Labour Market, Demographic Transition and Economic Growth Cycles

María José Roa(1)(*), Dulce Saura(2) and Francisco J. Vázquez(3)

(1) Centro de Investigación y Docencia Económicas, CIDE, Mexico
(2) Universidad de Zaragoza, Spain
(3) Universidad Autónoma de Madrid, Spain

Members of the IIES Francisco de Vitoria, Spain

The purpose of this paper is to study the dynamic interaction between economic growth, labour market characteristic and population growth in a general model of economic growth. The dynamical results show that unemployment rate and per capita income dynamics fluctuate along cycles of different periods, and they may even have aperiodic paths. The characteristic of labour market institutions is the endogenous source of instability. In particular, as the rigidity of the labour market increases, the possibility of irregular behaviour increases as well. Moreover, in the same line as wage bargaining models, we get the result that the higher workers' bargaining power, the lower both employment rate and per capita production. Next, the introduction of endogenous population growth generates a demographic transition that affects the dynamics of unemployment and economic growth rate. Specifically, instability decreases (rises) when the population growth rate is growing (decreasing). This dynamic result is related to the population's age structure. Our results show a potential role for state intervention and labour market institutions in order to enhance growth and reduce its instability

(*) Corresponding author: María José Roa García
División de Economía
Centro de Investigación y Docencia Económicas, CIDE
Carretera México-Toluca 3655, México D.F., 01210
Phone: +52-55-57279800, ext. 2712
E-mail: mariajose.roa@cide.edu

1 Introduction

The purpose of this paper is to study the dynamic interaction between economic growth, labour market characteristic and population growth in a general model of economic growth. Our results show a potential role for state intervention and labour market institutions in order to enhance growth and reduce its instability.

Economists and international organizations have argued that labour market institutions have produced rigidities that account for the persistently high unemployment experienced by much of the developed world over the last two decades. However, traditional theories of economic growth assume full employment; that the labour market is always in equilibrium. Another drawback of the neoclassical growth theory has been in considering an exogenous growth rate of the labour supply.

This paper aims to complement the neoclassical growth literature by elaborating on a general and simple framework of economic growth with unemployment. The labour market situation is introduced through a non-market real wage clearing modelled by a non-linear Phillips Curve. It is in the line with those theories of the labour market where wage determination depends in a general way on a bargaining context (Layard and Nickell, 1985, 1986). Then, by following the literature of endogenous population growth (Becker et al., 1990; Jones, 2001; Galor and Weil, 2000), it is assumed that labour supply is determined through micro-founded fertility choices of individuals.

The dynamical results of the model are as follows. Unemployment rate and per capita income dynamics fluctuate along cycles of different periods, and they may even have aperiodic paths. We show that characteristic of labour market institutions is the endogenous source of instability. In particular, as the rigidity of the labour market increases, the possibility of irregular behaviour increases as well. Moreover, in the same line as wage bargaining models, we get the result that the higher workers' bargaining power, the lower both employment rate and per capita production. Next, the introduction of endogenous population growth generates a demographic transition that affects the dynamics of unemployment and economic growth rate. Specifically, instability decreases (rises) when the population growth rate is growing (decreasing). This dynamic result is related to the population's age structure. A young labour force could smooth fluctuations caused by labour market rigidities.

The remainder of the paper is organized as follows. In section 2 we explain the model and its main hypothesis. In section 3 we analyze the dynamic behaviour of the model's variables. In Section 4, we extend the wage dynamics foundation with a more realistic assumption regarding wage behaviour. In Section 5, we introduce endogenous population growth in the model. The conclusions of the paper are summarized in section 6.

2 The model

2.1 Final Good and Knowledge Production

There is a single economy that produces a final good Y_t. The market for this good is perfectly competitive. The aggregate production function is:

$$Y_t = F(L_t, h_t) = \mu \left(\gamma h_t L_t\right)^{\alpha}, \quad 0 < \alpha < 1, 0 < \gamma < 1, \mu > 0. \tag{2.1}$$

The labour input is measured in efficiency units $h_t L_t$, where L_t denotes physical labour and h_t is the stock of knowledge, or labour-augmenting technical progress. μ is the sector productivity and γ is the fraction of time that people devote to the production of final goods. For simplicity, physical capital is constant (Aghion-Howitt, 1998), which could be included in the parameter μ.

Technological progress evolves according to

$$h_{t+1} = e^{\delta(1-\gamma)} h_t, \qquad 0 < \delta < 1, \tag{2.2}$$

where $(1-\gamma)$ is the fraction of time devoted to technological progress and δ is the productivity of the sector[1] .

Regarding, saving and investment decisions, if L_t is used in the production of final goods and if the economy does not invest in knowledge, the final production (potential product) would be $\tilde{Y}_t$:

$$\tilde{Y}_t = \mu(L_t h)^\alpha.$$

The diference between the production levels:

$$\mu(L_t h_t)^\alpha - \mu\left(\gamma L_t\right)^\alpha h_t^\alpha = (1-\gamma^\alpha)\, \tilde{Y}_t$$

shows the opportunity cost of the use of labour inputs in the production of knowledge instead of final goods. This diference is interpreted as the total savings of the economy.

This framework encompasses specific growth models as special cases. γ can be exogenous (Sollow, 1956; Cass-Koopmans, 1965) or endogenous (Romer, 1990; Lucas, 1988; Grossman and Helpman, 1990; Aghion and Howitt, 1992). On the other hand, h could be considered any factor that causes sustained growth: technical progress, knowledge in the tradition of endogenous growth models or human capital. For simplicity, γ is assumed

[1] Note that equation (2.2) is linear. It is an approximation of the general linear modelling of technical progress in the literature of economic growth:

$$h_{t+1} = h_t + h_t \delta\left(1-\gamma\right) \qquad 0 < \delta < 1.$$

Because δ represents the knowledge productivity in each period its value is close to zero

$$1 + \delta\left(1-\gamma\right) \sim e^{\delta(1-\gamma)},$$

so equation (2.2) is a good approximation to those. It is considered because it simplifies the later analysis considerably.

exogenous. So, in the tradition of Solow model (1956) an exogenous constant fraction of output is saved in each period.

2.2 Consumers-Workers

Each individual is endowed with one unit of time that can be used either in producing the final good and/or in producing knowledge. Labour is supplied ineslastically. At the end of each period, individuals receive all the income from labour supplied in these activities, and all labour is paid at the same wage[2] denoted by w_t. The income not previously saved, in the terms explained in the prior section, is spent on consumption[3] . So, in each period the total demand for the final good is equal to the wage income paid at the end of the previous period:

$$D_{t+1} = C_{t+1} = w_t L_t, \tag{2.3}$$

where D_{t+1} is total demand in period $t+1$, C_{t+1} is total consumtion in period $t+1$, L_t is total employment, and $w_t L_t$ is the total wage income paid at the end of period t.

