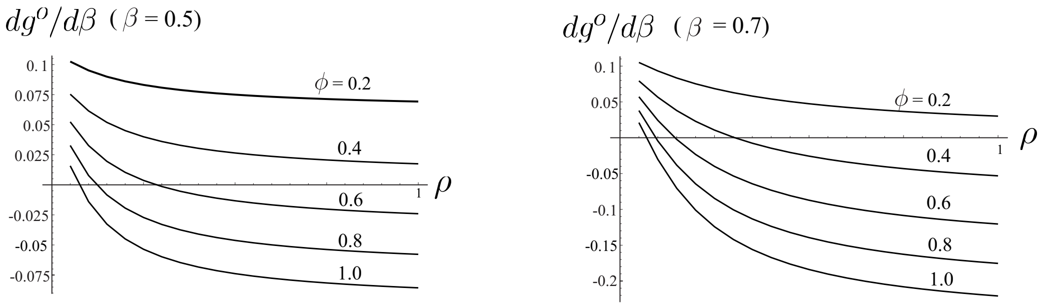

Fig. 3: The effect of changes in β on the growth rate as a function of ρ for various values of ϕ (Parameter values: $A = 0.13$, $\gamma = 2.50$, $\theta = 0.04$, and $\varepsilon = 0.70$)

are sufficiently small, the positive effect is likely to dominate. When ϕ is small, the technological level of abatement is high so that the marginal benefit of abatement activities is small, which leads to small m^o. This enhances the positive effect of an increase in β. However, as ρ increases, m^o is larger (Proposition 1), thus reducing the positive effect on the growth rate. Fig. 3 illustrates the threshold values at which the positive effect is offset by the negative effect. For example, in our simulation with $\phi = 0.6$, the threshold values are $\rho = 0.30$ if $\beta = 0.50$, and $\rho = 0.15$ if $\beta = 0.70$.

Consider next the effect of technological change on the steady-state growth rate. The effect of a change in ϕ on m^o is obtained by invoking the implicit function theorem on the optimality allocation rule for abatement activities (22):

$$\frac{dm^o}{d\phi} = -\frac{-(\varepsilon\chi)/m^o}{\partial MAC/\partial m^o - \partial MAB/\partial m^o} > 0, \tag{23}$$

where the sign of the denominator is discussed in Appendix B. Differentiating (21), we see that the effect of a change in ϕ on the steady-state growth rate is given by

$$\frac{dg^o}{d\phi} = -\frac{1}{\gamma(1-\beta)+\beta} \cdot \frac{\partial m^o}{\partial \phi}, \tag{24}$$

which is negative, because of (23), and depends on β and ρ through m^o.

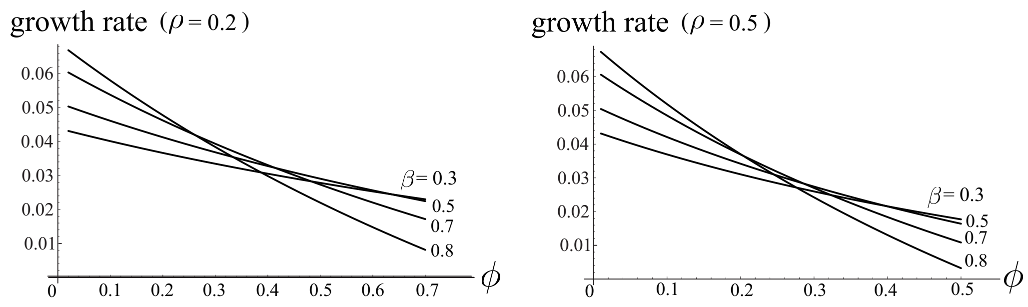

Fig. 4: The effect of changes in ϕ on the growth rate with various value of β and ρ (Parameter values: same as in Fig. 3)

The effect of technological changes in abatement on the optimal growth rate is shown in Fig. 4. As the technology of abatement is improved (ϕ is smaller), the growth rate is higher. With a low level of technology, the more agents care about the reference consumption, the lower is the optimal growth rate. However, as the technology of abatement improves, an economy whose agents care more about the reference consumption and have persistent habit stock will grow faster. In other words, if agents have persistent habits and care about their habits, more technological progress is required for sustained growth.

5 Concluding remarks

More and more people realize that climate change and environmental degradation are central issues of our time. Many governments have now introduced environmental policies which aim at making households and firms reduce emissions. If, however, people attach importance to their standard of living or tend to adhere to old habits, they may sacrifice environmental quality in order to carry on their way of life or pursue higher standards of living.