On the other hand, population N grows at the fixed exogenous rate n. Moreover, consistent with the empirical evidence[4] , the total population is a constant fraction of the labour force A, $N = \lambda A$ $(\lambda > 1)$. Thus,

$$\frac{A_{t+1}}{A_t} = \frac{N_{t+1}}{N_t} = 1 + n, \quad n > 0. \tag{2.4}$$

[2] The model can easily be rewritten in terms of a two sector economy, in which there exists a constant proportionality between the populations devoted to the production of final goods and knowledge, and workers are paid at the same wage.

[3] Under some parameters constant save and consume fractions result from an optimization household problem (Barro and Sala-i-Martin, 1995). The Solow model could be rewritten in terms of an optimization household problem.

[4] For example, in Spain this relation is rather stable, although less than in other European countries, except during particular periods such as when women were incorporated into the labour market.

2.3 Production of Final Good and Labour Demand

The productive sector produces whatever individuals demand. The production process for the final good takes one period. In each period, final production is equal to the total demand in the next period, which we assumed is known by the productive sector,

$$Y_t = D_{t+1}.$$

Substituting the demand and supply expressions

$$\mu\left(\gamma L_t h_t\right)^{\alpha} = w_t L_t, \tag{2.5}$$

and solving for L_t, we get the total labour demand of the economy

$$L_t = \left(\frac{\mu(\gamma h_t)^{\alpha}}{w_t}\right)^{\frac{1}{1-\alpha}}.$$

At the beginning of each period, the productive sector demands labour in such a way that supply of goods equals demand in the next period. Increasing knowledge raises employment because of the increase in demand, and thus in production, more than the increase in productivity. For this reason, employment depends positively on knowledge parameters[5] .

2.4 Wage Dynamics

The labour market is in disequilibrium and wage dynamics is determined by this disequilibrium. In particular, we assume a Phillips equation, where wages are increasing with the rate of employment

$$\frac{w_{t+1}}{w_t} = h\left(l_t\right), \qquad h'(l) > 0,$$

[5] Most of the economic literature has shown the negative relation between growth and unemployment (Pissariades, 1990; Bean and Pissarides, 1993), except for the positive relation between unemployment and growth (technological unemployment) in "creative destruction" models (Aghion and Howitt, 1992), based on the ideas of Schumpeter (1934). Bean and Pissarides suggest that the relationship between both phenomena is ambiguous because both variables can be endogenously and jointly determined, which implies that the relationship between them depends on exactly which economic structure we are considering.

where $l_t = \frac{L_t}{A_t}$ is the rate of employment.

In particular, we consider the following non linear curve:

$$\frac{w_{t+1}}{w_t} = \exp\left(-a + bl_t\right), \ b > a > 0. \tag{2.6}$$

Since pioneer authors (Phillips, 1958; Lipsey, 1960), the nonlinearity of the Phillips Curve has been maintained. Near to full employment firms raise wage rates in order to attract the most suitable workers. But, when unemployment is high, workers are unwilling to accept wage reductions. For simplicity, most models introduce a linear approximation in line with Goodwin (1967). However, he states that the linear approximation is "quite satisfactory for moderate movements of l near to the point 1". So, we think that a nonlinear reformulation of the Phillips Curve is necessary[6] .

The log-linear formulation is the most simple, because the logarithm is linear; the habitual linear formulation is attained with a simple linearization. For this reason, it is assumed. The dynamic results of the model, shown in the next section, are independent of the specific log-linear modeling considered. Initially, we introduce equation (2.6) for analytical tractability. It will be discussed later.

The functional relationship between wage dynamics and the rate of employment is ilustrated in figure 1.

(Figure 1 about here)

Introducing imperfections in the labour market through wage dynamics is common in disequilibrium models[7] (Chiarella and Flaschel, 1999; Chiarella et al., 2000). This

[6] Also, the Goodwin model presents an economic inconsistency: l could be greater than one. Desai et al. (2003) prove that by introducing a nonlinear Phillips Curve, or assuming that a fraction of capitalist profits are not invested; $l < 1$ is guaranteed.

[7] There are two groups of growth models in which the possibility of unemployment is contemplated: the Regime Switching Models (Ito, 1978, 1980; Henin and Michel,1982) and Disequilibrium Macroeconomic Models with Money (Chiarella et al., 2000). Although different in purpose and ideas from our paper, both works have in common that labor market disequilibrium is introduced also by a Phillips Curve.

modelization tries to introduce, in a simple way, the ideas of the *New Keynesian Economics* literature (Mankiw and Romer, 1991). The labour market disequilibrium is caused by different imperfections which explain wage and price rigidity from the optimizing behavior of individuals: eficiency wage, insiders-outsiders, and the wage bargain between employers and unions[8] ,[9] .

Finally, regarding a and b we make the following assumptions. On the one hand, in order to guarantee that wages do not rise when $l = 0$, it is necessary that $a > 0$, that is, $e^{-a} < 1$. On the other hand, under full employment, the growth of wages must be high, so that $b - a$ is large; in particular we require $e^{b-a} > 1$. Therefore, we assume $b > a$ and $a > 0$. As will be explained, these parameters are usually characteristic of labour market (Pohjola, 1981).

3 Dynamic Analysis

3.1 The dynamics of l_t

Beginning with the decision regarding the production of final goods we get:

$$\frac{Y_{t+1}}{Y_t} = \frac{D_{t+2}}{D_{t+1}},$$

and substituting the expressions for production and demand:

$$\left(\frac{h_{t+1}}{h_t}\right)^{\alpha} \left(\frac{L_{t+1}}{L_t}\right)^{\alpha} = \frac{w_{t+1}}{w_t} \frac{L_{t+1}}{L_t}$$

Using equations (2.2) and (2.6), taking logarithms and then exponentials, we obtain

[8] Generally, the theoretical elements of bargaining models are modelled in terms of a “wage curve” (Blanchard and Oswald, 1995), which captures the inverse relation between employment and wages instead of their growth rate. This can be get from union-firm optimization problems. However, Chiarella et al. (2000) assert: “the theory based level form formulations of such wage and price equations should be reducible to rates of growth, possibly considering demand as well as cost pressure terms”.

[9] However, this wage determination can also be interpreted in terms of neoclassical labour market (according to Lipsey view (1960)) with a "natural rate of unemployment" where unemployment can be considered "voluntary" (Manfredi and Fanati, 2003).

the following nonlinear dynamic equation for the employment rate:

$$\frac{l_{t+1}}{l_t} = \frac{1}{1+n} \exp\left(\frac{\alpha\delta\left(1-\gamma\right)+a}{\left(1-\alpha\right)}\right) \cdot \exp\left(\frac{-bl_t}{\left(1-\alpha\right)}\right).$$

Letting $r = \frac{1}{1+n} \exp\left(\frac{\alpha\delta(1-\gamma)+a}{(1-\alpha)}\right)$ and $s = \frac{b}{(1-\alpha)}$ the equation is reduced to a one-dimensional, nonlinear discrete time dynamical system:

$$l_{t+1} = f(l_t) = r \exp\left(-sl_t\right) l_t. \tag{3.1}$$

Equation (3.1) is known as the Ricker-Moran equation (Ricker, 1954; Moran, 1950; Cook, 1965; Macfadyen, 1963), and has been employed in ecology, especially in the study of fish populations. The dynamics of this equation are well documented in the ecological and mathematical literature (May, 1975). We briefly summarize the main results (see May, 1975; May and Oster, 1976).

L_t may not equal labour supply, so we could get unemployment $L_t < A_t$. Similarly, we could get $L_t > A_t$, that is, the production sector's demand for labour could be rationed. Because our objective is to explain the dynamic relationship between growth and unemployment, this last case will not be analysed. Therefore, we make the neccessary assumptions in order that $L_t \leq A_t$, and the market for the final good is always in equilibrium (Pohjola, 1981). In particular, we assume $b+\alpha > 1$ and $r < es$ in order to guarantee that $L_t < A_t$ is always satisfied; that is $l < 1$.

Table 1 summarizes the dynamics for each value of r (the parameter s affects the value of equilibrium points, but does not affect the behaviour of the dynamical system).

(Table 1 about here)

This shows how increasing r gives rise to a sequence of stable points of period 2^n. Although this bifurcation process produces an infinite sequence of cycles of period 2^n it is bound by some parameter value ($r_c \approx 14.767075$). Beyond this limit point, we enter in the "chaotic" regime which is characterized[10] by a finite number of attracting fixed points, an infinite number of repelling fixed points, and an uncountable number of points (initial conditions) whose trajectories are totally aperiodic. Periodic doubling is an example of a "route to chaos"; the way in which the dynamics of a system change as a parameter is changed, leading ultimately to the appearance of chaos.

Figure 2 shows the erratic behaviour which is a characteristic of a regime of chaos generated by the Ricker-Moran equation for a value of r in the chaotic regime, $r = 18$.

(Figure 2 about here)

The Ricker-Moran equation (3.1) is generated only with log-linear Phillips Curves. As explained previously, this modelization is plausible from an economic point of view. In the following we show how the labor market situation, represented by the Phillips Curve, is the source of chaos.

By taking standard values for the parameters α, γ, δ and n (Lucas, 1988; Barro and Sala-i-Martin, 1995), the possibility of chaos is "reasonable" from an economic point of view only if a is bigh enough. In particular, for $\alpha = 0.8, \gamma = 0.7, \delta = 0.1$ and $n = 0.02$ must be bigger than 0.78372. The parameter a determines the elasticity of the wage's growth to the employment rate. As is known, the more sensitive the growth wage to changes in employment rates, the higher the flexibility in the labour market. Since a bigger a (that is r) implies a lower elasticity, the model predicts that the more imperfect the labour market[11] the greater the possibility of instability. So, given the standard values of the

[10] A sufficient condition for the existence of topological chaos may be established by the Li-Yorke theorem "Period Three Implies Chaos" (1975).

[11] The rigidity of the labour market is explained from different imperfections such as cost of mobility, informational imperfections, mismatch between the workers seeking jobs and, the vacancies available, minimum wage and union wage setting, etc.

other parameters, r could be interpreted to reflect labour market rigidity.

The parameter b also determines the elasticity of the wage, but it does not affect the dynamic behaviour of the unemployment rate. The greater the value of b, the greater $s = \frac{b}{1-\alpha}$, and the lower the employment rate. Because s can be interpreted as reflecting workers' bargaining power, since, given a, an increase in b (that is in s) implies a greater growth of the wage (Pohjola, 1981), the model implies (in the same way as bargaining wage models (Layard and Nickell, 1985, 1986)) that a lower unemployment rate goes along with greater bargaining power. Taking into account the parameters that determine r, the employment rate increases with the effectiveness of investment in human capital δ, the fraction of time devoted to knowledge accumulation $(1-\gamma)$, the production function returns α, and the parameter a. As was discussed earlier, in economies with excess demand, increasing knowledge raises employment because of the increase in demand, and thus in production, more than the increase in productivity. For this reason, the employment rate depends positively on knowledge parameters. As far as population growth is concerned, we find that the employment rate and irregular behaviour decrease with population growth.

3.2 The dynamics of y_t

The the rate of economic growth is

$$\frac{y_{t+1}}{y_t} = \left(\frac{h_{t+1}}{h_t}\right)^{\alpha} \left(\frac{l_{t+1}}{l_t}\right)^{\alpha} (1+n)^{\alpha-1} = z \cdot \left(\frac{l_{t+1}}{l_t}\right)^{\alpha}, \tag{3.2}$$

where $z = e^{\delta(1-\gamma)\alpha}(1+n)^{\alpha-1}$. So, the rate of economic growth depends of the evolution of the employment rate and the parameter z. The qualitative conclusions obtained concerning the dynamics of the employment rate apply directly to the growth rate.

The numerical simulations show that, whenever $z < 1$, the growth of knowledge productivity does not absorb population growth, and the per capita production of the economy

y_{t+1} tends to be zero. This is because we have assumed than h depends on the fraction of time that individuals devote to knowledge production instead of the total number of workers employed in this sector[12] .

However, in order for the economy to disappear, the values which must be taken by the rate of population growth are unrealistic[13] . Considering realistic values of n, the economy would disappear only if it does not invest in knowledge ($\gamma = 1$) or or if it does at an infinitesimal level. This result coincides with the Malthusian idea concerning the stagnation of the economy. As the population rises, in a non-industrialized economy and with limited resources, at some moment the amount of food falls below the subsistence level. This leads to a situation in which the economy is stagnant, and where the population no longer rises. So, in the same way as traditional models of growth, sustained growth is due to knowledge production or technological change, in particular, to the productivity δ and the fraction of time devoted to knowledge accumulation $(1 - \gamma)$.

Equation (3.2) also shows that, as far as workers' bargaining power b goes, lower levels of the rate of economic growth are associated with higher bargaining power; this is because greater bargaining power implies greater unemployment, and so, less resources are used (Davieri and Tabellini, 2000; Alonso, Echevarría and Tran, 2002).

If $z > 1$, per capita income shows a long term positive growth trend that is sustained by knowledge production, and its dynamics behaviour depends on the dynamics of the

[12] This assumption eliminates the so-called "scale effect": that population size affects human capital accumulation and economic growth positively (Romer, 1990; Grossman and Helpman,1991; Aghion and Howitt, 1992). In our model, if we consider that technological progress depends on the total number of workers employed in this sector, the scale effect would cause an overflowing of human accumulation, economic growth and employment when the population's size is large. Moreover, we must choose the initial population size, which is too arbitrary. For that reason we have chosen modeling (2).