There is much evidence that people do indeed attach importance to their standard of living and tend to adhere to old habits. Shove (2003), a sociologist, focusing on conventions of comfort, cleanliness, and convenience, reports that the amount of laundry in the USA has tripled in fifty years. While people used to take a bath once a week, they now shower daily or twice daily. The number of households with air-conditioned rooms is increasing. These and other findings show that our daily lives are getting more resource-demanding and increase pressure on the environment, due to our habitual behavior.

For a given degree of environmental concern of individuals, an economy with habit stock will be different from one without habit stock, in terms of economic growth and environmental quality. In this paper we study, in the context of an endogenous growth model, how habit formation in consumption affects pollution abatement activities and the steady-state growth, and how technological change in abatement affects the growth rate. We show that the degree of persistence of habit stock plays a key role on abatement activities and economic growth. When individuals adhere to their habits, the optimal level of abatement activities is lower, and a higher level of pollution and a higher growth rate will result. If individuals care more about their habits, the optimal level of abatement activities increases and the optimal pollution level decreases. The effect on the optimal growth rate is ambiguous, depending on the degree of persistence of habit and the level of technology of abatement.

We also show that when the level of technology is low, then the more agents care about habit stock, the lower will be the optimal growth rate. This tendency is enhanced by a strong persistence of habit. Therefore, if agents have persistent habit stock and care about their habit, more technological progress is required for sustained economic growth.

Since habit-forming individuals dislike large and rapid reductions in consumption, their response to a change in environmental policy may be sluggish. Therefore, the effect of a change in environmental policy may only become apparent after a considerable time lag. The transitional dynamics of the economy in response to a change in environmental policy is an important issue for future research.

Appendix A: Stability of the steady state

Linearizing the three equations (15), (16), and (17) around the steady state, we obtain

$$\begin{pmatrix} \dot{\chi} \\ \dot{q} \\ \dot{\omega} \end{pmatrix} \approx J^o \begin{pmatrix} \chi - \chi^o \\ q - q^o \\ \omega - \omega^o \end{pmatrix}$$

with

$$J^o := \begin{pmatrix} \chi^o \left(1 + \frac{\rho\beta}{\omega^o} - \rho q^o \phi\varepsilon\right) & \chi^o \rho \left(-\phi\varepsilon\chi^o + \frac{\Phi}{(1-\rho q^o)^2}\right) & -\frac{\rho\beta(\chi^o)^2}{(\omega^o)^2} \\ q^o(1-\rho q^o)\left(-\phi\varepsilon + \frac{\beta}{q^o\omega^o}\right) & q^o\chi^o\left(\phi\varepsilon\rho - \frac{\beta}{(q^o)^2\omega^o}\right) & -\frac{\beta(1-\rho q^o)\chi^o}{(\omega^o)^2} \\ \omega^o\left(\frac{\rho}{\omega^o} + 1 + \phi\varepsilon(1-\rho q^o)\right) & -\omega^o\phi\varepsilon\chi^o\rho & -\frac{\rho\chi^o}{\omega^o} \end{pmatrix}.$$

The eigenvalues of J^o are the solutions of its characteristic equation:

$$-\nu^3 + \mathrm{tr}(J^o)\nu^2 - S_2(J^o)\nu + \det(J^o) = 0, \tag{25}$$

where $\mathrm{tr}(J^o)$, $S_2(J^o)$, and $\det(J^o)$ denote the trace of J^o, the sum of its three principal minors or order two, and its determinant, respectively.

We find

$$\mathrm{tr}(J^o) = 2(A - m^o - g^o) > 0.$$

and

$$\det(J^o) = - \underbrace{\frac{\beta\rho(\chi^o)^2}{(1-\rho q^o)}\left(1 + \frac{\phi\varepsilon\chi^o}{A - m^o + \rho - \beta(g^o+\rho)}\right)}_{+}\underbrace{\left(-\phi\varepsilon\chi^o(1-\rho q^o)^2 + \Phi\right)}_{\pm}$$
$$- \underbrace{\frac{\rho(\chi^o)^3}{q^o(\omega^o)^2}}_{-}\underbrace{\left(\phi\varepsilon\rho(q^o)^2\omega^o - \beta\right)}_{-}\underbrace{\left((1-\beta)(1-\rho q^o\phi\varepsilon) - \beta\phi\varepsilon\right)}_{+} - \underbrace{\frac{\phi\varepsilon(\chi^o)^3\rho\beta(1-\rho q^o)}{\omega^o}}_{+}.$$