[13] See Kremer (1993) for the evolution of world population growth rate since one million years B.C. through 1990. The greatest rate of growth was 2.01% in 1960. So, for example, the parameters values that we have considered and $n = 2\%$, $z = 1.01822$.

employment rate[14] . The qualitative conclusions obtained about the dynamics of the employment rate apply directly to per capita income y_t. Again, the flexibility of the labour market determines the dynamics behaviour of per capita income. For example, for a value of r in the chaotic regime $r = 18$ (figure 3) the trajectory of y_t is quite irregular[15] . Once more, workers' bargaining power does not affect the dynamics of per capita income.

(Figure 3 about here)

The earlier works that show the possibility of irregular dynamics and chaos in neoclassical growth models (see the introduction) get a nonlinear discrete dynamical system for the economic growth rate very similar to the Ricker-Moran equation; the Logistic equation (May, 1975; May and Oster, 1976). In these works the possibility of chaos is also related to the existence of market imperfections.

Finally, most of the neoclassical theory of growth focuses on explaining the positive trend of income growth, and it does not normally take into account the existence of the periods in which income decreases, and in those cases when it does the decreases are generated exogenously. Although the model generates a positive income growth trend for all numerical simulations, in figure 6 we can see periods in which per capita income decreases $\frac{y_{t+1}}{y_t} < 1$, which is consistent with what we see in the real world.

4 Wage Dynamics Extension

Looking at the trajectories of employment and per capita income, the size of the oscillations of the time series are large when compared with real world behaviour. The reasons for this result are the simple nonlinear modelings in some of the economic relationships that

[14] For $z = 1$, the positive growth trend disappears. However, the parameter values for which $z = 1$, in particular the value of n, are not realistic for the same reason that we have already explained for $z < 1$.

[15] In figure 3, $a = 0.62$. The remaining parameter values are the standard values.

we have assumed, for example the wage dynamics. In this section we will now show how introducing more realistic assumptions about the nonlinearity of the Phillips Curve, in particular for l values with neither full employment nor total unemployment, the size of the oscillations decreases and are close to full employment. In particular, we consider the following alternative modeling of the Phillips Curve:

$$\frac{w_{t+1}}{w_t} = h(l_t) = \exp\left(-a + bl_t^{\sigma} + c\,(1-l_t)^{-\varepsilon}\right), \tag{4.1}$$

with $a, b, c > 0$, $0 < \sigma < 1$, $0 < \varepsilon < 1$, the resulting behaviour is illustred in figure 4.

(Figure 4 about here)

We have made the following assumptions. First, we assume there is a large area, corresponding to an interval $(\varepsilon_1, 1-\varepsilon_2)$ with ε_1 and ε_2 small, where wages are very insensitive to employment changes. This modeling considers, in a more precise way, wage behaviour far from either full employment or total unemployment, and it is based on Rose [16] (1967): "In some neigbourhood of the l at which unemployment is balanced by unfilled vacancies, frictions and imperfections weaken the responsiveness of wage inflation to changes in l".

Secondly, we assume a vertical tangency $l = 0$ in order to capture the rapid fall of wages[17] when the rate of unemployment is very close to 100% (l close to 0). Finally, we assume a vertical asymptote at $l = 1$. This asymptote is designed to reflect the fact that with near to full employment it is more and more difficult to hire workers[18] and the labour

[16] Rose introduces a kind of nonlinearity in Phillips Curve with the purpose of preventing local explosive dynamics. The qualitative behaviour which he obtained is similar to the dynamic behaviour that we get with the curve assumed here.

[17] Some macroeconomic growth models with money and disequilibrium (Chiarella et al., 2000; Chiarella and Flaschel, 1999) assume the so-called "kinked" Phillips Curve. This curve is an extreme case of the nonlinear Phillips Curve because it introduces downward rigidity in nominal wages; wages are constant if the growth rate is negative. They affirm that this curve is more realistic than assuming the possibility of wage decreases. The objective of this work is to extend the habitual nonlinearity of the Phillips Curve, a "kinked curve", and to use numerical simulations to show how the dynamical behaviour of the model changes drastically and would be able to generate chaotic paths.

[18] This assumption is very usual both in theoretical work which introduces the Phillips Curve (Phillips, 1958; Lipsey, 1960; Samuelson and Solow, 1960; Rose, 1967; Chiarella and Flaschel,1996) and in empirical

market pressure accelerates the growth of wages; with which near to full employment would be huge.

Taking into account the Phillips Curve (4.1), the dynamics of the employment rate are

$$l_{t+1} = f\left(l_t\right) = \frac{1}{1+n} \exp\left(\frac{(\alpha+\beta)\delta\left(1-\gamma\right) - q(l_t)}{(1-\alpha)}\right), \tag{4.2}$$

where now $q\left(l_t\right)$ is

$$q(l_t) = -a + bl_t^{\sigma} + c\left(1-l_t\right)^{-\varepsilon}.$$

In figure 5 we have drawn the intertemporal evolution of l_t for normal parameter values and set the parameters of the curve (4.1) at $a = 1.716, b = 1.455, \sigma = 0.01, \varepsilon = 0.1$. As in the previous section, the parameter values of the Phillips Curve have been choosen so that the case $l > 1$ is excluded. The resulting l_t dynamics is irregular, since we are now close to full employment values. The dynamics behaviour of (4.2) is qualitatively similar to the function (3.1). Both are unimodal functions which generate cyclical temporal evolution of the rate of employment. The difference is that the oscillations are now much more close to full employment values[19] .

(Figure 5 about here)

As in the same approach in the previous section, the instability is transferred to per capita production dynamics (see figure 6):

(Figure 6 about here)

In conclusion, as we postulated at the outset, the simple adaptation of the Phillips Curve to theoretical patterns that fit better, its assumed behaviour is enough to generate more realistic intertemporal evolutions of the employment rate and per capita production.

work (Chadha, Masson and Meredith, 1992; Laxton, Meredith and Rose, 1995; Clark, Laxton and Rose, 1996).

[19] We emphasize that the Ricker-Moran equation is generated independently of the log-linearity considered.

5 Endogenous population growth

Following the standard models of endogenous population (Becker et al., 1990; Jones, 2001; Galor and Weil, 2000), it is assumed that labour supply is determined through micro-founded fertility choices of individuals, in which households choose the number of children.

Let us assume that the representative family's preferences are represented by the following standard log-linear utility function:

$$U(c_{t+1}, b_{t+1}) = c_{t+1}^{1-\epsilon} b_{t+1}^{\epsilon}, \qquad 0 < \epsilon < 1,$$

where c is the per capita consumption and ϵ measures the preference for children. At every point of time t each family choice determines the number of children b, who will be born in the next period $t+1$.

The cost of childrearing E_t includes both the cost of raising a child, regardless of quality, θ_1, and education cost $e_t = \theta_2 h_t^{\rho}$:

$$E_t = \theta_1 + \theta_2 h_t^{\rho}, \quad 0 < \rho < 1.$$

θ_1 is considered as a fixed "maintenance" cost (e.g. food, clothes), and the education cost e_t depends on the medium level of knowledge[20] .

Given these assumptions, each family's static optimization problem[21] is

$$\begin{aligned} \max U(c_{t+1}, b_{t+1}) &= c_{t+1}^{1-\epsilon} b_{t+1}^{\epsilon} \\ s.t.\ c_{t+1} + (\theta_1 + \theta_2 h_t^{\rho}) b_{t+1} &= y_t. \end{aligned} \tag{5.1}$$

[20] Schultz (1975) claimed that new technology will create a demand for skilled workers to analyze new production processes. So, education cost would depend on knowledge h.

[21] Some models of endogenous population growth introduce altruism in the parent's utility function (Becker et al. 1990). This assumption implies both of the dynastic utility functions considered, and solves dynamic optimization problems of the Bellman equation type. By considering some very simple assumptions: "the more standard dynamic optimization problem reduces to the sequence of static problems" (see Jones, 2001, p. 5)

The last equation is the income constraint, where $y_t = \frac{Y_t}{A_t}$ is the per capita income. Solving problem (5.1) we get that the optimal number of children for a representative family at each time[22] t

$$b_{t+1} = \frac{y_t \epsilon}{\theta_1 + \theta_2 h_t^{\rho}}, \tag{5.2}$$

and b can be interpreted as the fertility rate. In line with endogenous population growth literature it depends positively on the per capita income and negatively on the medium level of knowledge. As individuals receive all income from labour supplied, $y_t = w_t l_t$, the fertility rate can be written as:

$$b_{t+1} = \frac{w_t l_t \epsilon}{\theta_1 + \theta_2 h_t^{\rho}}.$$

Finally, in order to obtain the population growth rate, we assume a constant mortality rate[23] π. Taking this rate into account, the population growth rate n is given by:

$$n_t = \frac{A_{t+1}}{A_t} - 1 = \frac{N_{t+1}}{N_t} - 1 = -\pi + \frac{1}{\lambda} b_{t+1}. \tag{5.3}$$

5.1 The dynamics with endogenous n

The economic growth model with unemployment and endogenous population growth is:

$$\begin{aligned}
l_{t+1} &= f(l_t) = \frac{1}{1+n_t} l_t \exp\left(\frac{\alpha\delta(1-\gamma)}{(1-\alpha)}\right) [h(l_t)]^{\frac{1}{1-\alpha}} \qquad (5.4)\\
h_{t+1} &= \exp(\delta(1-\gamma)) h_t \\
\frac{w_{t+1}}{w_t} &= h(l_t) = \exp\left(-a + b l_t^{\sigma} + c(1-l_t)^{-\varepsilon}\right) \\
\frac{y_{t+1}}{y_t} &= \exp(\delta(1-\gamma)\alpha)\left(\frac{1}{1+n_t}\right)^{1-\alpha}\left(\frac{l_{t+1}}{l_t}\right)^{\alpha} \\
n_t &= -\pi + \frac{w_t l_t \epsilon}{\lambda(\theta_1 + \theta_2 h_t^{\rho})}.
\end{aligned}$$

[22] In the endogenous population models every period usually is considered a period of life (generation). But according to the logic of production, investment, employment and wages that we are assuming, the periods will be shorter than periods of life. Parent determine the flow of births per unit time, rather than the overall stock of children during its lifetime.

[23] Blanchard (1985) states that "Evidence on mortality rates suggests low and approximately constant probabilities from age 20 to age 40," after this, mortality rates depend inversely and exponentially on individual age (see "Gomperty's Law," Wetterstrand, 1981).

Because of the high analytical complexity of the model we limit its study to some numerical simulations (by using MATLAB). Like in the previous sections, the parameter values have been choosen excluding $l > 1$. There are no empirical studies that reveal standard values for population problem parameters. In general, they are chosen ad hoc in theoretical studies. In order to present the dynamic results, we set $\epsilon = 0.02$, $\theta_1 = 0.3$, $\theta_2 = 0.01$, $\rho = 0.6$, $\pi = 0.01, \lambda = 1.6$, and the initial values $y_0 = 1$, $h_0 = 10, l_0 = 0.9$. We get a wide enough interval of values for the temporal evolution of the variables satisfying $l < 1$. All the simulations made show the same behaviour in this interval of values which, due to the restriction of labour market disequilibrium, could be relatively small.

The dynamic analysis shows the endogenous emergence of a logistic behaviour in the population growth rate describing its historical evolution (see figure 7). The large number of simulations carried out showed that this is a robust property of the model for acceptable changes in parameter values.

(Figure 7 about here)

The evolution is characterized by a stage habitually named Post Malthusian, where population growth rate increases and, after a transition phase, a modern growth stage follows where n decreases and tends to stagnate (Galor and Weil, 2000; Galor 2005). For low levels of knowledge development the education cost is small and income growth raises the fertility rate. But for high levels of knowledge development education cost rises inducing parents to have fewer, but more high-quality children. It generates a demographic transition, that is, an economy transitions from a high fertility and low knowledge accumulation stage to a low fertility and rapid knowledge accumulation stage[24] .

[24] Because human capital is not a family choice, the model is abstracted from the so-called quantity-quality trade off in the literature of endogenous population growth (Galor and Weil, 2000; Galor, 2005).

Besides the logistic behaviour of population growth, Figure 7 shows the behaviour of n during the different phases. In the stage previous to the transition n oscillates regularly or periodically, in the transition stage it shows a very regular behaviour, and in the last stage it oscillates in an erratic way. The reasons for this result could be the simple modelization in the birth rate that we have assumed. The model attempts to be representative of a developed economy, and some assumptions are not appropriate for explaining the transition from an agricultural economy to an industrial one[25] . More realistic assumptions regarding the birth rate (e.g. endogenous mortality) and the behaviour of the model's variables during different stages would generate more realistic intertemporal evolutions of the population growth rate[26] .

This type of dynamic evolution is not exclusive of the population growth rate. In figure 8 we have drawn the intertemporal evolution of l and we show the same behaviour that was generated in n. The evolution of per capita production would show the same behaviour.

(Figure 8 about here)

So, by considering an endogenous population growth rate that evolves along time, the dynamics behaviour of employment changes. Current research on the impact of demography on growth shows that changes in the population's age structure play a significant role (Canning and Bloom, 1999, 2005). Although age structure has not been analyzed or modelized in our model, we could relate it with population growth. This relationship is

[25] Galor and Weil (2000) develop a model that captures the historical evolution of population, technology and output. Recently, the model has been calibrated by Lagerlof (2006).