We cannot determine the sign of the term $(\pm)$ since it depends on how large ϕ is. We assume that ϕ is small enough so that $\det(J^o)$ is negative. We determine the signs of the real parts of the roots of (25) from Theorem 1 of Benhabib and Perli (1994), which states that the number of roots of the polynomial (25) with positive real parts is equal to the number of variations of sign in the scheme

$$-1 \qquad \mathrm{tr}(J^o) \qquad - S_2(J^o) + \frac{\det(J^o)}{\mathrm{tr}(J^o)} \qquad \det(J^o).$$

Since $\mathrm{tr}(J^o) > 0$ and $\det(J^o) < 0$, we have two sign changes in this scheme, independent of the sign of the third (very complicated) term: $-++-$ or $-+--$. Hence, by Benhabib-Perli's result, the polynomial (25) has two roots with positive real parts. The differential equations system has one predetermined (ω) and two jump variables (χ and q). The linearized dynamic system above exhibits a saddle-point property.

Appendix B: Proofs of the three propositions

Proof of Proposition 1:

(i) The curves of $MAC(m^o)$ and $MAB(m^o)$ are hyperbola with

$$\lim_{m^o \to 0} MAB(m^o) = \pm\infty, \quad \lim_{m^o \to \infty} MAB(m^o) = -\frac{\phi\varepsilon(\gamma-1)(1-\beta)}{\gamma(1-\beta)+\beta},$$

$$\lim_{m^o \to A+\rho} MAC(m^o) = \pm\infty, \quad \lim_{m^o \to \infty} MAC(m^o) = \frac{\gamma(1-\beta)}{\gamma(1-\beta)+\beta},$$

and

$$MAC(0) = \frac{A\gamma(1-\beta) + \beta\theta + (1-\beta)\rho(\gamma(1-\beta)+\beta)}{(A+\rho)(\gamma(1-\beta)+\beta)}.$$

We distinguish between two cases. First, when $\gamma < 1 + \theta/\rho(1-\beta)$, then $MAC(m^o)$ is increasing in m^o. Then, $MAC(m^o)$ and $MAB(m^o)$ have two intersections, one at each of the intervals $m^o < A+\rho$ and $m^o > A+\rho$. However, the intersection at the interval $m^o > A+\rho$ is not feasible because it implies a negative growth rate. Hence there is a unique solution. (Notice that if $\gamma < 1$, $MAB(m^o)$ is increasing in m^o so that both curves have only one intersection at the interval $m^o > A+\rho$. Thus, there can be no optimal level of abatement.)

Second, when $\gamma > 1 + \theta/\rho(1-\beta)$, then $MAC(m^o)$ is decreasing in m^o. Since $MAC(A) > MAB(A)$, the two curves intersect at the interval $m^o < A$ and also at the interval $m^o > A$. However, when $m^o > A$, the growth rate becomes negative. Therefore, for any γ, there exists a unique positive level of abatement m^o associated with a positive optimal growth rate.

(ii) Differentiating $MAC(m^o, \rho; \beta) = MAB(m^o, \rho; \beta)$ yields

$$\frac{dm}{d\rho} = \frac{\partial MAB/\partial\rho - \partial MAC/\partial\rho}{\partial MAC/\partial m^o - \partial MAB/\partial m^o},$$

where

$$\frac{\partial MAB(\cdot,\cdot)}{\partial\rho} - \frac{\partial MAC(\cdot,\cdot)}{\partial\rho} = 0 - \frac{(-\beta)}{(A+\rho-m^o)^2}\left(A - m^o - \frac{A-\theta-m^o}{\gamma(1-\beta)+\beta}\right) > 0,$$

and

$$\frac{\partial MAC(\cdot,\cdot)}{\partial m^o} - \frac{\partial MAB(\cdot,\cdot)}{\partial m^o} = \frac{-\beta\left(\rho(\gamma-1)(1-\beta)-\theta\right)}{\left(\gamma(1-\beta)+\beta\right)(A+\rho-m^o)^2} + \frac{\phi\varepsilon\left(A(\gamma-1)(1-\beta)+\theta\right)}{\left(\gamma(1-\beta)+\beta\right)(m^o)^2}.$$

From Fig. 2 we see that an increase in ρ leads to an increase in m^o: $dm^o/d\rho > 0$. Therefore,

$$\partial MAC(\cdot,\cdot)/\partial m^o - \partial MAB(\cdot,\cdot)/\partial m^o > 0.$$