[26] For example, in the same line as demographic and industrial transition models (Lucas, 1998; Hansen and Prescott, 1999; Kögel and Prskawetz, 2001; Tamura, 2002) we could model a productive sector for every stage: agricultural and industrial. Moreover instead of a Phillips Curve it should consider an appropriate wage dynamic in the agricultural stage.

direct if population begins to grow because young populations change. Then, a possible justification for the stability of employment during the transition could be the following. At the beginning of the transition, a young population (resulting from the typical "baby boom" of the post Malthusian stage) joins the labor force. A young labour force satisfies the needs of the labour market more easily (e.g. training, geographic and labor mobility) than an adult population (corresponding to a low population growth), characteristic of developed economies (the modern economic growth stage). So, a young labour force could smooth fluctuations caused by labour market rigidities.

Finally, introducing endogenous population growth does not affect the result of the previous section about how changes of the model's parameter values affect employment and per capita income dynamics. Regarding the parameters of endogenous population rate of growth, because of the inverse relationship between population and economic growth: a) the greater is the value of child preference ϵ the lower per capita income, and b) the lower both childrearing cost and mortality rate the greater per capita income.

6 Conclusions

Our work complements the theoretical results of neoclassical economic growth models and cycles literature. The internal dynamics of a simple economy where the labour market is in disequilibrium generates both the positive growth trend and the irregular behaviour, which characterizes the intertemporal evolution of per capita income. The dynamics behaviour depends on the labour market characteristic. Along the lines of wage bargaining models, we find that the higher the workers' bargaining power, the lower the employment rate, the per capita production, and the population growth.

In addition, we get an inverse relation between population and economic growth that reflects the Malthusian idea that the greater the population growth, the lower the per capita income. Then, when we assume endogenous population growth by means of optimal fertility choices, the model endogenously generates a logistic behaviour in the population growth rate that changes the dynamics of employment. Specifically, the higher the population growth rate the lower the instability. This dynamic result could be related to the population's age structure.

Although the model has not been explicitly applied to policy analysis, some important conclusions can be extracted. Policies which improve the flexibility of the labour market will decrease instability. On the other hand, as in models of wage bargaining, we find that greater levels of workers' bargaining power are associated with lower levels of both the employment rate and per capita production. Thus, policies which reduce workers' bargaining power will have a positive effect on economic growth, increasing the employment rate. Lastly, with regard to demographic variables, on the one hand, an increase in the population growth rate leads to a decrease in the growth rate of income, and policies which reduce incentives for fertility may positively affect long run economic growth.

Additionally, the possibility of chaos has some interesting implications in terms of economic policy. In our model, persistent oscillations emerge solely because of the internal structure of the economy. Therefore, the undesired fluctuations could be eliminated or reduced through appropriate structural interventions rather than the traditional business cycle theory policies that are focused on attenuating exogenous shocks to economic conditions (temporary changes in technologies and preferences). However, this does not mean that oscillations are purely deterministic, but the endogenous shocks are not the only source of economic irregularity. We find these preliminary results encouraging enough to perform further work, both at the theoretical and empirical level.

7 References

Aghion, P., Howitt, P., 1992. A Model of Growth through Creative Destruction. Econometrica 60, 323-351.

Aghion, P., Howitt, P., 1998. Endogenous Growth Theory. MIT Press, Cambridge Mass.

Alonso, A., Echevarría, C., Tran, K., 2002. Long Run Economic Performance and the Labour Market. Southern Economic Journal, 70, 905-919.

Barro, R.J., Sala-i-Martin, X., 1995. Economic Growth. McGraw-Hill, New York.

Bean, C., Pissarides, C., 1993. Unemployment, consumption and growth. European Economic Review 37, 837-859.

Benhabib, J. y Nishimura, K., 1985, Competitive equilibrium cycles, Journal of Economic Theory 35, 284-306.

Becker, G.S, Murphy, K.M., and Tamura, R.F., 1990. Human Capital, Fertility, and Economic Growth, Journal of Political Economy, 98, S12-S37.

Blanchard, O.J., 1985. Debt, Deficits, and Finite Horizons, Journal of Political Economy, 93, 2, 223-247.

Blanchard, O.J., Oswald, A.J., 1995. An Introduction to the Wage Curve. The Journal of Economics Perspectives 9, 153-167.

Bloom, D.E., and Canning, D., 1999. Economic Development and Demographic Transition: The Role of Cumulative Causality, CAER II Discussion Paper, 51.

—. 2005. Global Demographic Change: Dimensions and Economic Significance, Harvard Initiative for Global Health. Working Paper 1.

Boldrin, M. y Deneckere, R., 1987, Simple macroeconomics models with a very complicated behavior. UCLA Economics Working Papers 527, UCLA Department of Economics,

Clark, P., Laxton, D., Rose, D., 1996. Asymetry in the U.S. Output-Inflation Nexus. IMF

Staff Papers 43, 216-251.

Cook, L.M., 1965. Oscillation in the Simple Logistic Growth Model. Nature 207, 316.

Chadha, B., Masson, P., Meredith, G., 1992. Models of Inflation and the Costs of Disinflation. IMF Staff Papers 39, 395-431.

Chiarella, C.P., Flaschel, G., 1996. Real and Monetary cycles in models of Keynes-Wicksell type. Journal of Economic Behavior & Organization 30, 327-351.

Chiarella, C.P, Flaschel, G., 1999, The Dynamics of Keynesian Monetary Growth: Macro Foundations. Cambridge University Press.

Chiarella, C.P, Flaschel, G., Groh, G., Semmler, W., 2000. Disequilibrium, Growth and Labor Market Dynamics: Macroperspectives. Springer-Verlag, Heidelberg.

Daveri, F., Tabellini, G., 2000. Unemployment, Growth and Taxation in Industrial Countries. Economic Policy 30, 49-104.

Deneckere, R. y Pelikan, S., 1986, Competitive chaos, Journal of Economic Theory 40, 13-25.

Day, R.H., 1982, Irregular growth cycles, American Economic Review 72, 406-14.

Desai, M., Henry, B., Mosley, A., Pemberton, M., 2003. On an Incosistency in the Goodwin Model of the Growth Cycle. London School of Economics.

Galor, O., Weil, D.N., 2000. Population, technology, and growth: From the Malthusian regime to the demographic transition and beyond. American Economic Review 90, 806-828.

Galor, O., 2005. From Stagnation to Growth: Unified Growth Theory. In: Aghion, P. and Durlauf, S.N. (Eds)., Handbook of Economic Growth, volume 1A, chapter 4, 171-293, Elsevier.

Goodwin, R.M., 1967. A Growth Cycle, in: Feinstein, C.H. (Ed), Socialism, Capitalism and Economic Growth, Cambridge University Press, pp.54-58.