Similarly, differentiating $MAC(m^o, \beta; \rho) = MAB(m^o, \beta; \rho)$ yields

$$\frac{dm^o}{d\beta} = \frac{\partial MAB/\partial\beta - \partial MAC/\partial\beta}{\partial MAC/\partial m^o - \partial MAB/\partial m^o},$$

where

$$\frac{\partial MAB(\cdot,\cdot)}{\partial\beta} - \frac{\partial MAC(\cdot,\cdot)}{\partial\beta} = \frac{(A-\theta-m^o)\left(-\phi\varepsilon(\gamma-1)(A+\rho-m^o) + m^o\gamma\right) + \rho m^o\left(\gamma(1-\beta)+\beta\right)^2}{(A+\rho-m^o)\left(\gamma(1-\beta)+\beta\right)^2 m^o}.$$

The sign of $dm^o/d\beta$ is the same as the sign of $\partial MAB/\partial\beta - \partial MAC/\partial\beta$. The numerator of the above equation is positive if and only if

$$\phi < \Omega(m^o) := \frac{m^o}{\varepsilon(A+\rho-m^o)} \cdot \frac{\rho\left(\gamma(1-\beta)+\beta\right)^2 + \gamma(A-\theta-m^o)}{(\gamma-1)(A-\theta-m^o)}. \tag{26}$$

From the optimality allocation rule for abatement activities (22), ϕ can be rewritten as a function of m^o as follows:

$$\widetilde{\phi}(m^o) = \frac{m^o}{\varepsilon(A+\rho-m^o)} \cdot \frac{A-m^o+\rho(1-\beta) - \frac{\beta(A-\theta-m^o)}{\gamma(1-\beta)+\beta}}{A-m^o - \frac{A-\theta-m^o}{\gamma(1-\beta)+\beta}}. \tag{27}$$

If $\widetilde{\phi}(m^o) < \Omega(m^o)$, then condition (26) holds on the steady-state growth path. Comparing $\widetilde{\phi}(m^o)$ and $\Omega(m^o)$, we see that

$$\begin{aligned}
&\text{sign}\left[\Omega(m^o) - \widetilde{\phi}(m^o)\right] \\
&= \text{sign}\left[\frac{\rho\left(\gamma(1-\beta)+\beta\right)^2 + \gamma(A-\theta-m^o)}{(\gamma-1)(A-\theta-m^o)} - \frac{A-m^o+\rho(1-\beta) - \frac{\beta(A-\theta-m^o)}{\gamma(1-\beta)+\beta}}{A-m^o - \frac{A-\theta-m^o}{\gamma(1-\beta)+\beta}}\right] \\
&= \text{sign}\left[\rho(1-\beta)^2(\gamma-1)^2(A-m^o) + \theta(A-\theta-m^o) + \rho\theta\Big((1-\beta)(2\gamma-1)+\beta\Big)\right] \\
&> 0.
\end{aligned}$$

We conclude that $\partial MAB/\partial\beta - \partial MAC/\partial\beta > 0$, so that an increase in β leads to an increase in m^o.

Proof of Proposition 2: This follows directly from Proposition 1.

Proof of Proposition 3: The fact that $\partial g^o\left(m^o(\rho;\beta)\right)/\partial\rho < 0$ follows directly from (21) and Proposition 1(ii).

Let us next examine the effect of β on the optimal growth rate. Differentiating the optimal growth rate (21) yields

$$\frac{\partial g^o(m^o)}{\partial\beta} = -\frac{1}{\gamma(1-\beta)+\beta}\frac{\partial m^o}{\partial\beta} + \frac{(A-\theta-m^o)(\gamma-1)}{(\gamma(1-\beta)+\beta)^2}$$

and

$$\begin{aligned}
\text{sign}\left[\frac{\partial g^o(m^o)}{\partial\beta}\right] &= \text{sign}\Big[(A-\theta-m^o)(\gamma-1) \\
&- \frac{(A-\theta-m^o)\big(-\phi\varepsilon(\gamma-1)(A+\rho-m^o) + \rho\gamma\big(\gamma(1-\beta)+\beta\big)^2}{\frac{-\beta\big(\rho(\gamma-1)(1-\beta)-\theta\big)(m^o)^2 + \phi\varepsilon\big(A(\gamma-1)(1-\beta)+\theta\big)(A+\rho-m^o)^2}{m^o(A+\rho-m^o)}}\Big].
\end{aligned}$$