Grossman, G. M., Helpman E., 1991. Innovation and Growth in the Global Economy. Cambridge MA, MIT Press.

Hansen,G., Prescott, E.C., 1999. From Malthus to Solow. Federal Reserve Bank of Minneapolis. Research Department Staff Report 257.

Henin, P-Y, Michel, P., 1982, Harrodian and Neoclassical Paths in a Constrained Growth Model, Economics Letters 10, 237-242.

Ito, T., 1978. A note on Disequilibrium Growth Theory. Economics Letters 1, 45-49.

Ito, T., 1980. Disequilibrium Growth Theory. Journal of Economic Theory 23, 380-409.

Jones, I.C., 2001. Was a Industrial Revolution Inevitable? Economic Growth over the Very Long Run, Advances in Macroeconomics, 1, n^o 2, art. 1. Kögel, T., Prskawetz, A., 2001. Agriculture productivity growth and escape from the Malthusian trap. Journal of Economic Growth, 6, 337-357.

Kremer, M., 1993. Population Growth and Technological Change: One Million B.C. to 1990. Quaterly Journal of Economics 108, 681-716.

Lagerlof, N. 2006. The Galor-Weil model revisited: A quantitative exploration. Review of Economic Dynamics 9, 116-142.

Laxton, D., Meredith, G., Rose, D. 1995. Asymmetric effects of economic activity on Inflation: Evidence and policy implications. IMF Staff Papers, 42, 2.

Layard, R., Nickell, S., 1985. The Causes of British Unemployment. National Institute Economic Review 111, 62-85.

Layard, R., Nickell, S., 1986. Unemployment in Britain. Economica 53 (Supplement), S121-S169.

Li, T.Y., Yorke, J.A., 1975. Period Three Implies Chaos. American Mathematical Monthly

82, 271-271.

Lucas, R.E. Jr., 1988. On the Mechanics of Development Planning. Journal of Monetary Economics 22, 3-42.

Lucas, R.E. Jr., 1998. The Industrial Revolution: Past and Future. Mimeo, University of Chicago. Published in: Lucas, R.E.Jr., (Ed), Lectures on Economic Growth Cambridge: Harvard University Press, 2002.

Lipsey, R.G., 1960. The Relation between Unemployment and he Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957: A Further Analysis. Economica 27, 1-31.

Macfadyen, A., 1963. Animal Ecology. London: Pitman.

Mankiw, N.G., Romer, D., (Eds), 1991. New Keynesian Economics, Vol. I, II. MIT Press, Cambridge.

May, R.M., 1975. Biological Populations Obeying Difference Equations: Stable Points, Stable Cycles, and Chaos. Journal of Theoretical Biology 51, 511-524.

May, R.M., Oster, G.F., 1976. Bifurcations and dynamic complexity in simple ecologicals models. American Naturalist 110, 573-599.

Moran, P.A.P., 1950. Some Remarks on Animal Population Dynamics. Biometrics 6, 250-258.

Phillips, A.W., 1958, The Relation between Unemployment and he Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957, Economica 25,100, 283-299.

Pissarides, C. A., 1990. Equilibrium unemployment theory. Basil Blackwell, Oxford.

Pohjola, M.J., 1981. Stable and Chaotic Growth: the Dynamics of a Discrete Version of Goodwin's Growth Cycle Model. Zeitschrift für Nationalökonomie 41, 27-38.

Ricker, W.E., 1954. Stock and Recruitment. Journal Fisheries Research Board of Canada

11, 559-623.

Romer, P., 1990. Endogenous Technological Change. Journal of Political Economy 98, part II, S71-S102.

Rose, H., 1967. On the non-linear theory of the employment cycle. Review of Economic Studies 34, 153-173.

Samuelson, P.A., Solow R.M., 1960. Analytical Aspects of Anti-Inflation Policy. The American Economic Review 50, 177-194.

Schumpeter, J.A., 1934. The Theory of Economic Development. Cambridge MA, Harvard University Press.

Schultz, T.W., 1975. The Value of the Ability to Deal with Desequilibria, Journal of Economic Literature 13(3), 827-846.

Solow, R. M., 1956, A Contribution to the Theory of Economic Growth, Quarterly Journal of Economics 70, 1 (February), 65-94.

Tamura, R., 2002. Journal of Economic Dynamics and Control, 27, 207-242.

Wetterstrand, W.H., 1981. Parametric Models for the Life Insurance Mortality Data: Gomperty's Law over Time, Transc. Soc. Actuaries, 33, 159-179

Woodford, M., 1988, Imperfect financial intermediation and complex dynamics, en W. Barnett, J. Geweke y K. Shell (eds), Chaos, sunspot, bubbles and nonlinearity. Cambridge University Press.

	Values of r
Stable equilibrium point	$1 < r < e^2$
Stable cycle of period 2	$e^2 < r < 12.5039$
Stable cycle of period 4	$12.5039 < r < 14.2392$
Stable cycle of period 8	$14.2392 < r < 14.6582$
Stable cycles of period 16, 32, 64,...	$14.6582 < r < 14.7611$
Chaos	$r > 14.7611$

Table 1. Dynamics behaviour

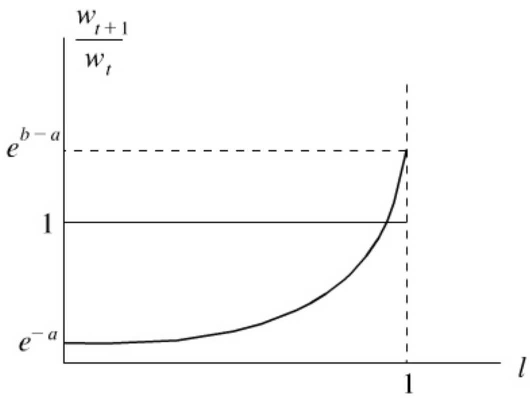

Figure 1: Wage dynamics and rate of employment

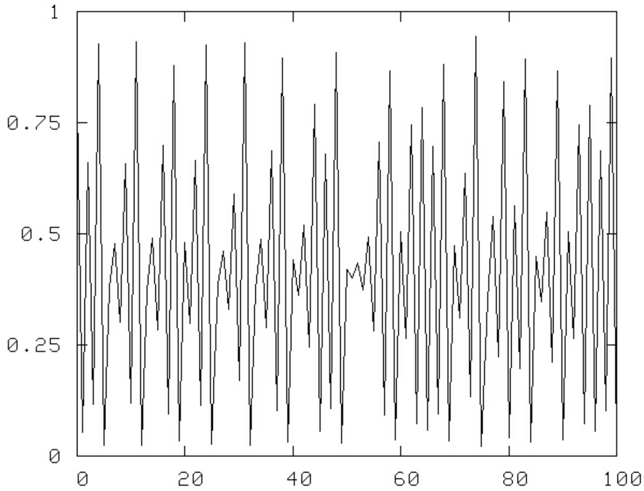

Figure 2: Chaotic time series of l

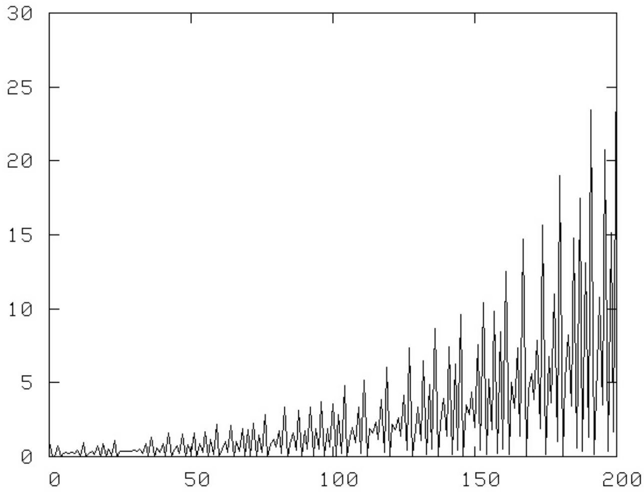

Figure 3: Chaotic time serie of y

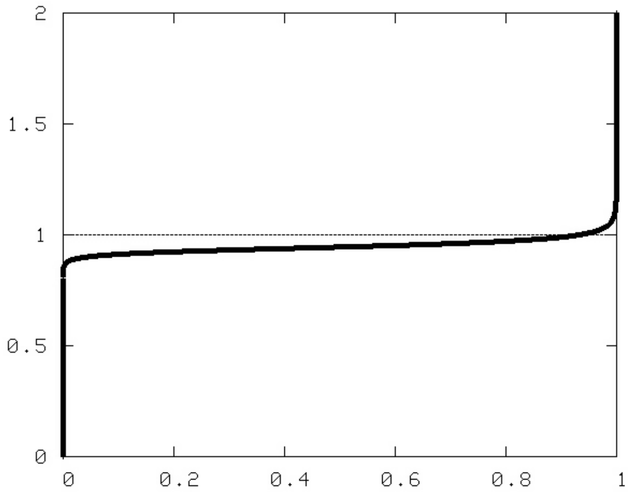

Figure 4: Phillips Curve

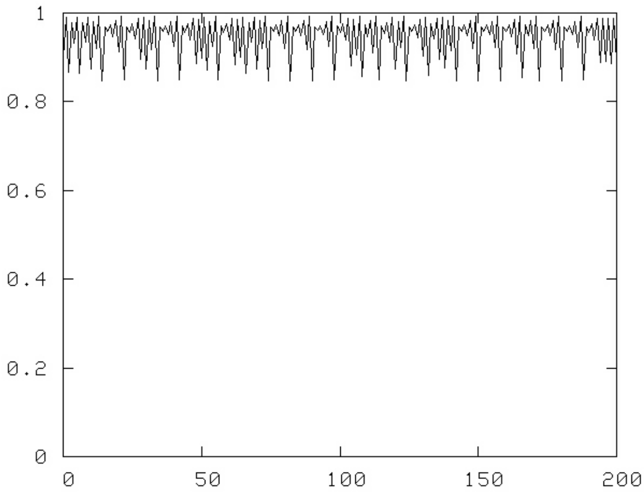

Figure 5: Intertemporal evolution of l

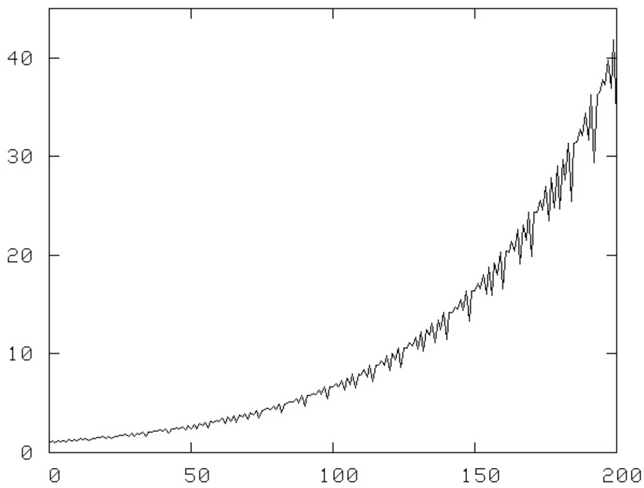

Figure 6: Intertemporal evolution of y

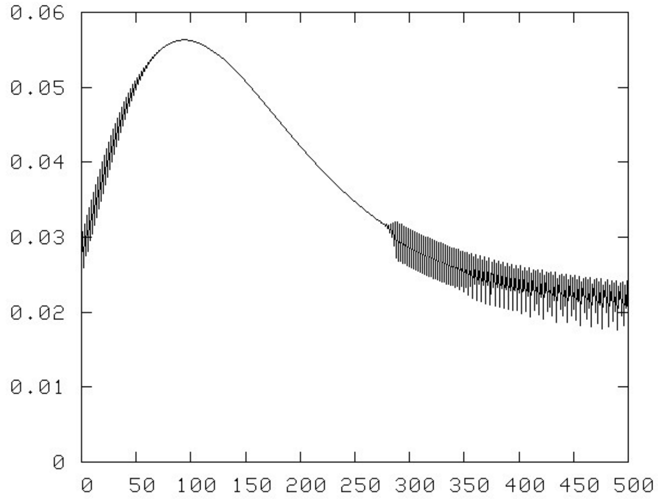

Figure 7: Logistic evolution in the population growth rate

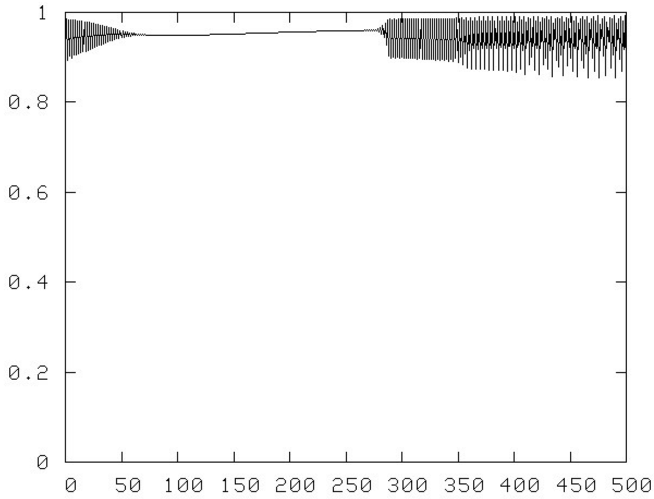

Figure 8: Intertemporal evolution of l