From the optimality allocation rule for abatement activities (22), the sign of $\partial g^o(m^o)/\partial\beta$ is positive if and only if $\hat{\phi}(m^o) > \Lambda(m^o)$, where $\hat{\phi}(m^o) := \widetilde{\phi}(m^o)\left(\varepsilon(A+\rho-m^o)/m^o\right)$

and

$$\Lambda(m^o) := m^o \cdot \frac{\frac{\beta(\rho(\gamma-1)(1-\beta)-\theta)}{A+\rho-m^o} + \frac{\gamma}{\gamma-1} + \frac{\rho(\gamma(1-\beta)+\beta)^2}{(\gamma-1)(A-\theta-m^o)}}{(A-\theta-m^o)\left(A(\gamma-1)(1-\beta)+\theta+m^o\right)}.$$

In addition, we know from (22) that the optimal abatement m^o is a solution of the equation

$$Opt(m^o) = -\lambda(m^o)^2 + \kappa m^o - \pi,$$

where

$$\lambda := \frac{(1-\beta)\left(\gamma+(\gamma-1)\phi\varepsilon\right)}{\gamma(1-\beta)+\beta},$$

$$\kappa := (1+2\phi\varepsilon)A + \rho(1-\beta+\phi\varepsilon) - \frac{\beta(A-\theta)+\phi\varepsilon(2A-\theta+\rho)}{\gamma(1-\beta)+\beta},$$

and $\pi := \phi\varepsilon\Big(A-(A-\theta)/(\gamma(1-\beta)+\beta)\Big)(A+\rho)$.

Let us investigate the graphs of $\hat{\phi}(m^o)$, $\Lambda(m^o)$ and $Opt(m^o)$. We have

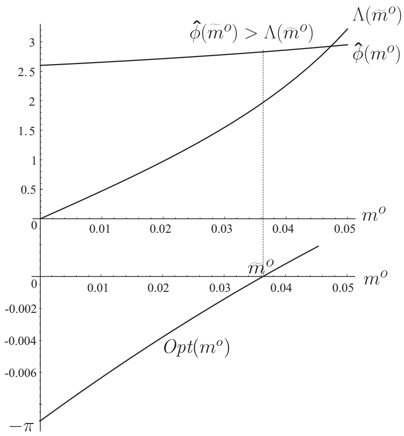

Fig. 5: The case of a positive effect of β on the optimal growth rate (Parameter values: $A = 0.13$, $\gamma = 2.50$, $\theta = 0.04$, $\rho = 0.20$, $\beta = 0.50$, $\varepsilon = 0.70$, $\phi = 0.50$)

$$\frac{\partial\hat{\phi}(m^o)}{\partial m^o}\Big|_{m^o=0} = \frac{(1-\beta)\left(\gamma(1-\beta)+\beta\right)\left(\rho(\gamma-1)(1-\beta)-\theta\right)}{\left(A(1-\beta)(\gamma-1)+\theta\right)^2} < (>)0$$

if ρ is small (large),

$$\frac{\partial\Lambda(m^o)}{\partial m^o}\Big|_{m^o=0} = \frac{\frac{\beta(\rho(\gamma-1)(1-\beta)-\theta)}{A+\rho} + \frac{\gamma}{\gamma-1} + \frac{\rho(\gamma(1-\beta)+\beta)^2}{(A-\theta)(\gamma-1)}}{A(\gamma-1)(1-\beta)+\theta} > 0,$$

and

$$\frac{\partial Opt(m^o)}{\partial m^o}\Big|_{m^o=0} = \kappa > 0,$$

and also

$$\hat{\phi}(0) = \frac{A\gamma(1-\beta)+\rho(1-\beta)\left(\gamma(1-\beta)+\beta\right)+\beta\theta}{A(\gamma-1)(1-\beta)+\theta} > 0,$$

$\Lambda(0) = 0$, and $Opt(0) = -\pi < 0$.

Since the equation $Opt(m^o) = 0$ is a quadratic function of m^o, it has two roots. Let $\widetilde{m}^o$ be the smaller root of the two roots, and let ρ and ϕ be sufficiently small, π is small (i.e., $-\pi$ is large), and κ (the slope of $Opt(m^o)$) small. Then, $\widetilde{\phi}(\widetilde{m}^o) > \Lambda(\widetilde{m}^o)$, as illustrated in Fig. 5. Thus we obtain that $\partial g^o(m^o)/\partial\beta > 0$ when ρ and ϕ are sufficiently small.

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